RFQ, Sequencing, and the Most Favorable Bargaining Outcome

Leon Yang Chu
Department of Data Science and Operations, Marshall School of Business, University of Southern California leonyzhu@usc.edu

Ying Rong
Department of Operations Management, Antai College of Economics and Management, Shanghai Jiao Tong University, China
yrong@sjtu.edu.cn

Huan Zheng
Department of Management Science, Antai College of Economics and Management, Shanghai Jiao Tong University, China
zhenghuan@sjtu.edu.cn

Inspired by prescription drug affordability practices, we study how a buyer may achieve cost reduction by combining a procurement process with quantity-dependent pricing contracts and exclusion clauses, the latter of which has been commonly adopted in contracting but rarely executed. We analyze the equilibrium outcomes when the buyer simultaneously or sequentially negotiates with imperfectly substitutable suppliers under a dual-sourcing setting. We show that quantity-dependent pricing contracts coordinate the supply chain, and introducing exclusion clauses leads to various equilibrium profit allocations. Surprisingly, the buyer can benefit from a request for quotation (RFQ) stage that precedes the negotiation stage even under a full information setting. Specifically, by endogenizing the sequence of negotiations via the quotations submitted in the RFQ stage, the buyer's equilibrium profit with an RFQ dominates the buyer's profit without an RFQ. The insights extend to an uncertain demand setting, in which the buyer first negotiates contracts with suppliers and then decides order quantities after demand realization.

Key words: Bilateral Bargaining, Request For Quotation (RFQ), Sourcing Strategy

---

1. Introduction

Individual customer needs and preferences are better understood and explored every day. Major retailers' shelf spaces present numerous substitutable goods from competing suppliers to better match varied customer tastes. Pharmaceutical companies offer drugs to treat the same disease with different efficacies due to human genetic variation and existing conditions. Responding to this trend, procurement professionals from supply chain management to health care have adopted more terms and flexibilities in contract negotiations, awarding contracts to multiple suppliers to meet the firm and its customers' needs (Raycraft 2016). This naturally poses an important question to procurement managers: How to best serve customers while leveraging supplier competition when the suppliers are imperfectly substitutable?

To secure price concessions in the prescription drug market, pharmacy benefit managers (PBMs) use sophisticated contract arrangements, including bulk discounts, market-share based rebates, and formularies (AMCP 2012, Reinke 2015). A formulary is a list of prescription drugs (Kouvelis et al. 2015). For drugs that are not on a formulary, PBMs may require patients to bear 100% of drug costs, that is, the drugs may be excluded from insurance benefits.

Responding to rising drug costs, PBMs have increased the use of exclusionary formularies (Langreth 2014). When Gilead Sciences, the maker of Sovaldi, a hepatitis C drug , refused to reduce the \$1,000 per pill tag price (\$84,000 for a course of treatment), Express Scripts, the largest PBM in the U.S., signed an exclusive contract with AbbVie, the maker of Viekira Pakits, a competitor of Sovaldi (Smolinski 2013). The threat of exclusion can successfully bring pharmaceutical manufacturers to the negotiation table and drive down PBMs' procurement cost, and very few drugs (in tens) are excluded by each PBM (Reinke 2015, Barrett et al. 2017).

The pharmaceutical supply chain relationship is comparable to what happens when a buyer procures from suppliers in a typical supply chain. The buyer can adopt various tools, such as quantity-dependent pricing schemes such as quantity discount, quantity flexibility, and sales rebates (e.g., Monahan 1984, Tsay 1999, Cachon 2003) to coordinate and reduce procurement costs. In addition, many buyers also set up preferred supplier programs that resemble either the exclusionary formulary or a tiered formulary.

The Operations Management community has long studied procurement and its impact on both supply chain performance and players' profits. The research has concentrated on auction design and contract coordination (e.g., see Beil 2010, Cachon 2003). In auction design, the focus is to arrive at efficient or optimal mapping from the solicited supplier information to the allocation and payment decision. In contract coordination, the focus is to arrive at a contract or a menu of contracts to better align incentives. In both stream of research, the exclusion clause and its implication have been neglected.

Another prevalent but seemingly overlooked practice in procurement is the Request for Quotation (RFQ) process. At the RFQ stage, a buyer typically specifies the quality, quantity, and/or other requirements, then solicits quotations or bids from the suppliers. The buyer can collect the sealed quotations in a one-shot fashion or request the quotations iteratively in multiple rounds. After receiving the quotations, the buyer can determine the winner and award contracts through negotiation or auction. In contract coordination literature, a RFQ stage is typically omitted. In auction design literature, a RFQ stage is typically modeled as a bid submission activity to solicit (cost) information. Nevertheless, Lovejoy (2010b) points out that under many circumstances, buyers continue to adopt RFQs despite having accurate cost estimations. Our manuscript provides a plausible explanation for this common practice.

1.1. Preview of the Results

In this paper, we focus on the procurement process with contract terms under a dual-sourcing setting when the suppliers are substitutable. Different from prior work (e.g., Hu and Qi 2017), we investigate a dual-sourcing problem under a bargaining framework to capture the threat of exclusion clauses. Even under a full information setting, we find that an RFQ process, along with sequential bargaining and exclusion clauses, is crucial for a buyer to achieve a desirable bargaining outcome. While an RFQ is commonly treated as a step to solicit private information in the procurement auction, we show that the buyer can leverage the RFQ process and improve her profit when the negotiation sequence is endogenously determined by the RFQ quotation stage. Because exclusion clauses may limit the profit of the supplier who is the Stackelberg follower, both suppliers prefer to be the Stackelberg leader in the negotiation sequence. Although the buyer's optimal negotiation sequence decision is fairly complicated (which is not a threshold policy), and a lower quotation does not automatically guarantee the supplier a leader position even in a symmetric setting, it turns out that the way to compete for the leader position is to quote a lower price in the RFQ stage and render more profit to the buyer. While our dual-sourcing framework is motivated by customer heterogeneity, the insights extend to other dual-sourcing settings, such as a buyer engaging in dual-sourcing to lower a disruption risk or avoid a potential monopolistic supplier.

A simplified procurement timeline for sequential bargaining with an RFQ is illustrated in Figure 1. The detailed timeline and formal model are presented in Section 2.4. The procurement has two stages: the RFQ stage and the negotiation stage. While we model the negotiation stage using a cooperative game framework, we model the RFQ stage using a non-cooperative game framework. This setting is motivated by the nature of the real-life procurement process. In the RFQ stage, the buyer first decides her quotation quantities and collects quotations from suppliers. In the negotiation stage, the buyer decides the negotiation sequence based on the RFQ quotations and then negotiates with suppliers. The buyer can either accept a quote and *renegotiate* a quantity-dependent procurement contract with exclusion clauses or reject a quotation and *negotiate* all terms of the contract. We use "she" to represent the buyer and "he" to represent a supplier. We use "*negotiate*" to denote the bargaining when the buyer rejects an offer and "*renegotiate*" to denote the bargaining when she accepts the offer in the remainder of the paper.

Following this process, we show that by endogenizing the bargaining sequence decision based on quotations, the buyer can intensify competition between suppliers with the threat of exclusion clauses. As a result, the RFQ process provides a strategic benefit to the buyer.

Our research contributes to the literature in the follow aspects:

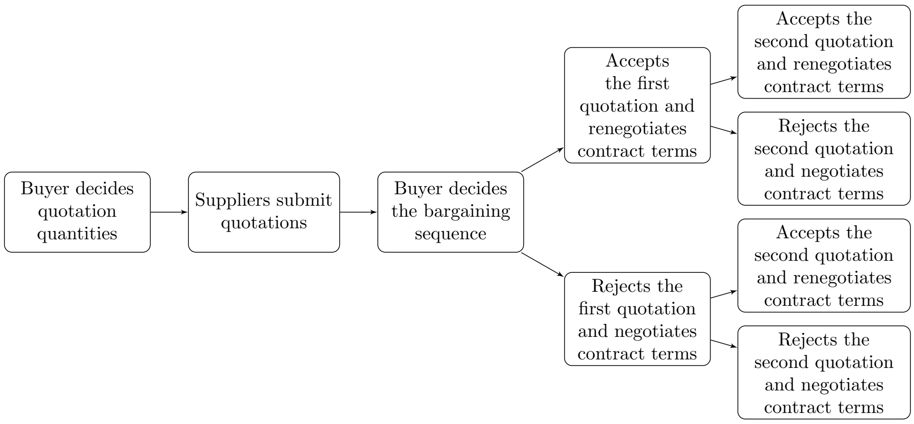


Figure 1 Event sequence.

1. We find that a quantity-dependent pricing contract can coordinate the supply chain. Under a simultaneous bargaining setting (defined in Section 2.2), we show that the buyer prefers dual-sourcing because each single-sourcing equilibrium is Pareto-dominated by a dual-sourcing equilibrium. Moreover, we prove that sequential bargaining settings (defined in Sections 2.3 and 2.4) lead to a dual-sourcing equilibrium. Furthermore, a dual-sourcing equilibrium in the aforementioned settings coordinates the supply chain (Theorems 1 and 2).

2. Under simultaneous bargaining, the inclusion of the exclusion clause creates a plethora of equilibria (Proposition 1). We define the "most favorable bargaining outcome" as the equilibrium that provides the buyer the highest profit in the simultaneous bargaining setting. This equilibrium offers the buyer a higher profit compared with the outcomes under sequential bargaining without an RFQ and some common mechanisms without the exclusion clause (Propositions 2 and 3).

3. Under sequential bargaining with an RFQ, we characterize the buyer's profit for given bargaining sequences and acceptance/rejection strategies (Lemmas 1-5), and find that the buyer's optimal bargaining sequence and acceptance/rejection strategy is a complicated function of the suppliers' quotations. Specifically, given one supplier's quotation, the buyer's optimal strategy is not a threshold policy with respect to the other supplier's quotation (Figure 9). Nevertheless, we show that the suppliers compete to be the Stackelberg leader in the negotiation stage by lowering their quotations in the RFQ stage (Lemma 9).

4. In equilibrium of either a one-shot simultaneous RFQ process or an iterative quotation process, the buyer accepts both offers and neither further bargaining nor the actual exclusion clauses need to be worked out (Theorem 4). Furthermore, the buyer's equilibrium profit under sequential bargaining with an RFQ is (weakly) higher than her profit under the most favorable equilibrium under the simultaneous bargaining setting (Theorem 5).

5. If the one-shot simultaneous RFQ process leads to an equilibrium, the resulting equilibrium has the same profit allocations as the most favorable equilibrium under the simultaneous bargaining

setting (Theorem 6). Nevertheless, the one-shot simultaneous RFQ process does not necessarily lead to an equilibrium (Figure 10). In such a case, we observe that the iterative quotation process provides the buyer a strictly higher profit compared with the most favorable equilibrium. This is driven by the fact that the buyer's optimal negotiation sequence decision is not a threshold policy and the supplier's optimal quotation response is not monotone with respect to the opponent's offer.

6. The cost-plus-fixed-fee contract format enables us to focus on profit allocation and use profit representation in the equilibrium analysis (Theorem 3). Furthermore, if the buyer can impose cost-plus-fixed-fee contracts and let suppliers quote their desired profit margins at the RFQ stage, the aforementioned contributions also hold in random demand settings (Proposition 4).

1.2. Literature Review

Our study lies at the intersection of two research streams: supplier sourcing and bilateral bargaining. It is natural for the buyer to diversify the supplier base due to supplier differences in cost, reliability, and leadtime. Tomlin and Wang (2005), Boute and Van Mieghem (2014), Xin and Goldberg (2016), and others study the optimal procurement quantity and/or timing of the dual-sourcing problem when the procurement cost is exogenously given. Wan and Beil (2009, 2014) and Li and Wan (2017) analyze the supply base design problem when the buyer optimally awards the contract to a single supplier. Chaturvedi and Martínez-de Albéniz (2011) and Yang et al. (2012) study supplier diversification using the optimal mechanism design framework when suppliers have private reliability/disruption information. Anton and Yao (1992) consider a split reward auction, which can be used to divide the full procurement among suppliers when the goods are identical and the total procurement quantity is set. In this paper, facing imperfect substitutable suppliers, the buyer's need to diversify comes from customers' idiosyncratic preferences.

A game with a single buyer and multiple suppliers belongs to the so-called "big boss games" (Muto et al. 1988), which are typically analyzed under a cooperative game framework in economics. Specifically, we utilize the theory of bargaining which stems from Nash Jr (1950) and Rubinstein (1982) to analyze the players' negotiations in this paper. Recently, there has been growing body of literature in operations management utilizing bargaining framework (e.g., Van Mieghem (1999), Plambeck and Taylor (2005), Gurnani and Shi (2006), Nagarajan and Sošić (2008), Huh and Park (2010), Lovejoy (2010a), Feng and Lu (2012, 2013b), Feng et al. (2014), Hsu et al. (2016)). When the bargaining process involves more than two parties, Rubinstein (1982) and Krishna and Serrano (1996) provide a framework for multilateral bargaining under which the offer proposed by any party is simultaneously and jointly judged by all remaining parties. However, under a dual-sourcing setting, antitrust law may prevent suppliers from jointly setting up a horizontal arrangement. Therefore, it is more appropriate to apply a bilateral bargaining framework in which a firm engages in multiple bilateral negotiations with individual suppliers.

Under a bilateral bargaining framework, the literature can be further divided into a simultaneous setting and a sequential setting based on the timing of the bilateral negotiations. Davidson (1988) and Horn and Wolinsky (1988) are among the first to study simultaneous bilateral bargaining. Collard-Wexler et al. (2017) study a setting under which the solution of an alternating-offers bargaining model converges with the Nash bargaining solution when multiple upstream firms trade with multiple downstream firms, and thus provides a micro-foundation to the Nash bargaining framework for a simultaneous bilateral bargaining setting. In operations management, Feng and Lu (2013a) compare Stackelberg games with simultaneous bilateral bargaining games under a wholesale-price contract and a two-part tariff contract. Aydin and Heese (2015) study the profit allocation problem under simultaneous bilateral bargaining games without explicit contract forms.

In a setting of one buyer and two suppliers with sequential bilateral bargaining, Aghion and Bolton (1987) show that the buyer and the supplier who negotiates first can extract the other supplier's surplus by a penalty clause. Marx and Shaffer (2002) find that the maximum joint profit is supported by an equilibrium under general conditions, although the individual payoffs depend on the negotiation sequence and the contract forms. Marx and Shaffer (2007) investigate the buyer's optimal negotiation sequence and find that the buyer starts from the supplier with less bargaining power or less stand-alone surplus, ceteris paribus. Marx and Shaffer (2010) analyze the impact of both suppliers' bargaining power on the bargaining sequence and the resulting payoffs. Nagarajan and Bassok (2008) consider an assembly setting (i.e., the suppliers are complementary) and assume that the suppliers compete for position by paying for the "favorable" negotiation position prior to the bargaining game.

Our paper shares a similar spirit with Hu and Qi (2017). The authors study a buyer's procurement problem for an assembly setting in both simultaneous and sequential manners, and follow the optimal mechanism design framework, which is the prevailing approach in the field of Operations Management. In contrast, we adopted a bargaining framework due to our focus of (the threat of) exclusion clauses. Furthermore, we study the ways that players' strategic moves might influence the bargaining outcome. Our key results illustrate how the buyer can leverage the RFQ in the procurement process with substitutable suppliers. We utilize the non-cooperative game framework for the RFQ stage and the cooperative game framework for the negotiation stage to address this question. This hybrid approach is also adopted in the strategy field (Brandenburger and Stuart 2007). We find that even under the full information setting, an RFQ provides the buyer a strategic benefit, which provides an explanation to Lovejoy (2010b)'s intriguing question of why the buyer engages in the RFQ process even when she knows the costs.

In the remaining sections of this paper, we introduce the notation and present the coordination results in Section 2. We analyze the simultaneous bargaining setting and the sequential bargaining with an RFQ process in Sections 3 and 4, respectively. We extend our results to a random demand setting in Section 5. In Section 6, we conclude the paper and suggest some directions for future research.

2. Bilateral Bargaining Model

2.1. Notations and Assumptions

We consider a two-tier supply chain with a monopoly buyer and two competing suppliers each producing a substitutable product. We label the buyer as player 0 and index the suppliers as players/suppliers 1 and 2. The supply chain's profit is denoted as $\Pi(q_1,q_2) \equiv V(q_1,q_2) - (C_1(q_1) + C_2(q_2))$, where V is the buyer's revenue function, C_i is supplier i's production cost function, and q_i is the buyer's order quantity to supplier i ($i = 1,2$). Without loss of generality (WLOG), the supply chain profit and costs with no procurement are normalized to 0–i.e., $\Pi(0,0) = V(0,0) = C_1(0) = C_2(0) = 0$. Denote $\Pi \equiv \max_{q_1,q_2 \geq 0} \Pi(q_1,q_2)$ as the maximum supply chain profit. We use $\Pi^{-i}(q_j)$ to denote the supply chain profit when supplier i fails to participate (i.e., $q_i = 0$) and the buyer procures q_j units exclusively from supplier j, and denote $\Pi^{-i} \equiv \max_{q_j} \Pi^{-i}(q_j)$ ($\{i,j\} = \{1,2\}$).

As the U.S. antitrust laws prevent supplier collusion, we thus study the setting in which the buyer conducts bilateral bargaining with each supplier. Under bilateral bargaining, the buyer and supplier i negotiate a contract $\mathcal{C}_i = (T_i, T_i^{-j})$. When the buyer procures from both suppliers, $T_i(q_i)$ is the quantity-dependent price scheme that specifies the payment from the buyer to supplier i for order quantity q_i. $T_i^{-j}(q_i)$ is the contingent payment scheme if supplier j is excluded and supplier i becomes the exclusive supplier. This exclusion clause essentially specifies a different quantity-dependent price scheme. In this paper, we use the contingent payment scheme and the exclusion clause to refer to $T_i^{-j}(q_i)$ interchangeably.

Given contract $(\mathcal{C}_1, \mathcal{C}_2)$, the buyer chooses the optimal order quantities. When the buyer chooses dual-sourcing, procurement quantities are obtained from the following optimization problem:

$$(q_1^*(\mathcal{C}_1,\mathcal{C}_2), q_2^*(\mathcal{C}_1,\mathcal{C}_2)) = \arg\max_{(q_1,q_2)>0}(V(q_1,q_2) - T_1(q_1) - T_2(q_2)),$$

while the buyer's profit $\pi_0|_{\mathcal{C}_1,\mathcal{C}_2}$ and supplier i's profit $\pi_i|_{\mathcal{C}_1,\mathcal{C}_2}$ are

$$\begin{aligned}
\pi_i|_{\mathcal{C}_1,\mathcal{C}_2} &= T_i(q_i^*(\mathcal{C}_1,\mathcal{C}_2)) - C_i(q_i^*(\mathcal{C}_1,\mathcal{C}_2)), \forall \{i,j\} = \{1,2\}, \\
\pi_0|_{\mathcal{C}_1,\mathcal{C}_2} &= V(q_1^*(\mathcal{C}_1,\mathcal{C}_2), q_2^*(\mathcal{C}_1,\mathcal{C}_2)) - T_1(q_1^*(\mathcal{C}_1,\mathcal{C}_2)) - T_2(q_2^*(\mathcal{C}_1,\mathcal{C}_2)) \\
&= \Pi(q_1^*(\mathcal{C}_1,\mathcal{C}_2), q_2^*(\mathcal{C}_1,\mathcal{C}_2)) - \pi_1|_{\mathcal{C}_1,\mathcal{C}_2} - \pi_2|_{\mathcal{C}_1,\mathcal{C}_2}, \qquad (1)
\end{aligned}$$

respectively. If the buyer procures exclusively from supplier i, the procurement quantity and the players' profit are

$$q_i^{-j*}(\mathcal{C}_i) = \arg\max_q(\Pi^{-j}(q) - (T_i^{-j}(q) - C_i(q)),$$

$$\begin{aligned}\pi_i^{-j}|_{\mathcal{C}_i} &= T_i^{-j}(q_i^{-j*}(\mathcal{C}_i)) - C_i(q_i^{-j*}(\mathcal{C}_i)),\\ \pi_0^{-j}|_{\mathcal{C}_i} &= \Pi^{-j}(q_i^{-j*}(\mathcal{C}_i)) - \pi_i^{-j}|_{\mathcal{C}_i},\end{aligned} \tag{2}$$

where $\pi_0^{-j}|_{\mathcal{C}_i}$ is the buyer's profit and $\pi_i^{-j}|_{\mathcal{C}_i}$ is supplier i's profit. Supplier j's profit is zero if the buyer procures exclusively from supplier i. The buyer chooses the most profitable options among dual-sourcing and single-sourcing from either supplier.

Three key assumptions are imposed:

1. $\Pi(q_1, q_2)$ is strictly jointly concave on $(q_1, q_2) \in \mathbb{R}_+^2$, with an internal maximizer (q_1^o, q_2^o).
2. $\frac{\partial^2 \Pi(q_1,q_2)}{\partial q_1 \partial q_2} < 0$ almost surely on $(q_1, q_2) \in \mathbb{R}_+^2$.
3. $\pi_j^{-i}|_{\mathcal{C}_j} \geq 0$ for $\{i, j\} = \{1, 2\}$.

Assumption 1 means that the marginal contribution of order quantities to a supply chain's profit is decreasing. This assumption simplifies the exposition and ensures that (q_1^o, q_2^o) are the unique efficient procurement quantities. As we subsequently show, these are the equilibrium procurement quantities. The internal maximizer restriction is imposed because we would like to focus on the ideal scenarios in which the buyer procures from both suppliers–i.e., $\Pi > \Pi^{-i}$, $(i = 1, 2)$.

Assumption 2 describes the substitutable nature between the suppliers in which the marginal contribution of ordering from one supplier is decreasing with the order quantity from the other supplier. This assumption implies that $\Pi(q_1, q_2)$ is submodular. Because $(q_1^o, q_2^o) > 0$, $\Pi = \Pi(q_1^o, q_2^o) + 0 < \Pi(q_1^o, 0) + \Pi(0, q_2^o) = \Pi^{-2}(q_1^o) + \Pi^{-1}(q_2^o)$, which implies that $\Pi^{-j}(q_i^o) > 0$ due to $\Pi > \Pi^{-i} \geq \Pi^{-i}(q_j^o)$ for $\{i, j\} = \{1, 2\}$. Assumptions 1 and 2 further imply that $\frac{d\Pi^{-i}(q_j)}{dq_j}|_{q_j = q_j^o} > 0$ and $\Pi^{-i} > \Pi^{-i}(q_j^o) > 0$.

Assumption 3 ensures non-negative payoffs for both suppliers under the exclusion clauses. Without this assumption, using the idea of Aghion and Bolton (1987), the buyer and supplier who negotiate first can fully extract the other supplier's surplus by selling at below-cost prices under the exclusion clause. Such a practice violates antitrust laws against predatory pricing (Marx and Shaffer 2002).

We next describe the various game setups in our study. We basically consider two types of settings: a simultaneous bargaining setting and a sequential bargaining setting. Under the simultaneous bargaining setting, the negotiations with the two suppliers happen at the same time, and we assume that the outcome of one negotiation will not be revealed to the players in the other negotiation (e.g. considering that the buyer sends two independent procurement teams to the two suppliers). We assume that the players in a negotiation will form a correct belief about the negotiation outcome of the other pair. Under the sequential bargaining setting, the buyer sequentially negotiates with suppliers, and thus the earlier negotiation outcome is revealed and it can impact the later negotiation outcome.

2.2. Setup of Simultaneous Bargaining

In this section, we outline the setup of simultaneous bargaining, under which two bilateral negotiations take place at the same time. The bargaining model is posed as a Nash bargaining problem (Osborne and Rubinstein 1990). The formulation along with explanations is provided below

$$\text{[Simu-1] } \mathcal{C}_1(\mathcal{C}_2) = \arg\max_{\mathcal{C}} (\pi_1|_{\mathcal{C},\mathcal{C}_2} - 0)^{\theta_1} (\pi_0|_{\mathcal{C},\mathcal{C}_2} - \pi_0^{-1}|_{\mathcal{C}_2})^{1-\theta_1}$$

$$s.t.\ \pi_1|_{\mathcal{C},\mathcal{C}_2} \geq 0, \pi_0|_{\mathcal{C},\mathcal{C}_2} \geq \pi_0^{-1}|_{\mathcal{C}_2};$$

$$\text{[Simu-2] } \mathcal{C}_2(\mathcal{C}_1) = \arg\max_{\mathcal{C}} (\pi_2|_{\mathcal{C}_1,\mathcal{C}} - 0)^{\theta_2} (\pi_0|_{\mathcal{C}_1,\mathcal{C}} - \pi_0^{-2}|_{\mathcal{C}_1})^{1-\theta_2}$$

$$s.t.\ \pi_2|_{\mathcal{C}_1,\mathcal{C}} \geq 0, \pi_0|_{\mathcal{C}_1,\mathcal{C}} \geq \pi_0^{-2}|_{\mathcal{C}_1}.$$

In each formulation, the objective is to maximize the product of players' negotiation surplus functions. The power coefficient $\theta_i \in [0,1]$ is defined as supplier i's bargaining power. A player's negotiation surplus is her/his profit minus her/his reservation utility. For the negotiation between the buyer and supplier i, supplier i's reservation utility is zero, and the buyer's reservation utility is $\pi_0^{-i}|_{\mathcal{C}_j}$ because if the negotiation with supplier i breaks up, the buyer may procure exclusively from supplier j based on contract $\mathcal{C}_j$ ($\{i,j\} = \{1,2\}$). These constraints ensure that the players' bargaining surpluses are non-negative, which are the individual rational conditions to participate in the game.

Recall that each contract has two payment schemes: the payment scheme to supplier i if the buyer sources from both suppliers (T_i), and the contingent payment scheme to supplier i if the buyer procures exclusively from supplier i (T_i^{-j}), $\{i,j\} = \{1,2\}$. It is worth noting that for a dual-sourcing equilibrium, in formulation [Simu-i] that determines $\mathcal{C}_i = (T_i, T_i^{-j})$, the buyer's profit depends on payment schemes (T_i, T_j) and the buyer's reservation utility depends on contingent payment scheme (T_j^{-i}), while contingent payment (T_i^{-j}) does not appear in the formulation. That is, [Simu-i] only limits the choice of payment scheme (T_i), but not contingent payment scheme (T_i^{-j}). As we will see, different pairs (T_1^{-2},T_2^{-1}) may arise under different dual-sourcing equilibrium, resulting in multiple equilibria ($\mathcal{C}_1$,$\mathcal{C}_2$) and different payoff combinations for the players. Because both negotiations occur simultaneously, a consistent belief about the reservation utilities is needed. This requires that at equilibrium, contract $\mathcal{C}_1 = (T_1, T_1^{-2})$ ($\mathcal{C}_2 = (T_2, T_2^{-1})$) provides the consistent reservation utility π_0^{-2} (π_0^{-1}) adopted in formulation [Simu-2] ([Simu-1]). The possibility of multiple exclusion clause pairs (T_1^{-2},T_2^{-1}) and different reservation utilities (π_0^{-1}, π_0^{-2}) pose a major challenge in analyzing the performance of a simultaneous bilateral bargaining setting.

When formulation [Simu-i] is infeasible, the buyer does not source from supplier i and we allow C_i to be either no deal, denoted by $\emptyset$, or any contract that induces single-sourcing from supplier

j in equilibrium. When the buyer fails to reach a deal with supplier i (i.e., $\mathcal{C}_i = \emptyset$), $\pi_0|_{\mathcal{C}_1,\mathcal{C}_2}$ and $\pi_j|_{\mathcal{C}_1,\mathcal{C}_2}$ reduce to $\pi_0^{-i}|_{\mathcal{C}_j}$ and $\pi_j^{-i}|_{\mathcal{C}_j}$, respectively, because the buyer can only source from supplier j; furthermore, the buyer's reservation utility $\pi_0^{-j}|_{\mathcal{C}_i}$ becomes zero. When the buyer and supplier i have agreed upon the contract $\mathcal{C}_i(\neq \emptyset)$, $\pi_0^{-j}|_{\mathcal{C}_i}$ can be positive even if the buyer procures exclusively from supplier j in equilibrium.

2.3. Setup of Sequential Bargaining

A natural alternative to the simultaneous bilateral bargaining setting is a sequential setting. We illustrate the formulation by assuming that the buyer first negotiates with supplier 1 and then with supplier 2.

We formulate the problem by backward induction. If the buyer and supplier 1 have agreed upon contract $\mathcal{C}_1$, we consider bilateral bargaining between the buyer and supplier 2. When contracting with supplier 2, the buyer's reservation utility is $\pi_0^{-2}|_{\mathcal{C}_1}$, and supplier 2's reservation utility is zero. The bilateral bargaining between the buyer and supplier 2 is modeled as follows:

$$\text{[Sequ-2]}\ \mathcal{C}_2(\mathcal{C}_1) = \arg\max_{\mathcal{C}} (\pi_2|_{\mathcal{C}_1,\mathcal{C}} - 0)^{\theta_2} (\pi_0|_{\mathcal{C}_1,\mathcal{C}} - \pi_0^{-2}|_{\mathcal{C}_1})^{1-\theta_2}$$
$$s.t.\ \pi_2|_{\mathcal{C}_1,\mathcal{C}} \geq 0, \pi_0|_{\mathcal{C}_1,\mathcal{C}} \geq \pi_0^{-2}|_{\mathcal{C}_1}.$$

Notice that formulation [Sequ-2] is identical to formulation [Simu-2] and both require $\mathcal{C}_2$ to be the optimal response given $\mathcal{C}_1$. If the buyer and supplier 1 fail to reach an agreement (i.e., $\mathcal{C}_1 = \emptyset$), the negotiation between the buyer and supplier 2 can be written as:

$$\mathcal{C}_2(\emptyset) = \arg\max_{\mathcal{C}} (\pi_2^{-1}|_{\mathcal{C}} - 0)^{\theta_2} (\pi_0^{-1}|_{\mathcal{C}} - 0)^{1-\theta_2}$$
$$s.t.\ \pi_2^{-1}|_{\mathcal{C}} \geq 0, \pi_0^{-1}|_{\mathcal{C}} \geq 0.$$

We now consider the bilateral bargaining between the buyer and supplier 1. Supplier 1's reservation utility is zero, and the buyer's reservation utility is her payoff when she fails to reach a deal with supplier 1. The bilateral bargaining between the buyer and supplier 1 is modeled as follows:

$$\text{[Sequ-1]}\ \mathcal{C}_1 = \arg\max_{\mathcal{C}} (\pi_1|_{\mathcal{C},\mathcal{C}_2(\mathcal{C})} - 0)^{\theta_1} (\pi_0|_{\mathcal{C},\mathcal{C}_2(\mathcal{C})} - \pi_0^{-1}|_{\mathcal{C}_2(\emptyset)})^{1-\theta_1}$$
$$s.t.\ \pi_1|_{\mathcal{C},\mathcal{C}_2(\mathcal{C})} \geq 0, \pi_0|_{\mathcal{C},\mathcal{C}_2(\mathcal{C})} \geq \pi_0^{-1}|_{\mathcal{C}_2(\emptyset)}.$$

We can also write out the formulation when the buyer first bargains with supplier 2 and then supplier 1 in a similar fashion. Notice that Marx and Shaffer (2007) study the optimal bargaining sequence from the buyer's perspective and show how the suppliers' bargaining power and the single-sourcing supply chain profit impact the buyer's optimal bargaining sequence decision. We will show that regardless of which supplier the buyer negotiates with first, her profit under sequential bargaining is no more than her profit of the most favorable bargaining outcome under the simultaneous bilateral bargaining (Proposition 3).

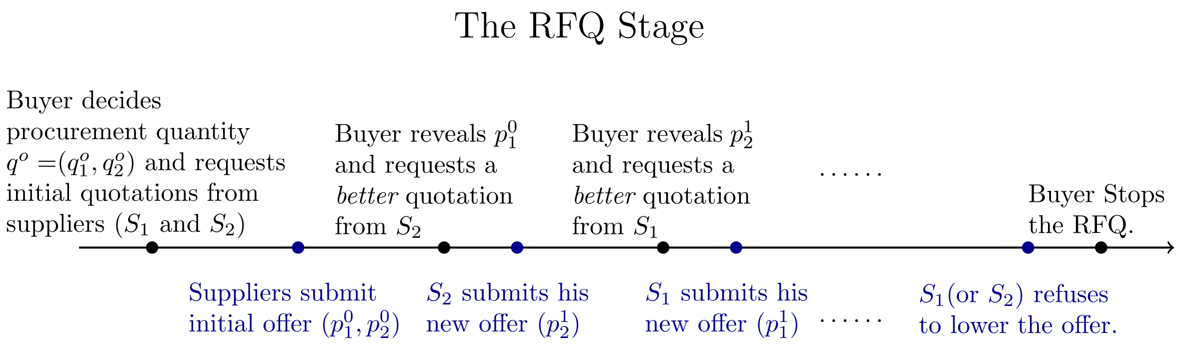
2.4. Setup of Sequential Bargaining Preceded by RFQ

We now introduce an RFQ stage ahead of the sequential bargaining stage. In this RFQ stage, the buyer only requests price quotations for the first-best procurement quantities from both suppliers. Notice that under a full information setting, the quotations do not solicit additional information for the buyer. In the negotiation stage, the buyer decides whether to accept the price quotations from the suppliers and then negotiates the contract terms with two suppliers in a sequential manner similar to the setting described in Section 2.3. We demonstrate the detailed settings for both the RFQ stage and the negotiation stage in the following.

2.4.1. Timeline. In the RFQ stage, the buyer can either request quotations for procurement quantities (q_1^o, q_2^o) in a one-shot simultaneous fashion under which each supplier only submits one quotation as shown in Figure 1, or she can iteratively request for quotations from suppliers as shown in Figure 2. Under the one-shot simultaneous RFQ process, we use $(\tilde{p}_1, \tilde{p}_2)$ to represent the price quotations. Under the iterative quotation process, the buyer asks for better quotations after receiving the initial prices. She reveals supplier 1's quotation price and asks for a new quotation from supplier 2. After receiving a quotation from supplier 2, she reveals the new price and asks for a quotation from supplier 1 to see whether he wants to reduce the price. She repeats the iterative quotation process as the suppliers monotonously decrease their quotation prices. When the suppliers no longer lower the price, the final quotation prices in the RFQ process are also denoted as $(\tilde{p}_1, \tilde{p}_2)$, and the negotiation stage begins.[1]

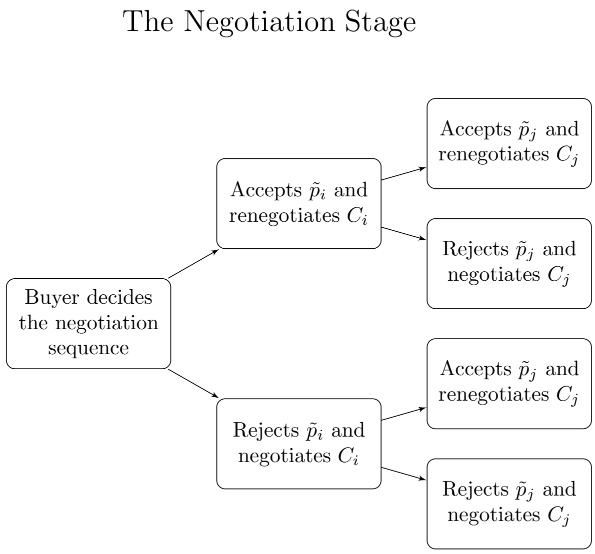
At the negotiation stage, Figure 3 illustrates how the buyer chooses the optimal negotiation sequence and quotation acceptance/rejection strategies to maximize her profit based on the quotation price $(\tilde{p}_1, \tilde{p}_2)$. WLOG, we assume the buyer first contracts with supplier i and then contracts with supplier j ($\{i, j\} = \{1, 2\}$). The buyer may accept or reject the offer from supplier i. If the buyer rejects the quotation, she *negotiates* contract $\mathcal{C}_i$ with supplier i. If the buyer accepts the quotation, she is obliged to procure q_i^o units at the quoted price $\tilde{p}_i$ from supplier i. At this point, the buyer and supplier i may *renegotiate* the contract. Notice that under the optimal sequence, after accepting supplier i's offer, it would be in the buyer's best interest to *renegotiate* with supplier i before contracting with supplier j; otherwise, the buyer would achieve a (weakly) higher profit by reversing the negotiation sequence and keeping the option on supplier i open. By working out the exclusion clause through *renegotiation*, the buyer and supplier i can limit supplier j's profit and increase their own profit.

[1] Notice that if the buyer can commit to specific negotiation sequences based on the realized quotations, the buyer can impose undue pressure on the suppliers during the quotation stage. For example, if the buyer commits to first negotiating with the supplier whose offer is no more than his profit under the most favorable bargaining outcome defined in Section 3, the most favorable bargaining outcome is an equilibrium. We study the sub-game perfect equilibrium of this procurement game without assuming such commitment power from the buyer.



* We use $\tilde{p}_i(\tilde{p}_j)$ to denote the final RFQ price $p_i^n(p_j^n)$ submitted by $S_i(S_j)$.

Figure 2 Event sequence in the RFQ stage.



* $\tilde{p}_i(\tilde{p}_j)$ denotes the final RFQ price submitted by Supplier $i(j)$.

Figure 3 Event sequence in the negotiation stage.

After contracting with supplier i, the buyer contracts with supplier j. The buyer may accept or reject the offer from supplier j. If the buyer rejects the quotation, she negotiates contract $\mathcal{C}_j$ with supplier j. If the buyer accepts the quotation, she is obliged to procure q_j^o units at the quoted price $\tilde{p}_j$ from supplier j. At this point, the buyer and supplier j may also renegotiate the contract.

If a supplier refuses to respond to the quotation, we can treat it as a prohibitively high price quotation so that the buyer would reject the supplier's quotation and negotiate with the supplier. In other words, one can view the sequential bargaining setting described in Section 2.3 as a special case of sequential bargaining with the RFQ when the quotations are set prohibitively high.

2.4.2. Formulation. Given suppliers' quotation $(\tilde{p}_1, \tilde{p}_2)$, we formulate the problem by backward induction similar to the setting in Section 2.3. We illustrate the formulation by assuming that the buyer first negotiates the contract with supplier 1 and then with supplier 2. If the buyer and supplier 1 have agreed upon contract $\mathcal{C}_1$, we consider the contracting between the buyer and supplier 2. The buyer may either reject or accept supplier 2's quotation.

If the buyer rejects supplier 2's offer, the buyer's reservation utility is $\pi_0^{-2}|_{\mathcal{C}_1}$, and supplier 2's reservation utility is zero. The bilateral bargaining between the buyer and supplier 2 is modeled as follows:

$$\text{[RFQ-R2] } \mathcal{C}_2(\mathcal{C}_1) = \arg\max_{\mathcal{C}} (\pi_2|_{\mathcal{C}_1,\mathcal{C}} - 0)^{\theta_2} (\pi_0|_{\mathcal{C}_1,\mathcal{C}} - \pi_0^{-2}|_{\mathcal{C}_1})^{1-\theta_2}$$
$$s.t.\ \pi_2|_{\mathcal{C}_1,\mathcal{C}} \geq 0, \pi_0|_{\mathcal{C}_1,\mathcal{C}} \geq \pi_0^{-2}|_{\mathcal{C}_1}.$$

Notice that formulation [RFQ-R2] is identical to formulation [Sequ-2] and both formulations require $\mathcal{C}_2$ to be the optimal response given $\mathcal{C}_1$. Furthermore, both formulations apply to the case in which the buyer and supplier 1 fail to reach a deal (i.e., $\mathcal{C}_1 = \emptyset$), and the resulting contract is $\mathcal{C}_2(\emptyset)$.

If the buyer accepts supplier 2's offer, we denote $\tilde{C}_2$ to be the contract between the buyer and supplier 2 that requires the buyer to procure q_2^o units from supplier 2 at price $\tilde{p}_2$ (i.e., $T_2(q_2^o) = T_2^{-1}(q_2^o) = \tilde{p}_2$ and $T_2(q) = T_2^{-1}(q) = \infty$ when $q \neq q_2^o$). Contract $\tilde{C}_2$ ensures supplier 2 a profit of $\tilde{p}_2 - C_2(q_2^o)$. The bilateral bargaining between the buyer and supplier 2 is modeled as follows:

$$\text{[RFQ-A2] } \tilde{\mathcal{C}}_2(\mathcal{C}_1, \tilde{p}_2) = \arg\max_{\mathcal{C}} (\pi_2|_{\mathcal{C}_1,\mathcal{C}} - \pi_2|_{\mathcal{C}_1,\tilde{\mathcal{C}}_2})^{\theta_2} (\pi_0|_{\mathcal{C}_1,\mathcal{C}} - \pi_0|_{\mathcal{C}_1,\tilde{\mathcal{C}}_2})^{1-\theta_2}$$
$$s.t.\ \pi_2|_{\mathcal{C}_1,\mathcal{C}} \geq \pi_2|_{\mathcal{C}_1,\tilde{\mathcal{C}}_2} = \tilde{p}_2 - C_2(q_2^o),\ \ \pi_0|_{\mathcal{C}_1,\mathcal{C}} \geq \pi_0|_{\mathcal{C}_1,\tilde{\mathcal{C}}_2}.$$

The buyer chooses the optimal acceptance/rejection strategy with respect to supplier 2's offer to maximize her profit. In equilibrium, the contract between her and supplier 2 is $\mathcal{C}_2(\mathcal{C}_1, \tilde{p}_2) = \arg\max_{\mathcal{C} \in \{\tilde{\mathcal{C}}_2(\mathcal{C}_1,\tilde{p}_2), \mathcal{C}_2(\mathcal{C}_1)\}} \pi_0|_{\mathcal{C}_1,\mathcal{C}}$.

We now consider the bilateral bargaining between the buyer and supplier 1. The buyer may either reject or accept supplier 1's quotation.

If the buyer rejects supplier 1's offer, supplier 1's reservation utility is zero, and the buyer's reservation utility is her payoff when she fails to reach a deal with supplier 1. The bilateral bargaining between the buyer and supplier 1 is modeled as follows:

$$\text{[RFQ-R1] } \mathcal{C}_1(\tilde{p}_2) = \arg\max_{\mathcal{C}} (\pi_1|_{\mathcal{C},\mathcal{C}_2(\mathcal{C},\tilde{p}_2)} - 0)^{\theta_1} (\pi_0|_{\mathcal{C},\mathcal{C}_2(\mathcal{C},\tilde{p}_2)} - \pi_0^{-1}|_{\mathcal{C}_2(\emptyset,\tilde{p}_2)})^{1-\theta_1}$$
$$s.t.\ \pi_1|_{\mathcal{C},\mathcal{C}_2(\mathcal{C},\tilde{p}_2)} \geq 0, \pi_0|_{\mathcal{C},\mathcal{C}_2(\mathcal{C},\tilde{p}_2)} \geq \pi_0^{-1}|_{\mathcal{C}_2(\emptyset,\tilde{p}_2)}.$$

If the buyer accepts supplier 1's offer, we denote $\tilde{C}_1$ to be the contract between the buyer and supplier 1 that requires the buyer to procure q_1^o units from supplier 1 at price $\tilde{p}_1$ (i.e., $T_1(q_1^o) =$

$T_1^{-2}(q_1^o) = \tilde{p}_1$ and $T_1(q) = T_1^{-2}(q) = \infty$ when $q \neq q_1^o$). Contract $\tilde{C}_1$ ensures supplier 1 a profit of $\tilde{p}_1 - C_1(q_1^o)$. The bilateral bargaining between the buyer and supplier 1 is modeled as follows:

$$[\text{RFQ-A1}]\ \mathcal{C}_1(\tilde{p}_1, \tilde{p}_2) = \arg\max_{\mathcal{C}} (\pi_1|_{\mathcal{C},\mathcal{C}_2(\mathcal{C},\tilde{p}_2)} - \pi_1|_{\tilde{\mathcal{C}}_1,\mathcal{C}_2(\tilde{\mathcal{C}}_1,\tilde{p}_2)})^{\theta_1} (\pi_0|_{\mathcal{C},\mathcal{C}_2(\mathcal{C},\tilde{p}_2)} - \pi_0|_{\tilde{\mathcal{C}}_1,\mathcal{C}_2(\tilde{\mathcal{C}}_1,\tilde{p}_2)})^{1-\theta_1}$$

$$s.t.\ \pi_1|_{\mathcal{C},\mathcal{C}_2(\mathcal{C},\tilde{p}_2)} \geq \pi_1|_{\tilde{\mathcal{C}}_1,\mathcal{C}_2(\tilde{\mathcal{C}}_1,\tilde{p}_2)} = \tilde{p}_1 - C_1(q_1^o), \pi_0|_{\mathcal{C},\mathcal{C}_2(\mathcal{C},\tilde{p}_2)} \geq \pi_0|_{\tilde{\mathcal{C}}_1,\mathcal{C}_2(\tilde{\mathcal{C}}_1,\tilde{p}_2)}.$$

The buyer chooses the optimal acceptance/rejection strategy with respect to supplier 1's offer to maximize her profit. In equilibrium, the contract between her and supplier 1 is $\arg\max_{\mathcal{C}\in\{\mathcal{C}_1(\tilde{p}_2),\mathcal{C}_1(\tilde{p}_1,\tilde{p}_2)\}} \pi_0|_{\mathcal{C},\mathcal{C}_2(\mathcal{C},\tilde{p}_2)}$. We can also write out the formulation when the buyer first bargains with supplier 2 and then supplier 1. The buyer may choose the optimal bargaining sequence by comparing the profit under the two alternatives.

2.5. Coordination Results and Welfare Representation

Given all the formulations in Sections 2.2-2.4, we establish some general equilibrium properties. In this section, we show that it suffices to focus on the dual-sourcing equilibrium under both the simultaneous bargaining setting and the sequential bargaining setting with given quotations. Furthermore, the quantity-dependent pricing contract with the exclusion clause can coordinate the supply chain in a dual-sourcing equilibrium. As a result, the equilibrium procurement quantities are the first-best procurement quantities, and we focus on how the players split the profit in equilibrium.

Theorem 1. *For each single-sourcing equilibrium under the simultaneous bargaining setting, we can find a Pareto-improving, dual-sourcing equilibrium. Moreover, the sequential bargaining setting with given quotations leads to dual-sourcing in equilibrium.*

Theorem 1 builds on the intuition that by inducing supplier j's participation, the players can split the incremental gain $(\Pi - \Pi^{-j})$. We next show that the players maximize the supply chain profit in a dual-sourcing equilibrium.

Theorem 2. *The dual-sourcing equilibrium procurement quantities are (q_1^o, q_2^o) under both the simultaneous bargaining setting and sequential bargaining settings with given quotations. That is, the maximum supply chain profit is achieved in a dual-sourcing equilibrium.*

Theorem 2 implies that with quantity-dependent pricing contracts, the buyer and suppliers can achieve supply chain coordination in a dual-sourcing equilibrium. By combining Theorems 1 and 2, we focus on profit allocation in the analysis and ignore the procurement quantity decisions. Notice that for a given equilibrium payoff combination, multiple equilibrium contracts may support the outcome. We next show that it suffices to focus on the so-called cost-plus-fixed-fee contract (CPFF), which has been widely used in practice (Weitzman 1980). The CPFF is one of the four contract types commonly used by the U.S. Department of Defense (Rogerson 1992). Under a CPFF contract, the supplier's profit is a constant independent of the procurement quantity (i.e., $T_i(q) = C_i(q) + \alpha_i$ and $T_i^{-j}(q^{-j}) = C_i(q^{-j}) + \alpha_i^{-j}$, where α_i and α_i^{-j} are constants obtained through negotiation). Thus, the buyer is the residual claimant and chooses the procurement quantities that maximize the supply chain profit. In the next theorem, we show that any contract in a dual-sourcing equilibrium is equivalent to a CPFF contract in terms of the profit split among three players.

THEOREM 3. *Suppose that $(\hat{\mathcal{C}}_1 = (\hat{T}_1, \hat{T}_1^{-2}), \hat{\mathcal{C}}_2 = (\hat{T}_2, \hat{T}_2^{-1}))$ is a dual-sourcing equilibrium under the simultaneous bargaining setting (sequential bargaining with given quotations) with equilibrium profit $\hat{\pi}_0, \hat{\pi}_1, \hat{\pi}_2$ for the players. $(\mathcal{C}_1 = (T_1, T_1^{-2}), \mathcal{C}_2 = (T_2, T_2^{-1}))$ is also a dual-sourcing equilibrium under the simultaneous bargaining setting (sequential bargaining with given quotations) with the same equilibrium profit, when $T_1(q) = C_1(q) + \hat{\pi}_1$, $T_1^{-2}(q_1^{-2}) = C_1(q_1^{-2}) + \Pi^{-2} - \pi_0^{-2}|_{\hat{\mathcal{C}}_1}$, $T_2(q) = C_2(q) + \hat{\pi}_2$, and $T_2^{-1}(q_2^{-1}) = C_2(q_2^{-1}) + \Pi^{-1} - \pi_0^{-1}|_{\hat{\mathcal{C}}_2}$.*

The analysis of Theorem 3 shows that the contingent payment schemes allow us to separate the buyer's reservation utilities from the actual procurement payments in a dual-sourcing equilibrium. As a result, instead of discussing equilibrium contracts, we can focus on the equilibrium payoffs and the payoffs under the contingent payment schemes, and characterize an equilibrium by a profit vector $\boldsymbol{\pi} = (\pi_0, \pi_1, \pi_2, \pi_0^{-1}, \pi_0^{-2})$, which greatly simplifies the exposition. The flexibility of the quantity-dependent pricing contracts and the contingent payment schemes also enable us to capture the entire problem setting by parameters $(\Pi, \Pi^{-1}, \Pi^{-2}, \theta_1, \theta_2)$.[2] In the remainder of this paper, we focus on the players' equilibrium profit when the entire supply chain is coordinated (that is, one can implicitly assume that the buyer adopts CPFF contracts).

3. Simultaneous Bilateral Bargaining

Given the results in Section 2, we study the profit splits under simultaneous bilateral bargaining. By Theorem 2, we can rewrite the buyer's profit π_0 as $\Pi - \pi_1 - \pi_2$. The dual-sourcing equilibrium profit of the suppliers solves the following formulation:

$$\pi_i = \arg\max_{\pi} (\pi - 0)^{\theta_i} (\Pi - \pi_j - \pi - \pi_0^{-i})^{1-\theta_i} \text{ for } \{i, j\} = \{1, 2\},$$

$$s.t.\ \pi \geq 0, \pi_0 = \Pi - \pi_j - \pi \geq \pi_0^{-i}.$$

[2] Using alternative proofs, one can establish Theorems 1 and 2 using the quantity-dependent pricing contracts absent the contingent payment schemes. Nevertheless, without the contingent payment schemes, Theorem 3 will not hold and parameters $(\Pi, \Pi^{-1}, \Pi^{-2}, \theta_1, \theta_2)$ will not characterize the problem completely.

Under Nash bargaining, supplier i earns θ_i proportion of the negotiation surplus and the buyer obtains the remaining $1-\theta_i$ proportion–i.e. $\frac{\pi_i - 0}{\pi_0 - \pi_0^{-i}} = \frac{\theta_i}{1-\theta_i}$. By Assumption 3, $\pi_0^{-i} \leq \Pi^{-i}$. Furthermore, $\pi_0^{-i} \geq 0$ because we have normalized the supply chain profit and costs with no procurement to zero. The formulation is feasible if and only if $\pi_j + \pi_0^{-i} \leq \Pi$. As described in Section 2.2, the contingent profit of supplier i (i.e., $\pi_i^{-j} = \Pi^{-j} - \pi_0^{-j}$) does not appear in the formulation. Therefore, the formulation only determines supplier i's profit, but not the buyer's reservation utility based on the contingent payment scheme of supplier i. Similarly, the formulation between supplier j and the buyer does not determine the contingent payment scheme of supplier j. As a result, we face a plethora of equilibria. Proposition 1 describes the equilibrium set.

PROPOSITION 1. *$\boldsymbol{\pi} = (\pi_0, \pi_1, \pi_2, \pi_0^{-1}, \pi_0^{-2})$ is a dual-sourcing equilibrium if and only if $\pi_0 = \frac{(1-\theta_i)(1-\theta_j)\Pi + \theta_i(1-\theta_j)\pi_0^{-i} + (1-\theta_i)\theta_j\pi_0^{-j}}{1-\theta_i\theta_j}$, $\pi_i = \theta_i \frac{(1-\theta_j)\Pi + \theta_j\pi_0^{-j} - \pi_0^{-i}}{1-\theta_i\theta_j}$, $\pi_0^{-i} \leq \Pi^{-i}$ and $\pi_0^{-i} \leq (1-\theta_j)\Pi + \theta_j\pi_0^{-j}$, for $\{i,j\} = \{1,2\}$.*

The buyer prefers higher π_0^{-1} and π_0^{-2}, which lead to higher buyer's reservation utility under the bilateral negotiations and result in a higher buyer's profit. With slight abuse of the notation, we use $\boldsymbol{\pi}(\pi_0^{-i}, \pi_0^{-j})$ to represent the equilibrium solution with $\pi_0 = \frac{(1-\theta_i)(1-\theta_j)\Pi + \theta_i(1-\theta_j)\pi_0^{-i} + (1-\theta_i)\theta_j\pi_0^{-j}}{1-\theta_i\theta_j}$, $\pi_i = \theta_i \frac{(1-\theta_j)\Pi + \theta_j\pi_0^{-j} - \pi_0^{-i}}{1-\theta_i\theta_j}$, for a given (π_0^{-i}, π_0^{-j}) when $\pi_0^{-i} \leq \Pi^{-i}$ and $\pi_0^{-i} \leq (1-\theta_j)\Pi + \theta_j\pi_0^{-j}$, for $\{i,j\} = \{1,2\}$.

PROPOSITION 2. *The equilibrium solution $\boldsymbol{\pi}^* \equiv \boldsymbol{\pi}(\min\{\Pi^{-1}, (1-\theta_2)\Pi + \theta_2\Pi^{-2}\}, \min\{\Pi^{-2}, (1-\theta_1)\Pi + \theta_1\Pi^{-1}\})$ offers the buyer the maximum profit among all the dual-sourcing equilibria.*

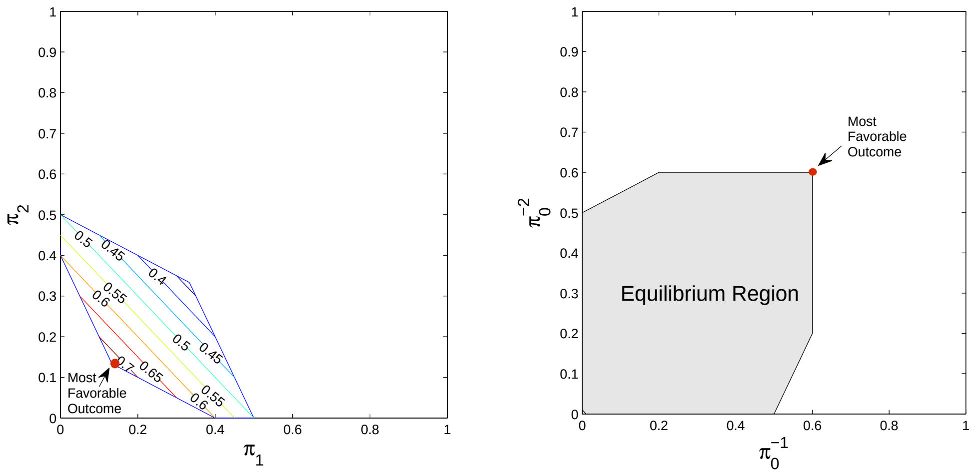
Proposition 2 characterizes the maximum buyer's profit under dual-sourcing. We call the corresponding equilibrium ($\boldsymbol{\pi}^*$) the most favorable bargaining outcome.[3] If π_0^{-1} (π_0^{-2}) is too high, the negotiation between the buyer and supplier 1 (supplier 2) may break down and the buyer may single-source from supplier 2 (supplier 1), which is Pareto dominated by some dual-soucing equilibrium by Theorem 1.

We further define two cases depending on parameters $(\Pi, \Pi^{-1}, \Pi^{-2}, \theta_1, \theta_2)$.

- **Regular Case**: $\Pi^{-1} < (1-\theta_2)\Pi + \theta_2\Pi^{-2}$ and $\Pi^{-2} < (1-\theta_1)\Pi + \theta_1\Pi^{-1}$.
- **Degenerate Case**: $\Pi^{-1} \geq (1-\theta_2)\Pi + \theta_2\Pi^{-2}$ or $\Pi^{-2} \geq (1-\theta_1)\Pi + \theta_1\Pi^{-1}$.

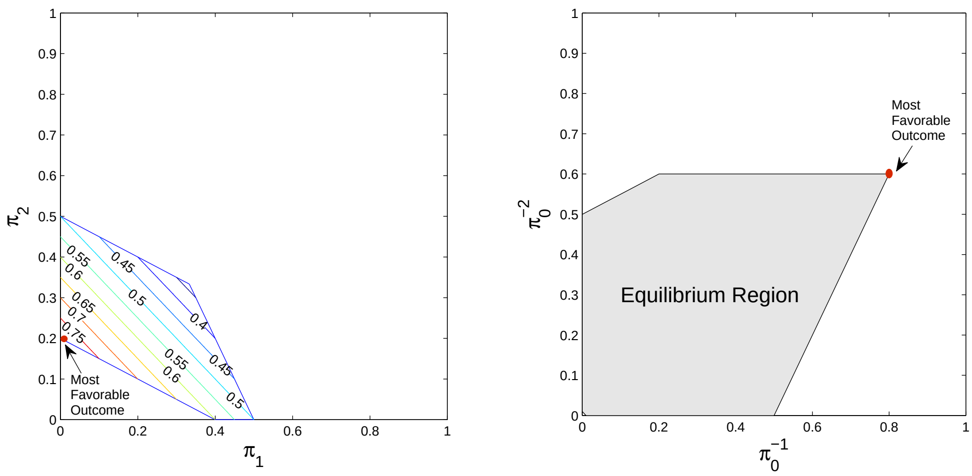
Figure 4 illustrates a regular case with $\theta_1 = \theta_2 = 0.5$, $\Pi^{-1} = \Pi^{-2} = 0.6\Pi$ with supply chain profit Π normalized to 1. Figure 4(a) shows how the buyer and suppliers split profits in dual-sourcing equilibria, and Figure 4(b) shows the corresponding exclusion clauses (π_0^{-1}, π_0^{-2}) in the

[3] For exogenously given reservation utilities (π_0^{-1}, π_0^{-2}), the Nash product form in our simultaneous bilateral bargaining setup can also be justified by a Rubinstein alternating offer process in the so-called *multiunit* bargaining model (Davidson 1988). Notice that if the buyer can specify (π_0^{-1}, π_0^{-2}) before the *multiunit* bilateral bargaining starts, the buyer would set reservation utilities at $(\min\{\Pi^{-1}, (1-\theta_2)\Pi + \theta_2\Pi^{-2}\}, \min\{\Pi^{-2}, (1-\theta_1)\Pi + \theta_1\Pi^{-1}\})$ at optimal, which would lead to the most favorable bargaining outcome. This implies that the buyer's profit in the most favorable bargaining outcome provides an upper bound for the profit in the corresponding Rubinstein alternating offer process.



(a) Simplex (b) Equilibria

Figure 4 A regular case ($\theta_1 = \theta_2 = 0.5,\ \Pi^{-1} = \Pi^{-2} = 0.6\Pi, \Pi = 1$).



(a) Simplex (b) Equilibria

Figure 5 A degenerate case ($\theta_1 = \theta_2 = 0.5,\ \Pi^{-1} = 0.8\Pi,\ \Pi^{-2} = 0.6\Pi, \Pi = 1$).

equilibrium region. Figure 4(a) is a contour plot of the buyer's profit, and the horizontal and vertical axes represent the profits of suppliers 1 and 2, respectively. The large dot represents the most favorable negotiation outcome (i.e., the buyers' most favorable profit (Figure 4(a)) and the corresponding exclusion clause (Figure 4(b)). Figure 5 shows a degenerate case with $\theta_1 = \theta_2 = 0.5,\ \Pi^{-1} = 0.8\Pi,\ \Pi^{-2} = 0.6\Pi$. The exclusion clauses (π_0^{-1}, π_0^{-2}) in the equilibrium region forms a hexagon in the regular case and a pentagon in the degenerate case. These two cases also result in different supplier profit patterns under the most favorable bargaining outcome.

Corollary 1. *Both suppliers obtain positive profits under the most favorable bargaining outcome if and only if the parameters fall in the regular case.*

The most favorable bargaining outcome is highly desirable from the buyer's view point. We formalize the results in the next proposition.

PROPOSITION 3. *The buyer's profit under the most favorable bargaining outcome* $\boldsymbol{\pi}^*$ *is higher than that of a sequential bargaining setting without an RFQ, or a bargaining setting that adopts CPFF contracts without contingent payment schemes in either a simultaneous or sequential manner.*

It is worth noting that under CPFF contracts without contingent payment schemes, each supplier obtains the same profit (a portion of his marginal contribution) in a bilateral bargaining setting regardless of whether the bargaining is conducted simultaneously or sequentially, of whether an RFQ stage precedes the actual bargaining. Proposition 3 illustrates the importance to consider contingent payment schemes explicitly and the dominance of the most favorable bargaining outcome. A natural question arises as to how the buyer can construct a procurement process to achieve her profit under the most favorable bargaining equilibrium. In the next section, we show that the buyer can actually achieve a (weakly) better outcome by adopting an RFQ.

4. Sequential Bilateral Bargaining with an RFQ

In this section, we study the profit split under sequential bilateral bargaining with an RFQ. Given the results in Section 2, we focus on the equilibrium profit allocation. In Section 4.1, we derive the profit expressions given the suppliers' quotations and the buyer's negotiation sequence and acceptance/rejection strategy. We analyze the buyer's optimal negotiation sequence and acceptance/rejection strategy in Sections 4.2 and the suppliers' quotation process in Section 4.3. The properties of the equilibrium outcome are summarized in Section 4.4.

4.1. Equilibrium Profit Expressions

When supplier i offers a quotation $\tilde{p}_i$, the demanded profit is $\tilde{\pi}_i = \tilde{p}_i - C_i(q_i^o)$, for $i = 1, 2$. To simplify the exposition, we use $(\tilde{\pi}_1, \tilde{\pi}_2)$ as the offers coming out of the RFQ process.

The buyer faces various choices in the negotiation sequence: (I) accepting both offers without renegotiation; (II) first accepting supplier i's offer and renegotiating with supplier i and then rejecting supplier j's offer and negotiating with supplier j; (III) first rejecting supplier i's offer and negotiating with supplier i and then rejecting supplier j's offer and negotiating with supplier j; and (IV) first rejecting supplier i's offer and negotiating with supplier i and then accepting supplier j's offer ($\{i, j\} = \{1, 2\}$). Notice that when the buyer accepts supplier j's offer, the buyer and supplier j need not go through the renegotiation for a complete contract because they have finalized the transaction price for the equilibrium procurement quantity by Theorem 2.

We assume that if some aforementioned choices provide the buyer the same profit, she prefers the one with more acceptances and prefers early acceptance over late acceptance. If the buyer still faces a tie, she breaks the tie by the lexicographic order, say, first contracting with supplier 1 and then supplier 2. For ease of reading, we call choices I, II, III, and IV A_iA_j, A_iR_j, R_iR_j, and R_iA_j, respectively.

We solve the suppliers' equilibrium behavior by backward induction. By Theorem 3, we focus on equilibrium profit vector $\boldsymbol{\pi} = (\pi_0, \pi_1, \pi_2, \pi_0^{-1}, \pi_0^{-2})$ in our analysis. By Theorem 2, $\pi_0 = \Pi - \pi_1 - \pi_2$. Recall that π_i represents player i's profit under dual-sourcing ($i = 0, 1, 2$) and π_0^{-j} is the buyer's profit when she fails to procure from supplier j ($j = 1, 2$). After contracting with supplier i, π_i and π_0^{-j} are known. The outcome of the ensuing negotiation between the buyer and supplier j is characterized by the following lemma if the buyer rejects supplier j's offer.

LEMMA 1. *Given* (π_i, π_0^{-j})*, the contract between supplier* j *and the buyer specifies* $\pi_j(\pi_i, \pi_0^{-j}) = \theta_j(\Pi - \pi_i - \pi_0^{-j})$ *and* $\pi_0(\pi_i, \pi_0^{-j}) = (1 - \theta_j)(\Pi - \pi_i) + \theta_j\pi_0^{-j}$ *when* $\Pi - \pi_i - \pi_0^{-j} \geq 0$ *and otherwise excludes supplier* j*, if the buyer rejects supplier* j*'s offer.*

WLOG, we assume $\Pi - \pi_i - \pi_0^{-j} \geq 0$ in equilibrium, because Theorem 1 shows that it is in the best interest of both the buyer and supplier i to induce supplier j's participation. This condition can be incorporated as a constraint in the first-stage bargaining model between the buyer and supplier i. Given (π_i, π_0^{-j}), the buyer accepts supplier j's offer if and only if $\tilde{\pi}_j \leq \pi_j(\pi_i, \pi_0^{-j}) \equiv \theta_j(\Pi - \pi_i - \pi_0^{-j})$.

The first-stage negotiation outcome depends on both the second-stage negotiation outcome (i.e., the buyer's acceptance/rejection strategy with supplier j) and the buyer's reservation utility D_0, which equals to the buyer's maximum possible utility by either accepting or rejecting supplier j's offer when she fails to negotiate/renegotiate with supplier i. In this subsection, we derive closed-form profit expressions assuming that in equilibrium, either $\tilde{\pi}_j < \pi_j(\pi_i, \pi_0^{-j})$ or $\tilde{\pi}_j > \pi_j(\pi_i, \pi_0^{-j})$, so that we know whether the buyer will accept or reject supplier j's offer after rectifying the exclusion clause with supplier i. The boundary case (i.e., $\tilde{\pi}_j = \pi_j(\pi_i, \pi_0^{-j})$) is examined in Section 4.2.

We now discuss the first-stage negotiation. Depending on whether the buyer accepts or rejects supplier i's quotation, we analyze two different bargaining models [AI] and [RI], respectively. We first discuss formulation [AI]. Suppose the buyer first accepts supplier i's offer at optimality. The optimality of this decision implies that $\tilde{\pi}_i \leq \Pi$. The buyer then renegotiates with supplier i, which is modeled as follows:

$$\text{[AI]:} \qquad (\pi_i, \pi_0^{-j}) = \arg\max_{(\pi, \pi')} (\pi - \tilde{\pi}_i)^{\theta_i} (\Pi - \pi - \min\{\tilde{\pi}_j, \pi_j(\pi, \pi')\} - D_0)^{1-\theta_i} \tag{3}$$

$$s.t. \quad \pi \geq \tilde{\pi}_i, \tag{4}$$

$$\Pi - \pi - \min\{\tilde{\pi}_j, \pi_j(\pi, \pi')\} \geq D_0, \tag{5}$$

$$\pi' \leq \Pi^{-j}, \tag{6}$$

$$\Pi - \pi - \pi' \geq 0, \tag{7}$$

where $D_0 = \max\{\Pi - \tilde{\pi}_i - \tilde{\pi}_j, \Pi - \tilde{\pi}_i - \pi_j(\tilde{\pi}_i, \Pi^{-j}(q_i^o) - \tilde{\pi}_i)\}$ is the buyer's reservation utility if the renegotiation with supplier i fails to reach an agreement and the buyer procures from supplier i based on quotation $\tilde{\pi}_i$. At this point, the buyer has the option to either accept supplier j's offer (and obtain $\Pi - \tilde{\pi}_i - \tilde{\pi}_j$) or reject his offer and negotiate (and obtain $\Pi - \tilde{\pi}_i - \pi_j(\tilde{\pi}_i, \Pi^{-j}(q_i^o) - \tilde{\pi}_i)$). The formulation is feasible because $(\pi_i, \pi_0^{-j}) = (\tilde{\pi}_i, (\Pi^{-j}(q_i^o) - \tilde{\pi}_i)^+)$ is a feasible solution when $\tilde{\pi}_i \leq \Pi$ (we define $(x)^+ \equiv \max\{x, 0\}$).

LEMMA 2. *Suppose that at optimality, the buyer first* **accepts** *the offer $\tilde{\pi}_i$. If the buyer* **accepts** *$\tilde{\pi}_j$, the profits of the buyer, supplier i, and supplier j are $(\pi_0, \pi_i, \pi_j) = (\Pi - \tilde{\pi}_i - \tilde{\pi}_j, \tilde{\pi}_i, \tilde{\pi}_j)$. If the buyer* **rejects** *$\tilde{\pi}_j$, the profits of the buyer and supplier j are $\pi_0 = \Pi - \pi_i - \pi_j$ and $\pi_j = \theta_j(\Pi - \pi_i - \Pi^{-j})^+$ respectively, where*

if $\tilde{\pi}_j \geq \theta_j(\Pi - \Pi^{-j}(q_i^o))$,

$$\pi_i = \begin{cases} \tilde{\pi}_i + \frac{\theta_i\theta_j}{1-\theta_j}(\Pi^{-j} - (\Pi^{-j}(q_i^o) - \tilde{\pi}_i)), & \text{if } \Pi \geq \tilde{\pi}_i + \Pi^{-j} + \frac{\theta_i\theta_j}{1-\theta_j}(\Pi^{-j} - (\Pi^{-j}(q_i^o) - \tilde{\pi}_i)); \\ \tilde{\pi}_i + \theta_i\theta_j(\Pi - \Pi^{-j}(q_i^o)), & \text{if } \Pi \leq \frac{1}{1-\theta_i\theta_j}(\tilde{\pi}_i + \Pi^{-j} - \theta_i\theta_j\Pi^{-j}(q_i^o)); \\ \Pi - \Pi^{-j}, & \text{otherwise;} \end{cases}$$

if $\tilde{\pi}_j < \theta_j(\Pi - \Pi^{-j}(q_i^o))$,

$$\pi_i = \begin{cases} \tilde{\pi}_i + \frac{\theta_i(\tilde{\pi}_j - \theta_j(\Pi - \tilde{\pi}_i - \Pi^{-j}))}{1-\theta_j}, & \text{if } \Pi \geq \Pi^{-j} + \tilde{\pi}_i + \frac{\theta_i}{1-\theta_j+\theta_i\theta_j}\tilde{\pi}_j; \\ \tilde{\pi}_i + \theta_i\tilde{\pi}_j, & \text{if } \Pi \leq \Pi^{-j} + \tilde{\pi}_i + \theta_i\tilde{\pi}_j; \\ \Pi - \Pi^{-j}, & \text{otherwise.} \end{cases}$$

Furthermore, both π_0 and π_j are continuous and (weakly) decreasing functions of $\tilde{\pi}_i$ and $\tilde{\pi}_j$ if the buyer rejects $\tilde{\pi}_j$.

We now discuss formulation [RI]. Suppose that the buyer first rejects supplier i's offer at optimality. The negotiation between the buyer and supplier i is then modeled as follows:

$$\text{[RI]:} \quad (\pi_i, \pi_0^{-j}) = \arg\max_{(\pi,\pi')}(\pi - 0)^{\theta_i}(\Pi - \pi - \min\{\tilde{\pi}_j, \pi_j(\pi, \pi')\} - D_0)^{1-\theta_i} \tag{8}$$

$$s.t. \quad \pi \geq 0, \tag{9}$$

$$\Pi - \pi - \min\{\tilde{\pi}_j, \pi_j(\pi, \pi')\} \geq D_0, \tag{10}$$

$$\pi' \leq \Pi^{-j}, \tag{11}$$

$$\Pi - \pi - \pi' \geq 0, \tag{12}$$

where $D_0 = \max\{(1 - \theta_j)\Pi^{-i}, (1 - \theta_j)\Pi^{-i} + \theta_j\Pi^{-i}(q_j^o) - \tilde{\pi}_j\}$ is the buyer's reservation utility if the negotiation with supplier i fails to reach an agreement and the buyer procures exclusively from supplier j. At this point, the buyer has the option to either reject his offer and negotiate (and obtain $(1-\theta_j)\Pi^{-i}$) or accept supplier j's offer (and obtain $\Pi^{-i}(q_j^o)-\tilde{\pi}_j$) and then renegotiate (and obtain additional $(1-\theta_j)(\Pi^{-i}-\Pi^{-i}(q_j^o))$). The formulation is feasible because $(\pi_i,\pi_0^{-j})=(0,0)$ is a feasible solution.

LEMMA 3. *Suppose that at optimality the buyer first* **rejects** *the offer $\tilde{\pi}_i$. If the buyer* **accepts** *$\tilde{\pi}_j$, the profits of the buyer, supplier i, and supplier j are $(\pi_0,\pi_i,\pi_j)=((1-\theta_i)(\Pi-\tilde{\pi}_j)+\theta_i D_0,\theta_i(\Pi-\tilde{\pi}_j-D_0),\tilde{\pi}_j)$ where $D_0=\max\{(1-\theta_j)\Pi^{-i},(1-\theta_j)\Pi^{-i}+\theta_j\Pi^{-i}(q_j^o)-\tilde{\pi}_j\}$. If the buyer* **rejects** *$\tilde{\pi}_j$, the profits of the buyer and supplier j are $\pi_0=\Pi-\pi_i-\pi_j$ and $\pi_j=\theta_j(\Pi-\pi_i-\Pi^{-j})^+$ respectively, where if $\tilde{\pi}_j>\theta_j\Pi^{-i}(q_j^o)$,*

$$\pi_i=\begin{cases}\theta_i(\Pi-\Pi^{-i})+\frac{\theta_i\theta_j}{1-\theta_j}\Pi^{-j}, & \text{if } \Pi\ge\frac{1}{1-\theta_i}\left(-\theta_i\Pi^{-i}+\left(1+\frac{\theta_i\theta_j}{1-\theta_j}\right)\Pi^{-j}\right);\\ \theta_i(\Pi-(1-\theta_j)\Pi^{-i}), & \text{if } \Pi\le\frac{1}{1-\theta_i}\left(\Pi^{-j}-\theta_i(1-\theta_j)\Pi^{-i}\right);\\ \Pi-\Pi^{-j}, & \text{otherwise;}\end{cases}$$

if $\tilde{\pi}_j\le\theta_j\Pi^{-i}(q_j^o)$,

$$\pi_i=\begin{cases}\frac{\theta_i((1-\theta_j)(\Pi-\Pi^{-i})+\tilde{\pi}_j-\theta_j\Pi^{-i}(q_j^o)+\theta_j\Pi^{-j})}{1-\theta_j}, & \text{if } \Pi\ge\frac{1}{1-\theta_i}\left(\left(1+\frac{\theta_i\theta_j}{1-\theta_j}\right)\Pi^{-j}-\frac{\theta_i}{1-\theta_j}((1-\theta_j)\Pi^{-i}+\theta_j\Pi^{-i}(q_j^o)-\tilde{\pi}_j)\right)\\ \theta_i(\Pi+\tilde{\pi}_j-\theta_j\Pi^{-i}(q_j^o)-(1-\theta_j)\Pi^{-i}), & \text{if } \Pi\le\frac{1}{1-\theta_i}\left(\Pi^{-j}+\theta_i(\tilde{\pi}_j-\theta_j\Pi^{-i}(q_j^o)-(1-\theta_j)\Pi^{-i})\right);\\ \Pi-\Pi^{-j}, & \text{otherwise.}\end{cases}$$

Furthermore, both π_0 and π_j are continuous (weakly) decreasing functions of $\tilde{\pi}_i$ and $\tilde{\pi}_j$ if the buyer rejects $\tilde{\pi}_j$.

4.2. The Negotiation Sequence

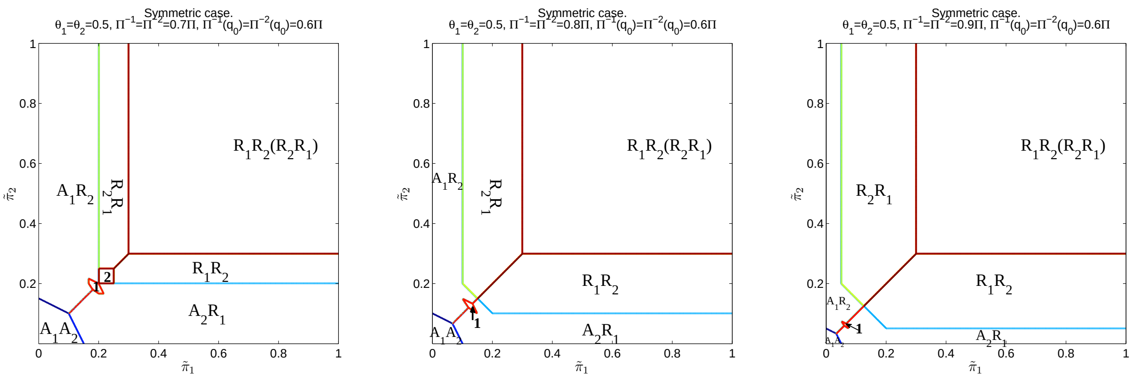
The profit expressions in Lemmas 2 and 3 are derived based on the assumption that we know whether the buyer's profit function takes the form $\Pi-\pi_i-\tilde{\pi}_j$ or $\Pi-\pi_i-\pi_j(\pi_i,\pi_0^{-j})$, despite the fact that the buyer's true profit function is $\Pi-\pi_i-\min\{\tilde{\pi}_j,\pi_j(\pi_i,\pi_0^{-j})\}$, where $\pi_j(\pi_i,\pi_0^{-j})=\theta_j(\Pi-\pi_i-\pi_0^{-j})\ge 0$. We now show that Lemma 2 provides the correct closed-form expressions and establishes the necessary and sufficient condition when the buyer accepts supplier i's offer for formulation [AI].

LEMMA 4. *Assuming that at optimality the buyer first accepts the offer $\tilde{\pi}_i$ from supplier i, Lemma 2 provides the correct profit expressions for formulation [AI]. Furthermore, it is optimal for the buyer to accept supplier j's offer $\tilde{\pi}_j$ if and only if $\tilde{\pi}_j\le\theta_j(\Pi-\tilde{\pi}_i-\Pi^{-j})^+$ ($\{i,j\}=\{1,2\}$).*

We now show that Lemma 3 provides the correct closed-form expressions for formulation [RI]. We know that up to one local optimum exists in each of the two half-spaces ($\tilde{\pi}_j\le\theta_j(\Pi-\pi_i-\pi_0^{-j})$ and $\tilde{\pi}_j>\theta_j(\Pi-\pi_i-\pi_0^{-j})$) by Lemma 3. During the bargaining, the buyer and supplier i need to find the solution to formulation [RI]. It is possible that the buyer and supplier i would compare two local optima to find the global optimum, or that no local optimum is valid in the respective half space and the global optimum is obtained on the boundary of the two half-spaces, $\tilde{\pi}_j = \theta_j(\Pi - \pi_i - \pi_0^{-j})$. We show that the latter scenario never occurs.

Lemma 5. *Assuming that at optimality the buyer first rejects the offer $\tilde{\pi}_i$ from supplier i, Lemma 3 provides the correct profit expressions for formulation [RI]. Furthermore, it is optimal for the buyer to reject supplier j's offer ($\{i,j\} = \{1,2\}$).*

Lemma 5 enables us to focus on scenario R_iR_j and ignore scenario R_iA_j when it is optimal for the buyer to first reject supplier i's offer. Assumption 2 (submodularity) plays a critical role in this result. When the suppliers are complementary, contracting with the second supplier can significantly increase the overall welfare. As a result, we can find examples under which the buyer's optimal negotiation sequence is R_iA_j when Assumption 2 is violated. With substitutable suppliers, the buyer is better off choosing R_iA_j over R_iR_j only if supplier j's offer $\tilde{\pi}_j$ is fairly small (because of submodularity, $\Pi \le \Pi^{-i}(q_j^o) + \Pi^{-j}(q_i^o)$–i.e., the marginal contribution of the additional supplier is relatively small.), under which we can show that the buyer prefers A_jR_i over R_iA_j.



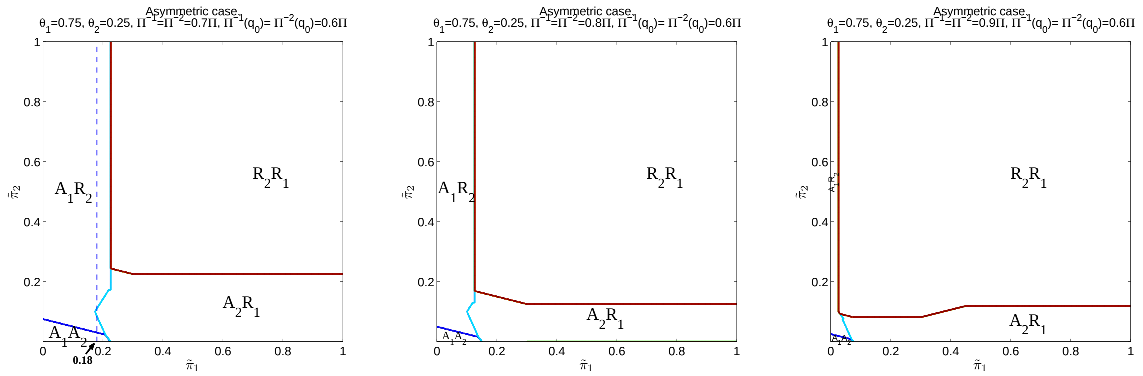
(a) $\Pi^{-1} = \Pi^{-2} = 0.7\Pi$ (b) $\Pi^{-1} = \Pi^{-2} = 0.8\Pi$ (c) $\Pi^{-1} = \Pi^{-2} = 0.9\Pi$

Figure 6 The buyer's bargaining strategies with symmetric settings ($\theta_1 = \theta_2 = 0.5$, $\Pi^{-1}(q_0) = \Pi^{-2}(q_0) = 0.6\Pi$).

Figure 6 illustrates the buyer's optimal negotiation sequence on different quotations $(\tilde{\pi}_1, \tilde{\pi}_2)$ in a symmetric setting with the same negotiation power ($\theta_1 = \theta_2 = 0.5$) and the same single-sourcing welfare ($\Pi^{-1} = \Pi^{-2}$, $\Pi^{-1}(q_2^o) = \Pi^{-2}(q_1^o) = 0.6\Pi$). WLOG, Π is normalized to 1 in Figures 6 to 8. Note that the regions with label "**1**" are the regions in which the buyer obtains the optimal profit with either A_1R_2 or A_2R_1. The region with label "**2**" is the region in which the buyer obtains the optimal profit with either R_1R_2 or R_2R_1 (under which the buyer's payoff equals $\Pi^{-1} = \Pi^{-2}$). The buyer will accept both suppliers' offers when both $\tilde{\pi}_1$ and $\tilde{\pi}_2$ are small. When only one supplier's offer is small, the buyer will first accept (and renegotiate with) the supplier with the lower quotation and then reject (and negotiate with) the other supplier. When neither supplier's offer is sufficiently small, the buyer will reject both suppliers and negotiate with the supplier of the higher quote first.

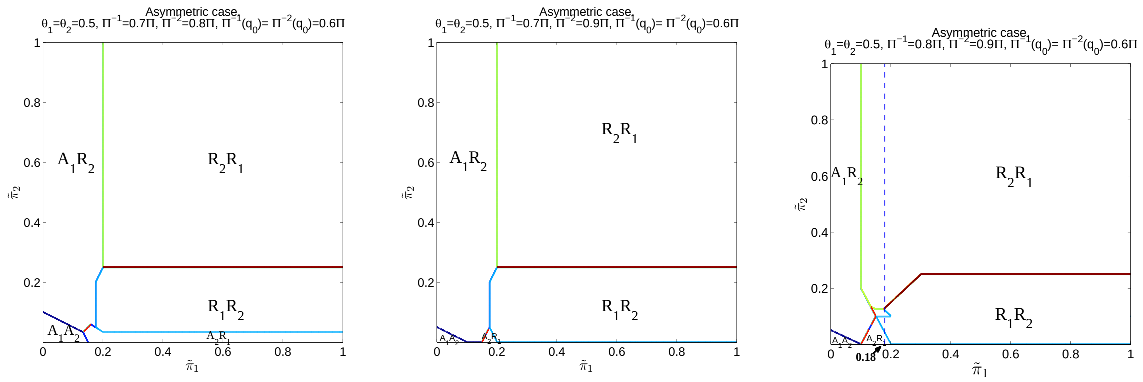
In doing so, the buyer has a higher reservation utility from the lower-quotation supplier. When both suppliers' offers are large, the buyer will reject them and obtain the same profit in either negotiation sequence.

When both single-sourcing welfare is high (i.e., $\Pi^{-1} = \Pi^{-2}$ is high), the substitution effect among suppliers is strong and the exclusion clause provides a powerful threat to the suppliers. As a result, the buyer tends to reject the suppliers' offers and the region of accepting both offers is small. The buyer is more prudent to reject the offers when $\Pi^{-1} = \Pi^{-2}$ is low.



(a) $\Pi^{-1} = \Pi^{-2} = 0.7\Pi$ (b) $\Pi^{-1} = \Pi^{-2} = 0.8\Pi$ (c) $\Pi^{-1} = \Pi^{-2} = 0.9\Pi$

Figure 7 The buyer's bargaining strategies with asymmetric settings ($\theta_1 = 0.75, \theta_2 = 0.25$, $\Pi^{-1}(q_0) = \Pi^{-2}(q_0) = 0.6\Pi$).



(a) $\Pi^{-1} = 0.7\Pi,\ \Pi^{-2} = 0.8\Pi$ (b) $\Pi^{-1} = 0.7\Pi,\ \Pi^{-2} = 0.9\Pi$ (c) $\Pi^{-1} = 0.8\Pi,\ \Pi^{-2} = 0.9\Pi$

Figure 8 The buyer's bargaining strategies with asymmetric settings ($\theta_1 = \theta_2 = 0.5$, $\Pi^{-1}(q_0) = \Pi^{-2}(q_0) = 0.6\Pi$).

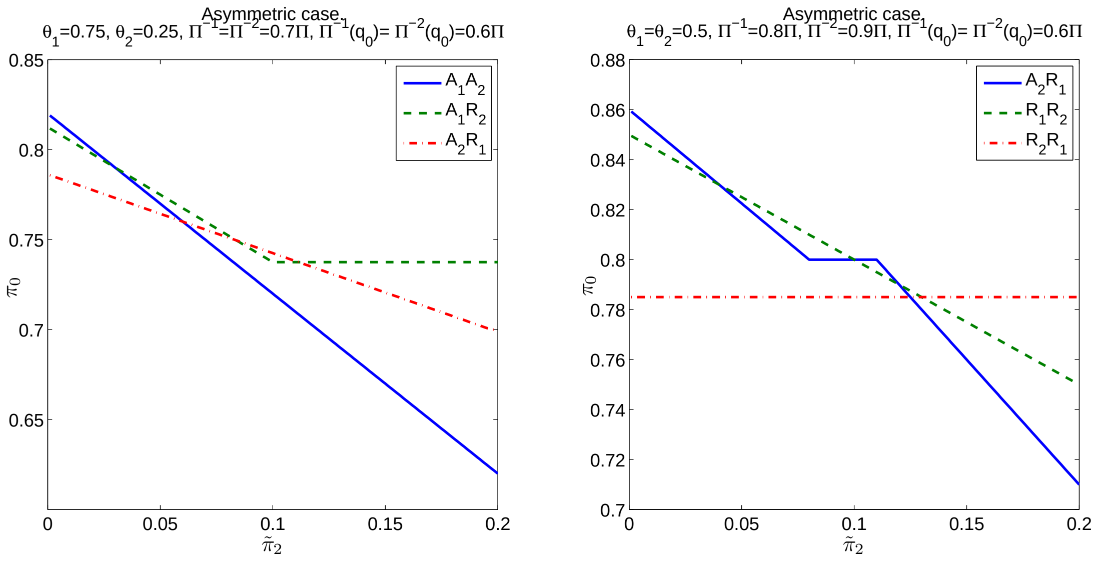
Figures 7 and 8 show the optimal negotiation sequences when suppliers are asymmetric. Figure 7 discusses scenarios in which supplier 1 has a higher negotiation power ($\theta_1 = 0.75$) than supplier 2 ($\theta_2 = 0.25$) when the single-sourcing welfare is the same. Based on negotiation power, supplier 1 is stronger than supplier 2. A buyer is more likely to reject an offer from a supplier with lower negotiation power. When both $\tilde{\pi}_1$ and $\tilde{\pi}_2$ are small, the buyer may accept supplier 1's offer, then reject supplier 2's offer even when $\tilde{\pi}_1 \geq \tilde{\pi}_2$. When both $\tilde{\pi}_1$ and $\tilde{\pi}_2$ are high, the buyer should reject both suppliers and negotiate first with supplier 2, who has lower bargaining power.

Figure 8 discusses cases in which the single-sourcing welfare with supplier 1 is higher than that of supplier 2 ($\Pi^{-2} \geq \Pi^{-1}$) while the negotiation power is the same ($\theta_1 = \theta_2 = 0.5$). Based on single-sourcing welfare, supplier 1 is stronger than supplier 2. When both $\tilde{\pi}_1$ and $\tilde{\pi}_2$ are small, the buyer is more likely to first accept the offer from supplier 1, who has higher single-sourcing welfare. When both $\tilde{\pi}_1$ and $\tilde{\pi}_2$ are high, the buyer rejects both offers and first negotiates with supplier 2, who has lower single-sourcing welfare. In doing so, the buyer is able to limit the profit of supplier 1, the stronger supplier, which is consistent with Marx and Shaffer (2007). Notice that Figures 8(b) and 8(c) are degenerate cases.

Figures 6 to 8 illustrate that the buyer's optimal bargaining sequence and acceptance/rejection strategy is a complicated function of the suppliers' quotations. Specifically, given one supplier's quotation, the buyer's optimal strategy is not "monotone" with respect to the other supplier's quotation. For example, if we fix $\tilde{\pi}_1$ at 0.18, Figures 7(a) and 9(a) show that the buyer's optimal strategy varies from A_1A_2, A_1R_2, A_2R_1, and back to A_1R_2 as $\tilde{\pi}_2$ increases from 0 to 1. Figures 8(c) and 9(b) show that the buyer's optimal strategy starts from A_2R_1, R_1R_2, then back to A_2R_1, R_1R_2, and goes to R_2R_1 as $\tilde{\pi}_2$ increases from 0 to 1. In other words, the buyer's optimal strategy is not a threshold policy.

Specifically, Figure 9 illustrates how the profit of each strategy varies as a function of $\tilde{\pi}_2$ when $\tilde{\pi}_1 = 0.18$ for the cases in Figures 7(a) and 8(c), respectively. By Lemmas 2 and 3, we know that the buyer's profit (weakly) decreases with quotation $\tilde{\pi}_2$ for a given acceptance/rejection strategy. Figure 9 demonstrates that the complexity of the buyer's optimal strategy decision arises from the fact that the decreasing rate of the buyer's profit varies under different strategies. Moreover, the decreasing rate of a single strategy also fluctuates depending on the binding constraints. In Figure 9(a), we can see that the buyer's profit in A_1A_2 decreases with $\tilde{\pi}_2$ at a rate of 1; the buyer's profit in A_1R_2 first decreases with $\tilde{\pi}_2$ at a rate less than 1 and then becomes independent of $\tilde{\pi}_2$; and the buyer's profit in A_2R_1 deceases with $\tilde{\pi}_2$ at an even smaller rate. In Figure 9(b), we can see that the buyer's profit in A_2R_1 is composed of three linear segments, with the middle flat region corresponding to a payoff that equals Π^{-1}; the buyer's profit in R_1R_2 decreases with $\tilde{\pi}_2$ at a rate less than 1 in the graph; and the buyer's profit in R_2R_1 is a constant given $\tilde{\pi}_1 = 0.18$.

This complication prevents us from obtaining a clean expression for the buyer's optimal strategy. Nevertheless, we can establish some structural results for the buyer's acceptance behavior and use such results to quantify the equilibrium behavior later.



(a) (b)

Figure 9 The buyer's profit curves in different bargaining sequences at $\tilde{\pi}_1 = 0.18$.

Lemma 6. *If $\tilde{\pi}_i = 0$, the buyer should accept supplier i's offer immediately* $(i = 1, 2)$.

Lemma 6 states that supplier i can assure the acceptance of his offer by submitting an offer with zero profit (his ultimate profit may still be positive due to renegotiation). Lemma 7 characterizes the offer combination under which the buyer accepts both offers immediately.

Lemma 7. *The buyer should accept both offers without renegotiation if and only if $\tilde{\pi}_j \leq \theta_j(\Pi - \tilde{\pi}_i - \Pi^{-j})^+$ for $\{i, j\} = \{1, 2\}$.*

4.3. The Quotation Process

Now we analyze the suppliers' quotation process. Given Lemma 6, a natural question is whether supplier i can assure the acceptance of his offer by submitting a small positive offer. As we will see, the answer is positive in the regular case, but negative in the degenerate case. This statement holds for both the one-shot simultaneous RFQ process and the iterative quotation process.

Lemma 8. *In the regular case, there exists $\delta > 0$ such that when supplier i quotes $\tilde{\pi}_i < \delta$, his offer is accepted immediately* $(i = 1, 2)$.

While in the regular case suppliers can secure positive profits by submitting a sufficiently low quotation, this is not true in the degenerate case. For instance, Figure 8(c) is a degenerate case because $\Pi^{-1} \geq (1 - \theta_2)\Pi + \theta_2\Pi^{-2}$ $(0.9\Pi \geq (1 - 0.5)\Pi + 0.5 \times 0.8\Pi)$. When $\tilde{\pi}_1 = 0.1$, A_1R_2 renders the optimal profit for the buyer, and supplier 2's profit is zero independent of his offer (even though supplier 2's offer impacts the buyer and supplier 1's profit).

LEMMA 9. *Suppose that the buyer's optimal acceptance/rejection strategy to* $(\tilde{\pi}_1, \tilde{\pi}_2)$ *is to first contract with supplier i and then reject supplier j's offer and negotiate with him, and supplier j's resulting profit is positive. Under the iterative quotation process, supplier i's optimal response is to keep his current offer price and supplier j can increase his final profit by reducing his offer price.*

We establish Lemma 9 under the iterative quotation process, which implies that under the one-shot simultaneous RFQ process, for any quotations outside region A_1A_2, the supplier who is the Stackelberg follower in the bargaining sequence would like to deviate. In the regular case, Lemma 8 shows that the suppliers can secure positive profits in equilibrium. Therefore, in the regular case, Lemma 9 implies that if a supplier is the Stackelber follower, his best response is to reduce his quotation to be the Stackelberg leader or to region A_1A_2 where the buyer accepts both quotations immediately. Otherwise, if the reduced quotation still keeps the supplier in the follower position, the best response of the leading supplier is to keep his quotation unchanged and terminate the quotation process. The quotation process may end if the follower supplier accepts his position in the bargaining sequence, which however contradicts to Lemma 9. That is, both suppliers would like to be the Stackelberg leader during the negotiation process in the regular case. To establish Lemma 9 under the iterative quotation process is challenging, because we need to show that when a Stackelberg follower reduces his quoted price, he not only gains in a myopic fashion, but also obtains a strictly higher profit that takes the competing supplier's future responses into consideration.

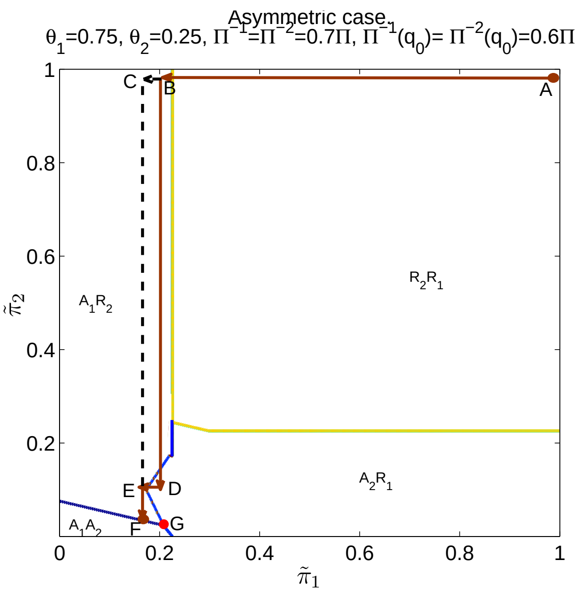


Figure 10 **An illustration of the quotation process.**

Replicating the setting of Figure 7(a), Figure 10 provides two possible quotation paths for the iterative quotation process. Suppose the initial quotation point is A with very high quotations, the buyer's optimal strategy is then R_2R_1. Supplier 2, being the Stackelberg leader, prefers to end the quotation process immediately by Lemma 9. In contrast, Supplier 1 wants to reduce his quotation and take over the leader position. He can do so by moving the quotation point to B, under which the buyer's optimal strategy is A_1R_2. By Lemma 9, it is in supplier 2's best interest to reduce his quotation and take over the leader position. If he responds by moving the quotation point to D, supplier 1 then has to respond by further lowering his quotation and moving to point E, the left-most point on the boundary of regions A_1R_2 and A_2R_1. This iterative quotation process will end at point F on the boundary of region A_1A_2, where the buyer accepts both suppliers' offers. Alternatively, if supplier 1 lowers his quotation from point A to point C in region A_1R_2, supplier 2 then lowers his quotation to point F on the boundary of region A_1A_2. Both iterative quotation processes (A-B-D-E-F and A-C-F) end at point F. In Section 4.4, we establish that point F is the unique equilibrium outcome of the iterative quotation process for this example.

4.4. The Equilibrium Outcome

Building onto Lemmas 8 and 9, we establish the buyer's equilibrium quotation acceptance/rejection strategies in the regular case.

THEOREM 4. *In equilibrium, the buyer accepts both offers in the regular case.*

Theorem 4 indicates that in the regular case, suppliers will reduce their prices to the point that the buyer accepts both offers immediately. This statement holds for both the one-shot simultaneous RFQ process and the iterative quotation process. If the buyer does not accept both offers in equilibrium, the buyer will work with the leading supplier on the exclusion clause to extract the follower's profit. The threat of exclusion clauses results in relatively low quotation prices in equilibrium in which neither further bargaining nor actual exclusion clauses need to be worked out.

THEOREM 5. *In equilibrium, the buyer's profit with the quotation process is no less than the profit of the most favorable bargaining outcome in the regular case.*

Theorem 5 indicates that the quotation process is weakly superior to the bilateral bargaining process without quotations in the regular case. A natural question is whether we simply recover the most favorable bargaining outcome through the quotation process. We show that the statement is true under the one-shot simultaneous RFQ process if an equilibrium exists.

THEOREM 6. *If an equilibrium exists under the one-shot simultaneous RFQ process in the regular case, the buyer's equilibrium profit is the profit of the most favorable bargaining outcome.*

The one-shot simultaneous RFQ process is highly desirable because it recovers the most favorable bargaining outcome. Nevertheless, Figure 10 illustrates an example under which no Nash equilibrium exists when both suppliers are asked to submit one quotation simultaneously. Both suppliers would like their offers to be accepted immediately, but in region A_1A_2, at least of one of the suppliers has an incentive to inflate the quotation. Specifically, at the most favorable bargaining outcome (i.e., point G at the upper-right corner of region A_1A_2), given supplier 1's quotation, supplier 2 has the incentive to inflate his quotation and obtain a higher profit by shifting into region A_2R_1. In other words, if an equilibrium does not exist under the one-shot simultaneous RFQ process in the regular case, one supplier can deviate from the most favorable outcome by raising his quotation and making the other as the Stackelberg follower in the bargaining sequence. This is driven by the fact that the buyer's optimal negotiation sequence decision is not a threshold policy as shown in Figure 9.

An iterative quotation process that gradually reduces the quotation prices assures the existence of an equilibrium. We further observe that if a supplier can benefit from deviating from the most favorable outcome under the one-shot simultaneous RFQ process in the regular case, he obtains strictly better equilibrium profit under the iterative quotation process than that of the most favorable outcome. This is because this supplier can force his opponent to commit a quotation which is strictly less than that under the most favorable outcome. As a result, the buyer and this supplier gain higher profits at the expense of the other supplier. Figure 10 provides an example in which the iterative quotation process leads to a strictly better outcome than the most favorable bargaining outcome for the buyer. Point F is the equilibrium of the iterative quotation process, which provides a buyer's profit higher than that of point G, the most favorable bargaining outcome.

The key to establishing Point F as the unique equilibrium is to recognize that supplier 1's equilibrium profit must equal his quotation at Point E ($(\tilde{\pi}_1, \tilde{\pi}_2) = (0.173, 0.1)$) as shown in Figure 10. We first show that supplier 1 can secure a profit of 0.173 by quoting $\tilde{\pi}_1 = 0.173$. When $\tilde{\pi}_1 = 0.173$, the buyer's optimal strategy is either A_1A_2 or A_1R_2, and supplier 1's quotation is always accepted immediately as described by Lemma 8, and his profit is no less than 0.173. To show that supplier 1 will earn no more than 0.173, we first show that an equilibrium of the iterative quotation process must be on the (northeast) boundary of region A_1A_2 because the buyer would continue to accept both offers if the suppliers further reduce the quotations. The two suppliers thus play the quotation game in the spirit of a zero-sum game, and it is the best interest of supplier 2 to limit the profit of supplier 1. We now show that supplier 2 can limit supplier 1's profit to 0.173 by quoting $\tilde{\pi}_2 = 0.1$. When supplier 1's quotation is greater than 0.173, the buyer's optimal strategy is A_2R_1, and it will be in supplier 1's best interest to reduce his quotation. When supplier 1's quotation is no more than 0.173, supplier 2 can respond by further lowering his quotation to the (northeast) boundary of region A_1A_2, thus locking in supplier 1's profit at his quotation.

Therefore, supplier 1's equilibrium profit is 0.173. In the iterative quotation process, supplier 1 may foresee this future and commit to an early quotation at this profit (e.g., Point C in Figure

10) or supplier 2 may force supplier 1 to make such a quotation (e.g., from Point D to Point E in Figure 10). In both cases, the equilibrium outcome of the iterative quotation process is Point F.

While the quotation process is superior to the bilateral bargaining process without quotations in the regular case, this dominance conclusion does not carry over to the degenerate case, in which a potential equilibrium may fall outside of region A_iA_j. In the degenerate case, one supplier may only obtain his reservation utility (i.e., zero) and thus cannot increase his profit by offering a lower quotation. Therefore, he has no incentive to compete aggressively in the quotation stage. An example is offered by Figure 8(c). Recall that supplier 1 can secure a profit of at least 0.1 by setting $\tilde{\pi}_1 = 0.1$. It is readily verified that when $\tilde{\pi}_1 = 0.1$, there are two sets of equilibria under the one-shot simultaneous RFQ process. One is $\tilde{\pi}_2 = 0$, which corresponds to the most favorable outcome with supplier 2's profit as zero, while the other is $\tilde{\pi}_2 \geq 0.2$ under which supplier 2 also gets zero profit. When $\tilde{\pi}_1 = 0.1$ and $\tilde{p}_2 \in (0, 0.2)$, supplier 1 would obtain a higher profit by inflating his quotation. Notice that in the second set, supplier 2 has no incentive to reduce the quotation. This observation implies that in the degenerate case, both the iterative quotation process and the one-shot simultaneous RFQ process may lead to multiple equilibrium profit allocations similar to that under the simultaneous bilateral bargaining setting.

5. Demand Uncertainty and Cost-plus Contracts

So far, we have assumed that all the players know the market conditions and the associated first-best procurement quantities (q_1^o, q_2^o). In this section, we show that our main insights continue to hold when the order quantities depend on the demand state.

Let ξ stand for the random demand state. We denote the supply chain profit as $\Pi(q_1, q_2|\xi) \equiv V(q_1, q_2|\xi) - (C_1(q_1) + C_2(q_2))$, which is assumed to be continuous in ξ on its compact support. The supply chain profit and costs with no procurement are normalized to 0 (i.e., $\Pi(0, 0|\xi) = V(0, 0|\xi) = C_1(0) = C_2(0) = 0$). Similarly, We use $\Pi^{-i}(q_j|\xi)$ to denote the supply chain profit when supplier i fails to participate (i.e., $q_i = 0$) and the buyer procures q_j units exclusively from supplier j under the demand state ξ ($\{i, j\} = \{1, 2\}$). We assume that the buyer needs to contract with the suppliers before the demand uncertainty is resolved. After signing the contract(s), ξ is realized, and the demand uncertainty is resolved. When the buyer procures from both suppliers, $\pi_0|_{\mathcal{C}_1, \mathcal{C}_2, \xi} = V(q_1, q_2|\xi) - (T_1(q_1) + T_2(q_2))$ and $\pi_i|_{\mathcal{C}_1, \mathcal{C}_2, \xi} = T_i(q_i) - C_i(q_i)$ for demand state ξ. When the buyer exclusively procures from supplier j, the buyer and supplier j then make a profit of $\pi_0^{-i}|_{\mathcal{C}_j, \xi} = \Pi^{-i}(q_j|\xi) - (T_j^{-i}(q_j) - C_j(q_j))$ and $\pi_j^{-i}|_{\mathcal{C}_j, \xi} = T_j^{-i}(q_j) - C_j(q_j)$, respectively. The buyer makes her optimal sourcing decision and chooses optimal procurement quantities based on contracts C_1 and C_2. As before, we assume that $(q_1^o, q_2^o|\xi) \equiv \arg\max_{q_1, q_2} \Pi(q_1, q_2|\xi) > 0$ for all ξ, and $E_\xi[\pi_j^{-i}|_{C_j, \xi}] \geq 0$. (The same conclusions hold under the alternative assumption $\pi_j^{-i}|_{C_j, \xi} \geq 0$ for all ξ.)

Proposition 4. *The equilibrium solution* $\boldsymbol{\pi}^* \equiv \boldsymbol{\pi}(\min\{\Pi^{-1}, (1-\theta_2)\Pi + \theta_2\Pi^{-2}\}, \min\{\Pi^{-2}, (1-\theta_1)\Pi + \theta_1\Pi^{-1}\})$ *offers the buyer the maximum profit among all the simultaneous bilateral bargaining equilibria, where* $\Pi \equiv E_\xi[\max_{q_1,q_2>0} \Pi(q_1,q_2|\xi)]$ *and* $\Pi^{-i} \equiv E_\xi[\max_{q_j} \Pi^{-i}(q_j)]$ *for* $\{i,j\} = \{1,2\}$.

Proposition 4 states that, after redefining the supply chain profits by taking expectation over ξ, we can identify the most favorable bargaining outcome under simultaneous bilateral bargaining using the expressions in Proposition 2. Specifically, the most favorable bargaining profits can be implemented when the contract type for both schemes is cost-plus-fixed-fee. Under this case, the supplier's contract and its exclusion clause can be simply represented by two parameters-the supplier's profit and her profit as an exclusive supplier, and the original analysis continues to hold.

With demand uncertainty, we can extend the main insights if the buyer requests CPFF contracts and asks the suppliers to quote their required margins iteratively over a quantity range (Lovejoy 2010b). When $\frac{\partial^2 \Pi(q_1,q_2|\xi)}{\partial q_1 \partial q_2} < 0$ for all ξ, we have

Corollary 2. *In the regular case, the buyer accepts both offers immediately in equilibrium.*

Corollary 3. *In the regular case, the buyer's profit with the quotation process is no less than the profit of the most favorable bargaining outcome.*

6. Conclusion

In this paper, we model an endogenous bargaining process based on quotations when a buyer dual sources from substitutable suppliers. Inspired by the practice, we allow contracts to have exclusion clauses, and show that a quotation process would help the buyer to reach an equilibrium that weakly dominates the most favorable bargaining outcome under the simultaneous bilateral bargaining setting in the regular case. Even in a full information setting, the buyer should leverage the quotation process to induce competition between suppliers, as both strive for the Stackelberg leader position in the bargaining process.

Solicitation provisions and contract clauses are important features of real-life procurement processes and contracts. As Coase (1992) commented in his 1991 Nobel Prize Lecture, "The process of contracting needs to be studied in a real world setting." We hope that our study constitutes a step towards this noble and challenging task. We foresee three streams of future research: on other commonly observed clauses, such as the most favored customer clause and the right of first refusal clause; on provisions and more detailed contracting processes; and on asymmetric information settings.

References

P. Aghion and P. Bolton. Contracts as a barrier to entry. *American Economic Review*, 28(3):388–401, 1987.

AMCP. Maintaining the affordability of the prescription drug benefit. *Academy of Managed Care Pharmacy*, http://www.amcp.org/WorkArea/DownloadAsset.aspx?id=9299, 2012.

J. J. Anton and D. A. Yao. Coordination in split award auctions. *The Quarterly Journal of Economics*, 107 (2):681–707, 1992.

G. Aydin and H. S. Heese. Bargaining for an assortment. *Management Science*, 61(3):542–559, 2015.

P. M. Barrett, R. Langreth, and J. Paton. The crazy math behind drug prices. *Bloomberg Businessweek*, July 3:14–16, 2017.

D. R. Beil. Procurement auctions. *INFORMS Tutorials in Operations Research*, pages 248–259, 2010.

R. N. Boute and J. A. Van Mieghem. Global dual sourcing and order smoothing: The impact of capacity and lead times. *Management Science*, 61(9):2080–2099, 2014.

A. Brandenburger and H. Stuart. Biform games. *Management science*, 53(4):537–549, 2007.

G. P. Cachon. Supply chain coordination with contracts. *Handbooks in Operations Research and Management Science*, 11:227–339, 2003.

A. Chaturvedi and V. Martínez-de Albéniz. Optimal procurement design in the presence of supply risk. *Manufacturing & Service Operations Management*, 13(2):227–243, 2011.

R. H. Coase. The institutional structure of production. *American Economic Review*, 82(4):713–719, 1992.

A. Collard-Wexler, G. Gowrisankaran, and R. S. Lee. "Nash-in-Nash" bargaining: A microfoundation for applied work. Technical report, National Bureau of Economic Research, 2017.

C. Davidson. Multiunit bargaining in oligopolistic industries. *Journal of Labor Economics*, 6(3):397–422, 1988.

Q. Feng and L. X. Lu. The strategic perils of low cost outsourcing. *Management Science*, 58(6):1196–1210, 2012.

Q. Feng and L. X. Lu. Supply chain contracting under competition: Bilateral bargaining vs. stackelberg. *Production and Operations Management*, 22(3):661–675, 2013a.

Q. Feng and L. X. Lu. The role of contract negotiation and industry structure in production outsourcing. *Production and Operations Management*, 22(5):1299–1319, 2013b.

Q. Feng, G. Lai, and L. X. Lu. Dynamic bargaining in a supply chain with asymmetric demand information. *Management Science*, 61(2):301–315, 2014.

H. Gurnani and M. Shi. A bargaining model for a first-time interaction under asymmetric beliefs of supply reliability. *Management Science*, 52(6):865–880, 2006.

H. Horn and A. Wolinsky. Bilateral monopolies and incentives for merger. *The RAND Journal of Economics*, 19(3):408–419, 1988.

V. N. Hsu, G. Lai, B. Niu, and W. Xiao. Leader-based collective bargaining: Cooperation mechanism and incentive analysis. *Manufacturing & Service Operations Management*, 19(1):72–83, 2016.

B. Hu and A. Qi. Optimal procurement in assembly supply chains. *Manufacturing & Service Operations Management*, Forthcoming, 2017.

W. T. Huh and K. S. Park. A sequential auction-bargaining procurement model. *Naval Research Logistics*, 57(1):13–32, 2010.

P. Kouvelis, Y. Xiao, and N. Yang. PBM competition in pharmaceutical supply chain: Formulary design and drug pricing. *Manufacturing & Service Operations Management*, 17(4):511–526, 2015.

V. Krishna and R. Serrano. Multilateral bargaining. *The Review of Economic Studies*, 63(1):61–80, 1996.

R. Langreth. More medicine goes off limits in drug-price showdown. *Bloomberg*, http://www.bloomberg.com/news/articles/2014-11-25/more-medicine-goes-off-limits-in-drug-price-showdown , 2014.

C. Li and Z. Wan. Supplier competition and cost improvement. *Management Science*, 63(8):2460–2477, 2017.

W. S. Lovejoy. Bargaining chains. *Management Science*, 56(12):2282–2301, 2010a.

W. S. Lovejoy. Conversations with supply chain managers. *Ross School of Business Paper*, 1145, 2010b.

L. M. Marx and G. Shaffer. Rent shifting and efficiency in sequential contracting. *University of Rochester, Working Paper*, 2002.

L. M. Marx and G. Shaffer. Rent shifting and the order of negotiations. *International Journal of Industrial Organization*, 25(5):1109–1125, 2007.

L. M. Marx and G. Shaffer. Break-up fees and bargaining power in sequential contracting. *International Journal of Industrial Organization*, 28(5):451–463, 2010.

J. P. Monahan. A quantity discount pricing model to increase vendor profits. *Management Science*, 30(6): 720–726, 1984.

S. Muto, M. Nakayama, J. Potters, and S. Tijs. On big boss games. *The Economic Studies Quarterly*, 39 (4):303–321, 1988.

M. Nagarajan and Y. Bassok. A bargaining framework in supply chains: The assembly problem. *Management Science*, 54(8):1482–1496, 2008.

M. Nagarajan and G. Sošić. Game-theoretic analysis of cooperation among supply chain agents: review and extensions. *European Journal of Operational Research*, 187(3):719–745, 2008.

J. F. Nash Jr. The bargaining problem. *Econometrica*, 18(2):155–162, 1950.

M. J. Osborne and A. Rubinstein. *Bargaining and Markets.* Academic Press, San Diego, CA., 1990.

E. L. Plambeck and T. A. Taylor. Sell the plant? the impact of contract manufacturing on innovation, capacity, and profitability. *Management Science*, 51(1):133–150, 2005.

N. Raycraft. Healthcare procurement: Recent trends in the world of buying for hospitals and other healthcare facilities. *PurchasingB2B*, http://www.purchasingb2b.ca/features/healthcare-procurement/, 2016.

T. Reinke. PBMs just say no to some drugs–but not to others. *Managed Care*, 24(4):24–25, 2015.

W. P. Rogerson. An economic framework for analyzing DoD profit policy. *Rand National Defense Research Institute Report R-3860-PA&E, Santa Monica, CA*, 1992.

A. Rubinstein. Perfect equilibrium in a bargaining model. *Econometrica*, 50(1):97–109, 1982.

I. Smolinski. Specialty formulary exclusions gaining traction. *Morning Consult*, https://morningconsult.com/opinions/specialty-formulary-exclusions-gaining-traction/ , 2013.

B. Tomlin and Y. Wang. On the value of mix flexibility and dual sourcing in unreliable newsvendor networks. *Manufacturing & Service Operations Management*, 7(1):37–57, 2005.

A. A. Tsay. The quantity flexibility contract and supplier-customer incentives. *Management Science*, 45(10): 1339–1358, 1999.

J. A. Van Mieghem. Coordinating investment, production, and subcontracting. *Management Science*, 45(7): 954–971, 1999.

Z. Wan and D. R. Beil. RFQ auctions with supplier qualification screening. *Operations Research*, 57(4): 934–949, 2009.

Z. Wan and D. R. Beil. Bid-taker power and supply base diversification. *Manufacturing & Service Operations Management*, 16(2):300–314, 2014.

M. L. Weitzman. Efficient incentive contracts. *The Quarterly Journal of Economics*, 94(4):719–730, 1980.

L. Xin and D. A. Goldberg. Asymptotic optimality of tailored base-surge policies in dual-sourcing inventory systems. *Management Science*, Forthcoming, 2016.

Z. Yang, G. Aydin, V. Babich, and D. R. Beil. Using a dual-sourcing option in the presence of asymmetric information about supplier reliability: Competition vs. diversification. *Manufacturing & Service Operations Management*, 14(2):202–217, 2012.

APPENDICES

EC.1. Proofs.

THEOREM 1. *For each single-sourcing equilibrium under the simultaneous bargaining setting, we can find a Pareto-improving, dual-sourcing equilibrium. Moreover, the sequential bargaining setting with given quotations leads to dual-sourcing in equilibrium.*

Proof of Theorem 1. Consider the equilibrium contract $(\hat{\mathcal{C}}_1, \hat{\mathcal{C}}_2)$ with $\hat{\mathcal{C}}_i = (\hat{T}_i, \hat{T}_i^{-j})$ for $\{i,j\} = \{1,2\}$. We allow $\mathcal{C}_i$ to be $\emptyset$ to simplify the presentation. Under this case, $\hat{T}_i(q) = \hat{T}_i^{-j}(q) = \infty$ for all q. Denote $\hat{\pi}_0$, $\hat{\pi}_i$, and $\hat{\pi}_j$ be the equilibrium profit of the buyer, supplier i, and supplier j, respectively.

We first show that under the simultaneous bargaining setting, for each single-sourcing equilibrium, we can construct a Pareto-improving, dual-sourcing equilibrium. WLOG, suppose supplier j is excluded in equilibrium (therefore, $\hat{\pi}_j = 0$).

It is readily verified that the bargaining between the buyer and supplier i maximizes the supply chain profit without supplier j, and the buyer and supplier i split the gain over $\pi_0^{-i}|_{\hat{\mathcal{C}}_j}$ according to their bargaining power. That is, $\hat{\pi}_0 = \theta_i \pi_0^{-i}|_{\hat{\mathcal{C}}_j} + (1-\theta_i)\Pi^{-j}$ and $\hat{\pi}_i = \theta_i(\Pi^{-j} - \pi_0^{-i}|_{\hat{\mathcal{C}}_j})$. We now propose contract pair $(\mathcal{C}_1, \mathcal{C}_2)$ that results in a Pareto-improving equilibrium under dual-sourcing.

We consider two possibilities.

If $\theta_i \pi_0^{-i}|_{\hat{\mathcal{C}}_j} + (1-\theta_i)\Pi \le \Pi^{-j}$, let $\mathcal{C}_i = (T_i, T_i^{-j})$, where $T_i(q_i) = C_i(q_i) + \theta_i(\Pi - \pi_0^{-i}|_{\hat{\mathcal{C}}_j})$ and $T_i^{-j}(q_i^{-j}) = C_i(q_i^{-j}) + \Pi^{-j} - (\theta_i \pi_0^{-i}|_{\hat{\mathcal{C}}_j} + (1-\theta_i)\Pi)$, and let $\mathcal{C}_j = (T_j, T_j^{-i})$, where $T_j(q_j) = C_j(q_j)$ and $T_j^{-i}(q_j^{-i}) = \hat{T}_j^{-i}(q_j^{-i})$.

Notice that because $T_j^{-i}(q_j^{-i}) = \hat{T}_j^{-i}(q_j^{-i})$, $\pi_0^{-i}|_{\hat{\mathcal{C}}_j} = \pi_0^{-i}|_{\mathcal{C}_j}$. Because $T_j(q_j) = C_j(q_j)$ and supplier j makes profit $\pi_j = 0$, formulation [Simu-i] is feasible. In equilibrium, the bargaining between the buyer and supplier i maximizes the supply chain profit, and the buyer and supplier i split the gain over $\pi_0^{-i}|_{\hat{\mathcal{C}}_j}$ according to their bargaining power. That is, $\pi_0 = \theta_i \pi_0^{-i}|_{\hat{\mathcal{C}}_j} + (1-\theta_i)\Pi$ and $\pi_i = \theta_i(\Pi - \pi_0^{-i}|_{\hat{\mathcal{C}}_j})$. Notice that, given contract $\mathcal{C}_j$, contract $\mathcal{C}_i$ is the optimal response and induces the aforementioned profit because $T_i(q_i) = C_i(q_i) + \theta_i(\Pi - \pi_0^{-i}|_{\hat{\mathcal{C}}_j})$.

We now show that given contract $\mathcal{C}_i$, contract $\mathcal{C}_j$ is the optimal response. We first show that formulation [Simu-j] is feasible. Given contract $\mathcal{C}_i$, $\pi_0^{-j}|_{\hat{\mathcal{C}}_i} = \theta_i \pi_0^{-i}|_{\hat{\mathcal{C}}_j} + (1-\theta_i)\Pi$ because $T_i^{-j}(q_i^{-j}) = C_i(q_i^{-j}) + \Pi^{-j} - (\theta_i \pi_0^{-i}|_{\hat{\mathcal{C}}_j} + (1-\theta_i)\Pi)$ and supplier i secures a constant profit under the contingent payment scheme and the buyer would choose the procurement quantity that maximizes the single-sourcing supply chain profit. Because $T_i(q_i) = C_i(q_i) + \theta_i(\Pi - \pi_0^{-i}|_{\hat{\mathcal{C}}_j})$ and supplier i makes a profit $\pi_i = \theta_i(\Pi - \pi_0^{-i}|_{\hat{\mathcal{C}}_j})$, $\pi_0^{-j}|_{\hat{\mathcal{C}}_i} + \pi_i = \Pi$. Therefore, formulation [Simu-j] is feasible. In equilibrium, supplier j makes zero profit and the buyer's profit is $\pi_0^{-j}|_{\hat{\mathcal{C}}_i}$, this can be implemented by contract $\mathcal{C}_j$.

Therefore, $(\mathcal{C}_1,\mathcal{C}_2)$ is a dual-sourcing equilibrium. Furthermore, in this equilibrium, supplier j maintains zero profit and supplier i and the buyer have strictly higher profit due to $\Pi > \Pi^{-j}$ (Assumption 1). Therefore, we find a Pareto-improving, dual-sourcing equilibrium.

If $\theta_i\pi_0^{-i}|_{\hat{\mathcal{C}}_j} + (1-\theta_i)\Pi > \Pi^{-j}$, let $\mathcal{C}_i = (T_i, T_i^{-j})$, where $T_i(q_i) = C_i(q_i) + \theta_i\frac{(1-\theta_j)\Pi+\theta_j\Pi^{-j}-\pi_0^{-i}|_{\hat{\mathcal{C}}_j}}{1-\theta_i\theta_j}$ and $T_i^{-j}(q_i^{-j}) = C_i(q_i^{-j})$, and let $\mathcal{C}_j = (T_j, T_j^{-i})$, where $T_j(q_j) = C_j(q_j) + \theta_j\frac{(1-\theta_i)\Pi+\theta_i\pi_0^{-i}|_{\hat{\mathcal{C}}_j}-\Pi^{-j}}{1-\theta_i\theta_j}$ and $T_j^{-i}(q_j^{-i}) = \hat{T}_j^{-i}(q_j^{-i})$. It is readily verified that $(\mathcal{C}_1,\mathcal{C}_2)$ is a dual-sourcing equilibrium that maximizes the supply chain profit. Moreover, it is straightforward to show $\pi_0^{-i}|_{\mathcal{C}_j} = \pi_0^{-i}|_{\hat{\mathcal{C}}_j}$ and $\pi_0^{-j}|_{\mathcal{C}_j} = \pi_0^{-j}$. In equilibrium, the players' profit is $\pi_0 = \frac{(1-\theta_i)(1-\theta_j)\Pi+\theta_i(1-\theta_j)\pi_0^{-i}|_{\hat{\mathcal{C}}_j}+(1-\theta_i)\theta_j\Pi^{-j}}{1-\theta_i\theta_j}$, $\pi_i = \theta_i\frac{(1-\theta_j)\Pi+\theta_j\Pi^{-j}-\pi_0^{-i}|_{\hat{\mathcal{C}}_j}}{1-\theta_i\theta_j}$, and $\pi_j = \theta_j\frac{(1-\theta_i)\Pi+\theta_i\pi_0^{-i}|_{\hat{\mathcal{C}}_j}-\Pi^{-j}}{1-\theta_i\theta_j}$, all of which is strictly improved. For detailed algebra, please refers to the analysis of Proposition 1.

Therefore, under the simultaneous bargaining setting, for each single-sourcing equilibrium, we can construct a Pareto-improving, dual-sourcing equilibrium.

We now consider the sequential bargaining setting with given quotations. WLOG, let us assume that the buyer first negotiates with supplier i and then supplier j. The previous analysis on the simultaneous bargaining setting implies that single-sourcing with supplier i cannot be an equilibrium solution because the profit of both the buyer and supplier i can be strictly higher by agreeing to the aforementioned contract $\mathcal{C}_i$, rejecting supplier j's quotation, and inducing contract $\mathcal{C}_j$. Therefore, the buyer must source from supplier j in equilibrium.

We now show that the buyer must source from supplier i in equilibrium for the sequential bargaining setting with given quotations. If the buyer sources exclusively from supplier j, it is readily verified that the bargaining between the buyer and supplier j maximizes the supply chain profit without supplier i, and the buyer and supplier j split the gain over $\pi_0^{-j}|_{\hat{\mathcal{C}}_i}$ according to their bargaining power. That is, $\hat{\pi}_0 = \theta_j\pi_0^{-j}|_{\hat{\mathcal{C}}_i} + (1-\theta_j)\Pi^{-i}$ and $\hat{\pi}_j = \theta_j(\Pi^{-i} - \pi_0^{-j}|_{\hat{\mathcal{C}}_i})$.

We now construct contract $\mathcal{C}_i$ so that in equilibrium, $\pi_i > \hat{\pi}_i = 0$ and $\pi_0 \geq \hat{\pi}_0 > 0$; therefore, we reach a contradiction and the buyer must procure from supplier i. Let $\mathcal{C}_i = (T_i, T_i^{-j})$, where $T_i(q_i) = C_i(q_i) + \Pi - \max\{\Pi^{-i}, \Pi^{-j}\}$ and $T_i^{-j}(q_i^{-j}) = \hat{T}_i^{-j}(q_i^{-j})$. It is readily verified that after signing contract $\mathcal{C}_i$, if the buyer rejects supplier j's quotation, the resulting profit of the players is $\pi_0 = \theta_j\pi_0^{-j}|_{\hat{\mathcal{C}}_i} + (1-\theta_j)\max\{\Pi^{-i}, \Pi^{-j}\} \geq \hat{\pi}_0$, $\pi_i = \Pi - \max\{\Pi^{-i}, \Pi^{-j}\} > 0$, and $\pi_j = \theta_j(\max\{\Pi^{-i}, \Pi^{-j}\} - \pi_0^{-j}|_{\hat{\mathcal{C}}_i}) \geq 0$, respectively; if the buyer accepts supplier j's quotation, she must obtain a (weakly) higher profit and supplier i's profit remains $\pi_i = \Pi - \max\{\Pi^{-i}, \Pi^{-j}\}$. Therefore, we reach a contradiction and the buyer must procure from supplier i. For detailed algebra, please refer to the analysis of Lemma 1.

Therefore, the sequential bargaining setting with given quotations leads to dual-sourcing in equilibrium. $\square$

THEOREM 2. *The dual-sourcing equilibrium procurement quantities are* (q_1^o, q_2^o) *under both simultaneous bargaining setting and sequential bargaining settings with given quotations. That is, the maximum supply chain profit is achieved in a dual-sourcing equilibrium.*

Proof of Theorem 2. Consider the equilibrium contract $(\hat{\mathcal{C}}_1, \hat{\mathcal{C}}_2)$ with $\hat{\mathcal{C}}_i = (\hat{T}_i, \hat{T}_i^{-j})$ for $\{i,j\} = \{1,2\}$. By Theorem 1, we focus on the dual-sourcing equilibrium and denote $(\hat{q}_1, \hat{q}_2) = (q_1^*(\hat{\mathcal{C}}_1, \hat{\mathcal{C}}_2), q_2^*(\hat{\mathcal{C}}_1, \hat{\mathcal{C}}_2))$ be the equilibrium procurement quantities. Denote $\hat{\pi}_0$, $\hat{\pi}_1$, and $\hat{\pi}_2$ be the equilibrium profit of the buyer, supplier 1, and supplier 2, respectively. Let $\hat{\Pi} \equiv \Pi(\hat{q}_1, \hat{q}_2) = \hat{\pi}_0 + \hat{\pi}_1 + \hat{\pi}_2$ be the equilibrium supply chain profit.

We prove this theorem by contradiction. Suppose that $(\hat{q}_1, \hat{q}_2) \neq (q_1^o, q_2^o)$. By the strict concavity of Π (Assumption 1), we define functions $q_1(q_2) \equiv \arg\max_q \Pi(q, q_2)$ and $q_2(q_1) \equiv \arg\max_q \Pi(q_1, q)$. That is, $q_i(q_j)$ is the system-wide optimal order quantity to supplier i given the order quantity to supplier j is fixed to q_j. Because $(\hat{q}_1, \hat{q}_2) \neq (q_1^o, q_2^o)$, either $q_1(\hat{q}_2) \neq \hat{q}_1$ or $q_2(\hat{q}_1) \neq \hat{q}_2$ due to the strict concavity.

First, we consider the simultaneous bargaining setting. WLOG, we assume $q_1(\hat{q}_2) \neq \hat{q}_1$. By the definition of $q_1(\hat{q}_2)$, $\Pi' \equiv \Pi(q_1(\hat{q}_2), \hat{q}_2) > \hat{\Pi}$. Now consider $\mathcal{C}_1 = (T_1, T_1^{-2})$, where $T_1(q) = C_1(q) + \hat{\pi}_1 + \varepsilon(\Pi' - \hat{\Pi})$ with $\varepsilon \in (0,1)$ and $T_1^{-2}(q_1^{-2}) = \hat{T}_1^{-2}(q_1^{-2})$. We show that given $\hat{\mathcal{C}}_2$, the profits of both supplier 1 and the buyer are strictly higher and the Nash product is strictly higher under $\mathcal{C}_1$ compared to $\hat{C}_1$. As a result, we reach a contraction that $(\hat{\mathcal{C}}_1, \hat{\mathcal{C}}_2)$ is an equilibrium.

Under $(\mathcal{C}_1, \hat{\mathcal{C}}_2)$, supplier 1's profit $\pi_1 = \hat{\pi}_1 + \varepsilon(\Pi' - \hat{\Pi}) > \hat{\pi}_1$ when the buyer procures from both suppliers. When ordering $(q_1(\hat{q}_2), \hat{q}_2)$, the buyer's profit is equal to $\Pi' - \pi_1 - \hat{\pi}_2 = (1-\varepsilon)(\Pi' - \hat{\Pi}) + \hat{\Pi} - \hat{\pi}_1 - \hat{\pi}_2 > \hat{\pi}_0$. Therefore, under $(\mathcal{C}_1, \hat{\mathcal{C}}_2)$, the buyer's profit π_0 is strictly higher than $\hat{\pi}_0$ because the buyer has the potential to further increase his profit by selecting procurement quantities (q_1, q_2) optimally. This also implies that the buyer procures from both suppliers under $(\mathcal{C}_1, \hat{\mathcal{C}}_2)$ because the payment amounts under contingency schemes are the same under $(\mathcal{C}_1, \hat{\mathcal{C}}_2)$ and $(\hat{\mathcal{C}}_1, \hat{\mathcal{C}}_2)$ and the buyer's profit under single-sourcing is no more than $\hat{\pi}_0$. As a result, $\pi_1|_{\mathcal{C}_1,\hat{\mathcal{C}}_2} > \pi_1|_{\hat{\mathcal{C}}_1,\hat{\mathcal{C}}_2}$, $\pi_0|_{\mathcal{C}_1,\hat{\mathcal{C}}_2} > \pi_0|_{\hat{\mathcal{C}}_1,\hat{\mathcal{C}}_2}$, and $(\pi_1|_{\mathcal{C}_1,\hat{\mathcal{C}}_2} - 0)^{\theta_1}(\pi_0|_{\mathcal{C}_1,\hat{\mathcal{C}}_2} - \pi_0^{-1}|_{\hat{\mathcal{C}}_2})^{(1-\theta_1)} > (\pi_1|_{\hat{\mathcal{C}}_1,\hat{\mathcal{C}}_2} - 0)^{\theta_1}(\pi_0|_{\hat{\mathcal{C}}_1,\mathcal{C}_2} - \pi_0^{-1}|_{\hat{\mathcal{C}}_2})^{(1-\theta_1)}$. Therefore, we reach a contradiction of the optimality of $\hat{\mathcal{C}}_1$.

We next consider the sequential bargaining setting with given quotations. WLOG, we assume that the buyer first negotiates with suppler i, and then supplier j in equilibrium ($\{i,j\} = \{1,2\}$). We must have $\hat{q}_j = q_j(\hat{q}_i)$; otherwise we can apply the analysis for the simultaneous setting and the Nash product for the bargaining between supplier j and the buyer would be strictly higher.

Therefore, $(\hat{q}_1, \hat{q}_2) \neq (q_1^o, q_2^o)$ implies that $\hat{q}_i \neq \hat{q}_i^o$. Now consider $\mathcal{C}_i = (T_i, T_i^{-j})$, where $T_i(q) = C_i(q) + \hat{\pi}_i + \varepsilon(\Pi - \hat{\Pi})$ with $\varepsilon \in (0,1)$ and $T_i^{-j}(q_i^{-j}) = \hat{T}_i^{-j}(q_i^{-j})$. We show that the profits of both supplier i and the buyer are strictly higher and that the Nash product is strictly higher under $\mathcal{C}_i$ compared to $\hat{C}_i$. As a result, we reach a contraction that $\hat{\mathcal{C}}_1$ is an equilibrium contract.

Under $\mathcal{C}_i$, supplier i's profit $\pi_i = \hat{\pi}_i + \varepsilon(\Pi - \hat{\Pi}) > \hat{\pi}_i$ when the buyer procures from both suppliers. Furthermore, because $\hat{\mathcal{C}}_i$ leads to a dual-sourcing equilibrium (Theorem 1), it is readily verified that given $\mathcal{C}_i$, the bargaining formulation between the buyer and supplier j is feasible and leads to dual-sourcing. To show that the buyer's profit also strictly increases under $\mathcal{C}_i$, we only need to consider that the buyer orders q_i^o to supplier i. Notice that supplier i will get the constant payoff under $\mathcal{C}_i$, and the buyer may potentially change order quantity to supplier i to further increase the profit. Therefore, if the contradiction can be reached at ordering q_i^o to supplier i, so does the optimal order quantity to supplier i under $\mathcal{C}_i$. When the buyer orders q_i^o to supplier i, the best response of the order quantity to supplier j is $q_j^o = q_j(q_i^o)$. Under this situation, the entire supply chain profit is Π.

We next consider whether the buyer accepts or rejects supplier j's quotation after the buyer and supplier i agree upon contract $\hat{\mathcal{C}}_i$ with ordering q_i^o.

(I) Suppose that the buyer accepts supplier j's quotation $\tilde{p}_j$ in equilibrium given $\hat{\mathcal{C}}_i$. Under this case, supplier j's profit $\hat{\pi}_j$ given $\hat{\mathcal{C}}_i$ is at least $\tilde{p}_j - C_j(q_j^o)$. After the buyer and supplier i agree upon contract $\mathcal{C}_i$, by procuring q_i^o from supplier i and accepting supplier j's quotation, the buyer's profit is equal to $\Pi - \pi_i - (\tilde{p}_j - C_j(q_j^o)) \geq \hat{\Pi} - \hat{\pi}_i - \hat{\pi}_j + (1-\varepsilon)(\Pi - \hat{\Pi}) > \hat{\pi}_0$. Therefore, the buyer's profit π_0 is strictly higher than $\hat{\pi}_0$ because the buyer also has the option of rejecting supplier j's quotation.

(II) Suppose that the buyer rejects supplier j's quotation $\tilde{p}_j$ in equilibrium given $\hat{\mathcal{C}}_i$. Under this case, it is readily verified that supplier j's profit $\hat{\pi}_j$ given $\hat{\mathcal{C}}_i$ is $\theta_j(\hat{\Pi} - \hat{\pi}_i - \pi_0^{-j}|_{\hat{\mathcal{C}}_i})$ and the buyer's profit $\hat{\pi}_0$ given $\hat{\mathcal{C}}_i$ is $\theta_j \pi_0^{-j'}|_{\hat{\mathcal{C}}_i} + (1-\theta_j)(\hat{\Pi} - \hat{\pi}_i)$ (Lemma 1). After the buyer and supplier i agree upon contract $\mathcal{C}_i$ by rejecting supplier j's quotation, the buyer's profit is equal to $\theta_j \pi_0^{-j'}|_{\mathcal{C}_i} + (1 - \theta_j)(\Pi - \hat{\pi}_i) > \theta_j \pi_0^{-j'}|_{\hat{\mathcal{C}}_i} + (1-\theta_j)(\hat{\Pi} - \hat{\pi}_i) = \hat{\pi}_0$. Therefore, the buyer's profit π_0 is strictly higher than $\hat{\pi}_0$ because the buyer also has the option of accepting supplier j's quotation. Therefore, we also reach a contradiction. $\square$

Theorem 3. *Suppose that $(\hat{\mathcal{C}}_1 = (\hat{T}_1, \hat{T}_1^{-2}), \hat{\mathcal{C}}_2 = (\hat{T}_2, \hat{T}_2^{-1}))$ is a dual-sourcing equilibrium under the simultaneous bargaining setting (sequential bargaining with given quotations) with equilibrium profit $\hat{\pi}_0, \hat{\pi}_1, \hat{\pi}_2$ for the players. $(\mathcal{C}_1 = (T_1, T_1^{-2}), \mathcal{C}_2 = (T_2, T_2^{-1}))$ is also a dual-sourcing equilibrium under the simultaneous bargaining setting (sequential bargaining with given quotations) with the same equilibrium profit when $T_1(q) = C_1(q) + \hat{\pi}_1$, $T_1^{-2}(q_1^{-2}) = C_1(q_1^{-2}) + \Pi^{-1} - \pi_0^{-2}|_{\hat{\mathcal{C}}_1}$, $T_2(q) = C_2(q) + \hat{\pi}_2$, and $T_2^{-1}(q_2^{-1}) = C_2(q_2^{-1}) + \Pi^{-1} - \pi_0^{-1}|_{\hat{\mathcal{C}}_2}$.*

Proof of Theorem 3. Notice that under $(\hat{\mathcal{C}}_1=(\hat{T}_1,\hat{T}_1^{-2}),\hat{\mathcal{C}}_2=(\hat{T}_2,\hat{T}_2^{-1}))$ and $(\mathcal{C}_1=(T_1,T_1^{-2}),\mathcal{C}_2=(T_2,T_2^{-1}))$, the buyer would obtain the same single-sourcing profit. This is because the suppliers secure a constant profit under $T_1^{-2}(q_1^{-2})=C_1(q_1^{-2})+\Pi^{-1}-\pi_0^{-2}|_{\hat{\mathcal{C}}_1}$ and $T_2^{-1}(q_2^{-1})=C_2(q_2^{-1})+\Pi^{-1}-\pi_0^{-1}|_{\hat{\mathcal{C}}_2}$, and the buyer would choose the procurement quantity that maximizes the single-sourcing supply chain profit. That is, $\pi_0^{-1}|_{\mathcal{C}_2}=\Pi^{-1}-(\Pi^{-1}-\pi_0^{-1}|_{\hat{\mathcal{C}}_2})=\pi_0^{-1}|_{\hat{\mathcal{C}}_2}$ and $\pi_0^{-2}|_{\mathcal{C}_1}=\Pi^{-2}-(\Pi^{-2}-\pi_0^{-2}|_{\hat{\mathcal{C}}_1})=\pi_0^{-2}|_{\hat{\mathcal{C}}_1}$. Because $(\hat{\mathcal{C}}_1,\hat{\mathcal{C}}_2)$ is a dual-sourcing equilibrium, $\hat{\pi}_0\geq\pi_0^{-1}|_{\hat{\mathcal{C}}_2}$ and $\hat{\pi}_0\geq\pi_0^{-2}|_{\hat{\mathcal{C}}_1}$. Furthermore, $\hat{\pi}_0+\hat{\pi}_1+\hat{\pi}_2=\Pi$ by Theorem 2. Therefore, given $(\mathcal{C}_1,\mathcal{C}_2\})$, the buyer's optimal dual-sourcing procurement strategies to procure the first-best quantities from both suppliers, which result in a profit of $\hat{\pi}_i$ for supplier i and a buyer's profit of $\Pi-\hat{\pi}_1-\hat{\pi}_2=\hat{\pi}_0$, which is no less than $\pi_0^{-1}|_{\mathcal{C}_2}$ and $\pi_0^{-2}|_{\mathcal{C}_1}$. Therefore, given $(\mathcal{C}_1,\mathcal{C}_2)$, we obtain the same players' payoffs.

We now show that $(\mathcal{C}_1,\mathcal{C}_2)$ is an equilibrium under the simultaneous bargaining setting. That is, given $\mathcal{C}_1$, $\mathcal{C}_2$ is the optimal response, and vice versa. Recall $(\hat{\mathcal{C}}_1,\hat{\mathcal{C}}_2)$ is a dual-sourcing equilibrium and the buyer procures the first-best quantities by Theorem 2. Therefore, we have

$$\hat{\pi}_2=\arg\max_{\pi}(\pi-0)^{\theta_2}(\Pi-\hat{\pi}_1-\pi-\pi_0^{-2}|_{\hat{\mathcal{C}}_1})^{1-\theta_2}$$

$$s.t.\ \pi\geq 0,\Pi-\hat{\pi}_1-\pi\geq\pi_0^{-2}|_{\hat{\mathcal{C}}_1}.$$

Notice that given $\mathcal{C}_1$, which secures a constant profit for supplier 1, it would be in the best interest of the buyer and supplier 2 to maximize the supply chain profit for the dual-sourcing solution. That is, the equilibrium profit π_2 must satisfy the following formulation:

$$\pi_2=\arg\max_{\pi}(\pi-0)^{\theta_2}(\Pi-\hat{\pi}_1-\pi-\pi_0^{-2}|_{\hat{\mathcal{C}}_1})^{1-\theta_2}$$

$$s.t.\ \pi\geq 0,\Pi-\hat{\pi}_1-\pi\geq\pi_0^{-2}|_{\mathcal{C}_1}.$$

Because $\pi_0^{-2}|_{\mathcal{C}_1}=\pi_0^{-2}|_{\hat{\mathcal{C}}_1}$, and the formulation is strictly concave, at equilibrium, $\pi_2=\hat{\pi}_2$, and contract $\mathcal{C}_2$ that supports the equilibrium profit is indeed the optimal response to $\mathcal{C}_1$. Similarly, contract $\mathcal{C}_1$ is the optimal response to $\mathcal{C}_2$. That is, $(\mathcal{C}_1=(T_1,T_1^{-2}),\mathcal{C}_2=(T_2,T_2^{-1}))$ is also a dual-sourcing equilibrium under the simultaneous bargaining setting.

We now show that $(\mathcal{C}_1,\mathcal{C}_2)$ is an equilibrium under sequential bargaining for given quotations. WLOG, we assume that the buyer first negotiates with suppler i, and then supplier j in equilibrium $(\{i,j\}=\{1,2\})$. Recall $(\hat{\mathcal{C}}_1,\hat{\mathcal{C}}_2)$ is a dual-sourcing equilibrium and the buyer procures the first-best quantities by Theorem 2. We first show that given $\mathcal{C}_i$, supplier j's payoff equals to $\hat{\pi}_j$.

Given $\hat{\mathcal{C}}_i$, suppose the buyer accepts supplier j's quotation. By Theorem 2, the renegotiation between the buyer and supplier j will not improve their profits because they have established the transaction price for the correct procurement quantity. Therefore, $\hat{\pi}_j = \tilde{p}_j - C_j(q_j^o)$. Suppose the buyer rejects supplier j's quotation, we have

$$\hat{\pi}_j = \arg\max_{\pi} (\pi - 0)^{\theta_j} (\Pi - \hat{\pi}_i - \pi - \pi_0^{-j}|_{\hat{\mathcal{C}}_i})^{1-\theta_j}$$
$$s.t.\ \pi \geq 0, \Pi - \hat{\pi}_i - \pi \geq \pi_0^{-j}|_{\hat{\mathcal{C}}_i}.$$

Notice that given $\mathcal{C}_i$, which secures a constant profit for supplier i, it would be in the best interest of the buyer and supplier j to maximize the supply chain profit for the dual-sourcing solution. Suppose the buyer accepts supplier j's quotation, supplier j's profit is $\pi_j = \tilde{p}_j - C_j(q_j^o)$. If the buyer rejects supplier j's quotation, the equilibrium profit π_2 must satisfy the following formulation:

$$\pi_j = \arg\max_{\pi} (\pi - 0)^{\theta_j} (\Pi - \hat{\pi}_i - \pi - \pi_0^{-j}|_{\hat{\mathcal{C}}_i})^{1-\theta_j}$$
$$s.t.\ \pi \geq 0, \Pi - \hat{\pi}_i - \pi \geq \pi_0^{-j}|_{\mathcal{C}_i}.$$

Notice that in both cases, the buyer decides the optimal acceptance/rejection strategy by maximizing her own payoff (which is equivalent to minimize supplier j's payoff). Therefore, if $\hat{\mathcal{C}}_j$ is the equilibrium contract given $\hat{\mathcal{C}}_i$, $\mathcal{C}_j$ is the equilibrium contract given $\mathcal{C}_i$.

We now show that if $\hat{\mathcal{C}}_i$ maximizes the Nash product for the bargaining between the buyer and supplier i after the buyer either accepts or rejects supplier i's quotation, $\mathcal{C}_i$ maximizes the Nash product for the bargaining between the buyer and supplier i after the buyer follows the same acceptance/rejection strategy regarding to supplier i's offer. This is true because both formulations have the identical reservation utilities for both the buyer and supplier i, $\mathcal{C}_i$ guarantees the constant profit $\hat{\pi}_i$ to supplier i, and we have shown that given $\mathcal{C}_i$, supplier j ends up with profit $\hat{\pi}_j$. Therefore, $(\mathcal{C}_1, \mathcal{C}_2)$ is an equilibrium under sequential bargaining for given quotations. $\square$

PROPOSITION 1. $\boldsymbol{\pi} = (\pi_0, \pi_1, \pi_2, \pi_0^{-1}, \pi_0^{-2})$ *is a dual-sourcing equilibrium if and only if* $\pi_0 = \frac{(1-\theta_i)(1-\theta_j)\Pi + \theta_i(1-\theta_j)\pi_0^{-i} + (1-\theta_i)\theta_j\pi_0^{-j}}{1-\theta_i\theta_j}$, $\pi_i = \theta_i \frac{(1-\theta_j)\Pi + \theta_j\pi_0^{-j} - \pi_0^{-i}}{1-\theta_i\theta_j}$, $\pi_0^{-i} \leq \Pi^{-i}$ *and* $\pi_0^{-i} \leq (1-\theta_j)\Pi + \theta_j\pi_0^{-j}$, *for* $\{i,j\} = \{1,2\}$.

Proof of Proposition 1. By solving the formulation, we obtain that $\pi_i = \theta_i(\Pi - \pi_j - \pi_0^{-i})$ for $\{i,j\} = \{1,2\}$. As a result, we can represent π_0, π_i, and π_j using π_0^{-i} and π_0^{-j}. Specifically,

$$\pi_0 = \frac{(1-\theta_i)(1-\theta_j)\Pi + \theta_i(1-\theta_j)\pi_0^{-i} + (1-\theta_i)\theta_j\pi_0^{-j}}{1-\theta_i\theta_j} \text{ and } \pi_i = \theta_i \frac{(1-\theta_j)\Pi - \pi_0^{-i} + \theta_j\pi_0^{-j}}{1-\theta_i\theta_j}.$$

The non-negativity of π_i implies that in equilibrium, $\pi_0^{-i} \leq (1-\theta_j)\Pi + \theta_j\pi_0^{-j}$. That is, an equilibrium would satisfy these two inequalities in addition to the aforementioned three equalities. It is readily verified that when all five conditions are satisfied, $\boldsymbol{\pi} = (\pi_0, \pi_1, \pi_2, \pi_0^{-1}, \pi_0^{-2})$ can be implemented by a quantity-dependent pricing contract with the exclusion clause (e.g., via cost-plus-fixed-fee contracts). $\square$

PROPOSITION 2. *The equilibrium solution* $\boldsymbol{\pi}^* \equiv \boldsymbol{\pi}(\min\{\Pi^{-1}, (1-\theta_2)\Pi + \theta_2\Pi^{-2}\}, \min\{\Pi^{-2}, (1-\theta_1)\Pi + \theta_1\Pi^{-1}\})$ *offers the buyer the maximum profit among all the dual-sourcing equilibria.*

Proof of Proposition 2. Notice that $\frac{\partial \pi_0}{\partial \pi_0^{-i}} > 0$ and $\pi_0^{-i} \le \Pi^{-i}$ for $i = 1, 2$. Therefore, if $\pi_0^{-i} \le (1-\theta_j)\Pi + \theta_j \pi_0^{-j}$ at $(\pi_0^{-1}, \pi_0^{-2}) = (\Pi^{-1}, \Pi^{-2})$ for $\{i, j\} = \{1, 2\}$, the buyer's maximum profit is obtained at $\boldsymbol{\pi}(\Pi^{-1}, \Pi^{-2})$.

Now suppose that at least one of above inequalities is violated at $(\pi_0^{-1}, \pi_0^{-2}) = (\Pi^{-1}, \Pi^{-2})$. Without loss of generality, let us assume $\Pi^{-1} \le \Pi^{-2}$. In this case, $\Pi^{-1} \le (1-\theta_2)\Pi + \theta_2\Pi^{-2}$ and therefore, $\Pi^{-2} > (1-\theta_1)\Pi + \theta_1\Pi^{-1}$. At a dual-sourcing equilibrium, $\pi_0^{-1} \le \Pi^{-1}$, $\pi_0^{-2} \le (1-\theta_1)\Pi + \theta_1\pi_0^{-1} \le (1-\theta_1)\Pi + \theta_1\Pi^{-1} < \Pi^{-2}$. Furthermore, $\Pi^{-1} \le (1-\theta_2)\Pi + \theta_2((1-\theta_1)\Pi + \theta_1\Pi^{-1}) = (1-\theta_1\theta_2)\Pi + \theta_1\theta_2\Pi^{-1}$. Thus, both inequalities are satisfied at $(\pi_0^{-1}, \pi_0^{-2}) = (\Pi^{-1}, (1-\theta_1)\Pi + \theta_1\Pi^{-1})$. The buyer's maximum profit is obtained at $\boldsymbol{\pi}(\Pi^{-1}, (1-\theta_1)\Pi + \theta_1\Pi^{-1}) = \boldsymbol{\pi}(\min\{\Pi^{-1}, (1-\theta_2)\Pi + \theta_2\Pi^{-2}\}, \min\{\Pi^{-2}, (1-\theta_1)\Pi + \theta_1\Pi^{-1}\})$. $\square$

PROPOSITION 3. *The buyer's profit under the most favorable bargaining outcome* $\boldsymbol{\pi}^*$ *is higher than that of a sequential bargaining setting without an RFQ, or in a bargaining setting that adopts CPFF contracts without contingent payment schemes in either a simultaneous or sequential manner.*

Proof of Proposition 3. By Proposition 1, the buyer's profit under the most favorable bargaining outcome is

$$\pi_0^{fav} = \frac{(1-\theta_i)(1-\theta_j)\Pi + \theta_i(1-\theta_j)\min\{\Pi^{-i}, (1-\theta_j)\Pi + \theta_j\Pi^{-j}\} + (1-\theta_i)\theta_j\min\{\Pi^{-j}, (1-\theta_i)\Pi + \theta_i\Pi^{-i}\}}{1-\theta_i\theta_j}.$$

Notice that if $\Pi^{-i} \le \Pi^{-j}$, $\Pi^{-i} \le (1-\theta_j)\Pi + \theta_j\Pi^{-j}$. We have three possible scenarios: either $\Pi^{-i} \le (1-\theta_j)\Pi + \theta_j\Pi^{-j}$ and $\Pi^{-j} \le (1-\theta_i)\Pi + \theta_i\Pi^{-i}$, or $\Pi^{-i} > (1-\theta_j)\Pi + \theta_j\Pi^{-j}$, or $\Pi^{-j} > (1-\theta_i)\Pi + \theta_i\Pi^{-i}$. These three cases are mutually exclusive and mutually exhaustive.

We now compare the buyer's profit under the most favorable bargaining outcome with her profit under a bilateral bargaining setting that adopts CPFF contracts, but without contingent payment schemes and a sequential bargaining setting without an RFQ, respectively.

I) Comparison with CPFF without contingent payment schemes

When the players adopt CPFF without contingent payment schemes, the bilateral bargaining between the buyer and supplier i in a simultaneous bargaining setting is modeled as follows:

$$\pi_i = \arg\max_{\pi}(\pi - 0)^{\theta_i}(\Pi - \pi_j - \pi - (\Pi^{-i} - \pi_j))^{1-\theta_i} \text{ for } \{i, j\} = \{1, 2\},$$

$$s.t.\ \pi \ge 0, \pi_0 = \Pi - \pi_j - \pi \ge \Pi^{-i} - \pi_j.$$

In equilibrium, supplier i's profit is $\theta_i(\Pi-\Pi^{-i})$ $(i=1,2)$. While we derive the expressions under the simultaneous bargaining setting, it is readily verified that supplier i's profit is still $\theta_i(\Pi-\Pi^{-i})$ in a sequential bargaining setting. Furthermore, if a RFQ stage precedes the actual bargaining, supplier i will demand a profit no less than $\theta_i(\Pi-\Pi^{-i})$ and the buyer will concede a profit no more than $\theta_i(\Pi-\Pi^{-i})$. Therefore, supplier i's profit is still $\theta_i(\Pi-\Pi^{-i})$ with the RFQ stage. That is, in equilibrium, the buyer's profit is $\pi_0^{NoC}=\Pi-\theta_i(\Pi-\Pi^{-i})-\theta_j(\Pi-\Pi^{-j})=(1-\theta_i-\theta_j)\Pi+\theta_i\Pi^{-i}+\theta_j\Pi^{-j}$. We consider two possibilities:

Ia) $\Pi^{-j}\le(1-\theta_i)\Pi+\theta_i\Pi^{-i}$. Under this case, the buyer's profit under the most favorable outcome is

$$\pi_0^{fav}=\frac{(1-\theta_i)(1-\theta_j)\Pi+\theta_i(1-\theta_j)\Pi^{-i}+(1-\theta_i)\theta_j\Pi^{-j}}{1-\theta_i\theta_j}.$$

Therefore, $\pi_0^{fav}>\pi_0^{NoC}\Leftrightarrow\theta_i\theta_j(1-\theta_i)\Pi+\theta_i\theta_j(1-\theta_j)\Pi>\theta_i\theta_j(1-\theta_i)\Pi^{-i}+\theta_i\theta_j(1-\theta_j)\Pi^{-j}$, which is true.

Ib) $\Pi^{-j}>(1-\theta_i)\Pi+\theta_i\Pi^{-i}$. Under this case, the buyer's profit under the most favorable outcome is

$$\pi_0^{fav}=\frac{(1-\theta_i)(1-\theta_j)\Pi+\theta_i(1-\theta_j)\Pi^{-i}+(1-\theta_i)\theta_j((1-\theta_i)\Pi+\theta_i\Pi^{-i})}{1-\theta_i\theta_j}=(1-\theta_i)\Pi+\theta_i\Pi^{-i},$$

which is greater than $(1-\theta_i-\theta_j)\Pi+\theta_i\Pi^{-i}+\theta_j\Pi^{-j}=\pi_0^{NoC}$.

II) Comparison with sequential bargaining without an RFQ

WLOG, we assume that the buyer first bargains with supplier i then supplier j. Because the sequential bargaining without an RFQ is equivalent to an RFQ with prohibitively high quotation prices, by Lemmas 3 and 5, the buyer's profit under sequential bargaining is

$$\pi_0^{seq}=\begin{cases}(1-\theta_i)(1-\theta_j)\Pi+\theta_i(1-\theta_j)\Pi^{-i}+\theta_j(1-\theta_i)\Pi^{-j}, & \text{if } \Pi\ge\frac{1}{1-\theta_i}\left(-\theta_i\Pi^{-i}+\left(1+\frac{\theta_i\theta_j}{1-\theta_j}\right)\Pi^{-j}\right);\\ (1-\theta_i)\Pi+\theta_i(1-\theta_j)\Pi^{-i}, & \text{if } \Pi\le\frac{1}{1-\theta_i}\left(\Pi^{-j}-\theta_i(1-\theta_j)\Pi^{-i}\right);\\ \Pi^{-j}, & \text{otherwise;}\end{cases}$$

IIa) $\Pi^{-i}\le(1-\theta_j)\Pi+\theta_j\Pi^{-j}$ and $\Pi^{-j}\le(1-\theta_i)\Pi+\theta_i\Pi^{-i}$. Under this case, the buyer's profit under the most favorable outcome is $\pi_0^{fav}=\frac{(1-\theta_i)(1-\theta_j)\Pi+\theta_i(1-\theta_j)\Pi^{-i}+(1-\theta_i)\theta_j\Pi^{-j}}{1-\theta_i\theta_j}$. When $\Pi\le\frac{1}{1-\theta_i}\left(-\theta_i\Pi^{-i}+\left(1+\frac{\theta_i\theta_j}{1-\theta_j}\right)\Pi^{-j}\right)$, $\pi_0^{fav}-\pi_0^{seq}\ge\pi_0^{fav}-\Pi^{-j}\ge\frac{(1-\theta_i)(1-\theta_j)\Pi+\theta_i(1-\theta_j)\Pi^{-i}+(1-\theta_i)\theta_j\Pi^{-j}}{1-\theta_i\theta_j}-\Pi^{-j}=\frac{(1-\theta_i)((1-\theta_i)\Pi+\theta_i\Pi^{-i}-\Pi^{-j})}{1-\theta_i\theta_j}\ge0$. Furthermore, when $\Pi>\frac{1}{1-\theta_i}\left(-\theta_i\Pi^{-i}+\left(1+\frac{\theta_i\theta_j}{1-\theta_j}\right)\Pi^{-j}\right)$, π_0^{seq} increases in Π in a rate of $(1-\theta_i)(1-\theta_j)$ and π_0^{fav} increases in Π in a rate of $\frac{(1-\theta_i)(1-\theta_j)}{1-\theta_i\theta_j}>(1-\theta_i)(1-\theta_j)$. Therefore, $\pi_0^{fav}>\pi_0^{seq}$.

IIb) $\Pi^{-i}>(1-\theta_j)\Pi+\theta_j\Pi^{-j}$. Under this case, the buyer's profit under the most favorable outcome is $\pi_0^{fav}=(1-\theta_j)\Pi+\theta_j\Pi^{-j}$. When $\Pi\le\frac{1}{1-\theta_i}\left(-\theta_i\Pi^{-i}+\left(1+\frac{\theta_i\theta_j}{1-\theta_j}\right)\Pi^{-j}\right)$, $\pi_0^{fav}-\pi_0^{seq}\ge\pi_0^{fav}-\Pi^{-j}\ge(1-\theta_j)\Pi+\theta_j\Pi^{-j}-\Pi^{-j}=(1-\theta_j)(\Pi-\Pi^{-j})>0$. Furthermore, when $\Pi>$

$\frac{1}{1-\theta_i}\left(-\theta_i\Pi^{-i}+\left(1+\frac{\theta_i\theta_j}{1-\theta_j}\right)\Pi^{-j}\right)$, π_0^{seq} increases in Π in a rate of $(1-\theta_i)(1-\theta_j)$ and π_0^{fav} increases in Π in a rate of $(1-\theta_j)>(1-\theta_i)(1-\theta_j)$. Therefore, $\pi_0^{fav}>\pi_0^{seq}$.

IIc) $\Pi^{-j}>(1-\theta_i)\Pi+\theta_i\Pi^{-i}$. Under this case, the buyer's profit under the most favorable outcome is $\pi_0^{fav}=(1-\theta_i)\Pi+\theta_i\Pi^{-i}$. Furthermore, $\Pi^{-j}>(1-\theta_i)\Pi+\theta_i\Pi^{-i}$ implies that $\Pi<\frac{1}{1-\theta_i}(\Pi^{-j}-\theta_i(1-\theta_j)\Pi^{-i})$ and $\pi_0^{seq}=(1-\theta_i)\Pi+\theta_i(1-\theta_j)\Pi^{-i}$. Therefore, $\pi_0^{fav}>\pi_0^{seq}$.

Therefore, the buyer achieves higher profit under the most favorable bargaining outcome. □

LEMMA 1. *Given (π_i,π_0^{-j}), the contract between supplier j and the buyer specifies $\pi_j(\pi_i,\pi_0^{-j})=\theta_j(\Pi-\pi_i-\pi_0^{-j})$ and $\pi_0(\pi_i,\pi_0^{-j})=(1-\theta_j)(\Pi-\pi_i)+\theta_j\pi_0^{-j}$ when $\Pi-\pi_i-\pi_0^{-j}\geq 0$ and excludes supplier j otherwise if the buyer rejects supplier j's offer.*

Proof of Lemma 1. When $\Pi<\pi_i+\pi_0^{-j}$, supplier j is excluded because the following formulation that characterizes the bargaining outcome between the buyer and supplier j is infeasible,

$$\pi_j=\arg\max_{\pi}(\pi-0)^{\theta_j}(\Pi-\pi_i-\pi-\pi_0^{-j})^{1-\theta_j}$$

$$s.t.\ \pi\geq 0, \pi_0=\Pi-\pi_i-\pi\geq\pi_0^{-j}.$$

When $\Pi\geq\pi_i+\pi_0^{-j}$, the optimal solution to the formulation specifies $\pi_j=\theta_j(\Pi-\pi_i-\pi_0^{-j})$; thus, $\pi_0=(1-\theta_j)(\Pi-\pi_i)+\theta_j\pi_0^{-j}$. □

LEMMA 2. *Suppose that at optimality the buyer first* **accepts** *the offer $\tilde{\pi}_i$. If the buyer* **accepts** *$\tilde{\pi}_j$, the profits of the buyer, supplier i and supplier j are $(\pi_0,\pi_i,\pi_j)=(\Pi-\tilde{\pi}_i-\tilde{\pi}_j,\tilde{\pi}_i,\tilde{\pi}_j)$. If the buyer* **rejects** *$\tilde{\pi}_j$, the profits of the buyer and supplier j are $\pi_0=\Pi-\pi_i-\pi_j$ and $\pi_j=\theta_j(\Pi-\pi_i-\Pi^{-j})^+$ respectively, where*

if $\tilde{\pi}_j\geq\theta_j(\Pi-\Pi^{-j}(q_i^o))$,

$$\pi_i=\begin{cases}\tilde{\pi}_i+\frac{\theta_i\theta_j}{1-\theta_j}(\Pi^{-j}-(\Pi^{-j}(q_i^o)-\tilde{\pi}_i)), & \textit{if } \Pi\geq\tilde{\pi}_i+\Pi^{-j}+\frac{\theta_i\theta_j}{1-\theta_j}(\Pi^{-j}-(\Pi^{-j}(q_i^o)-\tilde{\pi}_i));\\ \tilde{\pi}_i+\theta_i\theta_j(\Pi-\Pi^{-j}(q_i^o)), & \textit{if } \Pi\leq\frac{1}{1-\theta_i\theta_j}(\tilde{\pi}_i+\Pi^{-j}-\theta_i\theta_j\Pi^{-j}(q_i^o));\\ \Pi-\Pi^{-j}, & \textit{otherwise;}\end{cases}$$

if $\tilde{\pi}_j<\theta_j(\Pi-\Pi^{-j}(q_i^o))$,

$$\pi_i=\begin{cases}\tilde{\pi}_i+\frac{\theta_i(\tilde{\pi}_j-\theta_j(\Pi-\tilde{\pi}_i-\Pi^{-j}))}{1-\theta_j}, & \textit{if } \Pi\geq\Pi^{-j}+\tilde{\pi}_i+\frac{\theta_i}{1-\theta_j+\theta_i\theta_j}\tilde{\pi}_j;\\ \tilde{\pi}_i+\theta_i\tilde{\pi}_j, & \textit{if } \Pi\leq\Pi^{-j}+\tilde{\pi}_i+\theta_i\tilde{\pi}_j;\\ \Pi-\Pi^{-j}, & \textit{otherwise.}\end{cases}$$

Furthermore, both π_0 and π_j are continuous and (weakly) decreasing functions of $\tilde{\pi}_i$ and $\tilde{\pi}_j$ if the buyer rejects $\tilde{\pi}_j$.

Proof of Lemma 2. Suppose that the buyer accepts both offers, the reservation utility of the buyer is $\pi - \tilde{\pi}_i - \tilde{\pi}_j$. If a renegotiation happens, one of the three inequalities $\pi_i \geq \tilde{\pi}_i$, $\pi_j \geq \tilde{\pi}_j$, and $\pi_0 \geq \Pi - \tilde{\pi}_i - \tilde{\pi}_j$, must hold strictly, which implies that $\pi_0 + \pi_i + \pi_j > \Pi$. This leads to a contradiction. Therefore, when the buyer accepts both offers, a renegotiation will not happen and $(\pi_0, \pi_i, \pi_j) = (\Pi - \tilde{\pi}_i - \tilde{\pi}_j, \tilde{\pi}_i, \tilde{\pi}_j)$.

If the buyer chooses A_iR_j, the optimal π_0^{-j} equals to $\min(\Pi^{-j}, \Pi - \pi_i)$ because the objective is increasing in π_0^{-j}. We discuss subcases based on whether $\pi_0^{-j} \leq \Pi^{-j}$ or/and $\Pi - \pi_i - \pi_0^{-j} \geq 0$ is binding for different realizations of D_0.

We first consider the case that $\tilde{\pi}_j \geq \pi_j(\tilde{\pi}_i, \Pi^{-j}(q_i^o) - \tilde{\pi}_i)$. The objective becomes $\max(\pi_i - \tilde{\pi}_i)^{\theta_i}(\Pi - \pi_i - \pi_j(\pi_i, \pi_0^{-j}) - D_0)^{1-\theta_i}$, where $D_0 = \Pi - \tilde{\pi}_i - \pi_j(\tilde{\pi}_i, \Pi^{-j}(q_i^o) - \tilde{\pi}_i)$.

When $\Pi \geq \tilde{\pi}_i + \Pi^{-j} + \frac{\theta_i\theta_j}{1-\theta_j}(\Pi^{-j} - (\Pi^{-j}(q_i^o) - \tilde{\pi}_i))$, we relax all constraints except (6). The optimal solution of the relaxed problem is binding at $\pi_0^{-j} \leq \Pi^{-j}$ and satisfies all other relaxed constraints. Thus, it is an optimal solution to the original problem. At optimality, $\pi_i = \tilde{\pi}_i + \frac{\theta_i\theta_j}{1-\theta_j}(\Pi^{-j} - (\Pi^{-j}(q_i^o) - \tilde{\pi}_i))$, $\pi_j = \theta_j(\Pi - \pi_i - \pi_0^{-j}) = \theta_j(\Pi - \tilde{\pi}_i - \Pi^{-j} - \left(\frac{\theta_i\theta_j}{1-\theta_j}\right)(\Pi^{-j} - (\Pi^{-j}(q_i^o) - \tilde{\pi}_i)))$, $\pi_0 = \Pi - \pi_i - \pi_j = (1-\theta_j)(\Pi - \tilde{\pi}_i) + \theta_j(1-\theta_i)\Pi^{-j} + \theta_i\theta_j(\Pi^{-j}(q_i^o) - \tilde{\pi}_i)$.

When $\Pi \leq \frac{1}{1-\theta_i\theta_j}(\tilde{\pi}_i + \Pi^{-j} - \theta_i\theta_j\Pi^{-j}(q_i^o))$, we relax all constraints except (7). The optimal solution of the relaxed problem is binding at $\Pi - \pi_i - \pi_0^{-j} \geq 0$ and satisfies all other relaxed constraints. Thus, it is an optimal solution to the original problem. At optimality, $\pi_i = \tilde{\pi}_i + \theta_i(\Pi - \tilde{\pi}_i - D_0) = \tilde{\pi}_i + \theta_i\theta_j(\Pi - \Pi^{-j}(q_i^o))$, $\pi_j = 0$, $\pi_0 = \Pi - \pi_i - \pi_j = \Pi - \tilde{\pi}_i - \theta_i\theta_j(\Pi - \Pi^{-j}(q_i^o))$.

When $\Pi \in \left(\tilde{\pi}_i + \frac{1}{1-\theta_i\theta_j}(\Pi^{-j} - \theta_i\theta_j(\Pi^{-j}(q_i^o) - \tilde{\pi}_i)), \tilde{\pi}_i + \Pi^{-j} + \frac{\theta_i\theta_j}{1-\theta_j}(\Pi^{-j} - (\Pi^{-j}(q_i^o) - \tilde{\pi}_i))\right)$, the optimal solution of the relaxed problem will violate (7) if we only keep constraint (6). If we only keep constraint (7), the optimal solution of the relaxed problem will violate (6). When we impose both (6) and (7), the optimal solution of the relaxed problem, satisfying (4) and (5), are binding at both $\pi_0^{-j} \leq \Pi^{-j}$ and $\Pi - \pi_i - \pi_0^{-j} \geq 0$. Thus, it is an optimal solution to the original problem, that is, $\pi_i = \Pi - \Pi^{-j}$, $\pi_j = 0$, $\pi_0 = \Pi^{-j}$.

We then discuss the case that $\tilde{\pi}_j \leq \pi_j(\tilde{\pi}_i, \Pi^{-j}(q_i^o) - \tilde{\pi}_i)$. The objective becomes $\max(\pi_i - \tilde{\pi}_i)^{\theta_i}(\Pi - \pi_i - \pi_j(\pi_i, \pi_0^{-j}) - D_0)^{1-\theta_i}$, where $D_0 = \Pi - \tilde{\pi}_i - \tilde{\pi}_j$.

When $\Pi \geq \Pi^{-j} + \tilde{\pi}_i + \frac{\theta_i}{1-\theta_j+\theta_i\theta_j}\tilde{\pi}_j$, we relax all constraints except (6). The optimal solution of the relaxed problem is binding at $\pi_0^{-j} \leq \Pi^{-j}$ and satisfies all other relaxed constraints. Thus, it is an optimal solution to the original problem. At optimality, $\pi_i = \tilde{\pi}_i + \frac{\theta_i(\tilde{\pi}_j - \theta_j(\Pi - \tilde{\pi}_i - \Pi^{-j}))}{1-\theta_j}$, $\pi_j = \theta_j(\Pi - \pi_i - \pi_0^{-j}) = \theta_j(\Pi - \Pi^{-j} - \tilde{\pi}_i - \frac{\theta_i(\tilde{\pi}_j - \theta_j(\Pi - \Pi^{-j} - \tilde{\pi}_i))}{1-\theta_j}) = \frac{\theta_j}{1-\theta_j}((1-\theta_j+\theta_i\theta_j)(\Pi - \Pi^{-j} - \tilde{\pi}_i) - \theta_i\tilde{\pi}_j)$, $\pi_0 = \Pi - \pi_i - \pi_j = \Pi^{-j} + (1-\theta_j+\theta_i\theta_j)(\Pi - \tilde{\pi}_i - \Pi^{-j}) - \theta_i\tilde{\pi}_j = \theta_j(1-\theta_i)\Pi^{-j} + (1-\theta_j+\theta_i\theta_j)(\Pi - \tilde{\pi}_i) - \theta_i\tilde{\pi}_j$.

When $\Pi \leq \Pi^{-j} + \tilde{\pi}_i + \theta_i\tilde{\pi}_j$, we relax all constraints except (7). The optimal solution of the relaxed problem is binding at $\Pi - \pi_i - \pi_0^{-j} \geq 0$ and satisfies all other relaxed constraints. Thus, it is an optimal solution to the original problem. At optimality, $\pi_i = \tilde{\pi}_i + \theta_i(\Pi - \tilde{\pi}_i - D_0) = \tilde{\pi}_i + \theta_i\tilde{\pi}_j$, $\pi_j = 0$, $\pi_0 = \Pi - \pi_i - \pi_j = \Pi - \tilde{\pi}_i - \theta_i\tilde{\pi}_j$.

When $\Pi \in \left(\Pi^{-j} + \tilde{\pi}_i + \theta_i\tilde{\pi}_j, \Pi^{-j} + \tilde{\pi}_i + \frac{\theta_i}{1-\theta_j+\theta_i\theta_j}\tilde{\pi}_j\right)$, the optimal solution of the relaxed problem will violate (7) if we only keep constraint (6). If we only keep constraint (7), the optimal solution of the relaxed problem will violate (6). When we impose both (6) and (7), the optimal solution of the relaxed problem, satisfying (4) and (5), are binding at both $\pi_0^{-j} \le \Pi^{-j}$ and $\Pi - \pi_i - \pi_0^{-j} \ge 0$. Thus, it is an optimal solution to the original problem, (i.e., $\pi_i = \Pi - \Pi^{-j}$, $\pi_j = 0$, $\pi_0 = \Pi^{-j}$).

By checking the boundaries, it is easy to show that both supplier j and the buyer's profit is continuous (weakly) decreasing functions of $\tilde{\pi}_i$ and $\tilde{\pi}_j$. $\square$

LEMMA 3. *Suppose that at optimality the buyer first* **rejects** *the offer $\tilde{\pi}_i$. If the buyer* **accepts** *$\tilde{\pi}_j$, the profits of the buyer, supplier i and supplier j are $(\pi_0, \pi_i, \pi_j) = ((1-\theta_i)(\Pi - \tilde{\pi}_j) + \theta_i D_0, \theta_i(\Pi - \tilde{\pi}_j - D_0), \tilde{\pi}_j)$ where $D_0 = \max\{(1-\theta_j)\Pi^{-i}, (1-\theta_j)\Pi^{-i} + \theta_j\Pi^{-i}(q_j^o) - \tilde{\pi}_j\}$. If the buyer* **rejects** *$\tilde{\pi}_j$, the profits of the buyer and supplier j are $\pi_0 = \Pi - \pi_i - \pi_j$ and $\pi_j = \theta_j(\Pi - \pi_i - \Pi^{-j})^+$ respectively, where if $\tilde{\pi}_j > \theta_j\Pi^{-i}(q_j^o)$,*

$$\pi_i = \begin{cases} \theta_i(\Pi - \Pi^{-i}) + \frac{\theta_i\theta_j}{1-\theta_j}\Pi^{-j}, & \textit{if } \Pi \ge \frac{1}{1-\theta_i}\left(-\theta_i\Pi^{-i} + \left(1 + \frac{\theta_i\theta_j}{1-\theta_j}\right)\Pi^{-j}\right); \\ \theta_i(\Pi - (1-\theta_j)\Pi^{-i}), & \textit{if } \Pi \le \frac{1}{1-\theta_i}\left(\Pi^{-j} - \theta_i(1-\theta_j)\Pi^{-i}\right); \\ \Pi - \Pi^{-j}, & \textit{otherwise;} \end{cases}$$

if $\tilde{\pi}_j \le \theta_j\Pi^{-i}(q_j^o)$,

$$\pi_i = \begin{cases} \frac{\theta_i((1-\theta_j)(\Pi - \Pi^{-i}) + \tilde{\pi}_j - \theta_j\Pi^{-i}(q_j^o) + \theta_j\Pi^{-j})}{1-\theta_j}, & \textit{if } \Pi \ge \frac{1}{1-\theta_i}\left(\left(1 + \frac{\theta_i\theta_j}{1-\theta_j}\right)\Pi^{-j} - \frac{\theta_i}{1-\theta_j}((1-\theta_j)\Pi^{-i} + \theta_j\Pi^{-i}(q_j^o) - \tilde{\pi}_j)\right) \\ \theta_i(\Pi + \tilde{\pi}_j - \theta_j\Pi^{-i}(q_j^o) - (1-\theta_j)\Pi^{-i}), & \textit{if } \Pi \le \frac{1}{1-\theta_i}\left(\Pi^{-j} + \theta_i(\tilde{\pi}_j - \theta_j\Pi^{-i}(q_j^o) - (1-\theta_j)\Pi^{-i})\right); \\ \Pi - \Pi^{-j}, & \textit{otherwise.} \end{cases}$$

Furthermore, both π_0 and π_j are continuous (weakly) decreasing functions of $\tilde{\pi}_i$ and $\tilde{\pi}_j$ if the buyer rejects $\tilde{\pi}_j$.

Proof of Lemma 3. If the buyer chooses R_iA_j, the objective becomes $\max_{\pi_i}(\pi_i - 0)^{\theta_i}(\Pi - \pi_i - \tilde{\pi}_j - D_0)^{1-\theta_i}$ where $D_0 = \max\{(1-\theta_j)\Pi^{-i}, (1-\theta_j)\Pi^{-i} + \theta_j\Pi^{-i}(q_j^o) - \tilde{\pi}_j\}$. The formulation is feasible if and only if $\Pi - \tilde{\pi}_j - D_0 \ge 0$. The optimal solution is $\pi_i = \theta_i(\Pi - \tilde{\pi}_j - D_0)$, $\pi_j = \tilde{\pi}_j$, and $\pi_0 = (1-\theta_i)(\Pi - \tilde{\pi}_j) + \theta_i D_0$. If the formulation is infeasible, the buyer cannot commit to accept supplier j's offer later and procure from supplier i.

If the buyer chooses R_iR_j, the optimal π_0^{-j} equals to $\min(\Pi^{-j}, \Pi - \pi_i)$ because the objective is increasing in π_0^{-j}. We discuss subcases based on whether $\pi_0^{-j} \le \Pi^{-j}$ or/and $\Pi - \pi_i - \pi_0^{-j} \ge 0$ is binding for different realizations of D_0.

We first discuss the case that $\tilde{\pi}_j \ge \theta_j\Pi^{-i}(q_j^o)$. The objective becomes $max(\pi_i - 0)^{\theta_i}(\Pi - \pi_i - \pi_j(\pi_i, \pi_0^{-j}) - D_0)^{1-\theta_i}$, where $D_0 = (1-\theta_j)\Pi^{-i}$.

When $\Pi \geq \frac{1}{1-\theta_i}\left(-\theta_i \Pi^{-i} + \left(1+\frac{\theta_i\theta_j}{1-\theta_j}\right)\Pi^{-j}\right)$, we relax all constraints except (11). The optimal solution of the relaxed problem is binding at $\pi_0^{-j} \leq \Pi^{-j}$ and satisfies all other relaxed constraints. Thus, it is an optimal solution to the original problem. At optimality, $\pi_i = \theta_i(\Pi - \Pi^{-i}) + \frac{\theta_i\theta_j}{1-\theta_j}\Pi^{-j}$, $\pi_j = \theta_j(\Pi - \pi_i - \pi_0^{-j}) = \theta_j((1-\theta_i)\Pi + \theta_i\Pi^{-i} - \left(1+\frac{\theta_i\theta_j}{1-\theta_j}\right)\Pi^{-j})$, $\pi_0 = \Pi - \pi_i - \pi_j = (1-\theta_i)(1-\theta_j)\Pi + \theta_i(1-\theta_j)\Pi^{-i} + \theta_j(1-\theta_i)\Pi^{-j}$.

When $\Pi \leq \frac{1}{1-\theta_i}(\Pi^{-j} - \theta_i(1-\theta_j)\Pi^{-i})$, we relax all constraints except (12). The optimal solution of the relaxed problem is binding at $\Pi - \pi_i - \pi_0^{-j} \geq 0$ and satisfies all other relaxed constraints. Thus, it is an optimal solution to the original problem. At optimality, $\pi_i = \theta_i(\Pi - D_0) = \theta_i(\Pi - (1-\theta_j)\Pi^{-i})$, $\pi_j = 0$, $\pi_0 = \Pi - \pi_i - \pi_j = (1-\theta_i)\Pi + \theta_i(1-\theta_j)\Pi^{-i}$.

When $\Pi \in \left(\frac{1}{1-\theta_i}(\Pi^{-j} - \theta_i(1-\theta_j)\Pi^{-i}), \frac{1}{1-\theta_i}\left(-\theta_i\Pi^{-i} + \left(1+\frac{\theta_i\theta_j}{1-\theta_j}\right)\Pi^{-j}\right)\right)$, the optimal solution of the relaxed problem will violate (12) if we only keep constraint (11). If we only keep constraint (12), the optimal solution of the relaxed problem will violate (11). When we impose both (11) and (12), the optimal solution of the relaxed problem, satisfying (9) and (10), are binding at both $\pi_0^{-j} \leq \Pi^{-j}$ and $\Pi - \pi_i - \pi_0^{-j} \geq 0$. Thus, it is an optimal solution to the original problem, (i.e., $\pi_i = \Pi - \Pi^{-j}$, $\pi_j = 0$, $\pi_0 = \Pi^{-j}$).

The above three subcases are mutually exclusive and collectively exhaustive because $\frac{1}{1-\theta_i}\left(-\theta_i\Pi^{-i} + \left(1+\frac{\theta_i\theta_j}{1-\theta_j}\right)\Pi^{-j}\right) \leq \Pi \leq \frac{1}{1-\theta_i}(\Pi^{-j} - \theta_i(1-\theta_j)\Pi^{-i})$ is impossible. If the inequality holds, $\Pi^{-j} < \Pi \leq \frac{1}{1-\theta_i}(\Pi^{-j} - \theta_i(1-\theta_j)\Pi^{-i})$ leads to $\Pi^{-j} > (1-\theta_j)\Pi^{-i}$. We thus have $\frac{1}{1-\theta_i}(\Pi^{-j} - \theta_i(1-\theta_j)\Pi^{-i}) - \frac{1}{1-\theta_i}\left(-\theta_i\Pi^{-i} + \left(1+\frac{\theta_i\theta_j}{1-\theta_j}\right)\Pi^{-j}\right) = -\frac{\theta_i\theta_j(\Pi^{-j}-(1-\theta_j)\Pi^{-i})}{(1-\theta_i)(1-\theta_j)} < 0$, a contradiction.

We then discuss the case that $\tilde{\pi}_j \leq \theta_j\Pi^{-i}(q_j^o)$. The objective becomes $max(\pi_i - 0)^{\theta_i}(\Pi - \pi_i - \pi_j(\pi_i, \pi_0^{-j}) - D_0)^{1-\theta_i}$, where $D_0 = (1-\theta_j)\Pi^{-i} + \theta_j\Pi^{-i}(q_j^o) - \tilde{\pi}_j$. $\Pi - \pi_i - \pi_j(\pi_i, \pi_0^{-j}) - D_0 = \Pi - (1-\theta_j)\Pi^{-i} - \theta_j\Pi^{-i}(q_j^o) - \pi_i + \tilde{\pi}_j - \theta_j(\Pi - \pi_i - \pi_0^{-j}) = (1-\theta_j)(\Pi - \Pi^{-i} - \pi_i) + \tilde{\pi}_j - \theta_j\Pi^{-i}(q_j^o) + \theta_j\pi_0^{-j} \geq 0$ because the formulation is feasible. Notice that $\pi_i \geq 0$ and $\Pi^{-j} \geq \pi_0^{-j}$ lead to $(1-\theta_j)(\Pi - \Pi^{-i}) + \tilde{\pi}_j - \theta_j\Pi^{-i}(q_j^o) + \theta_j\Pi^{-j} \geq (1-\theta_j)(\Pi - \Pi^{-i} - \pi_i) + \tilde{\pi}_j - \theta_j\Pi^{-i}(q_j^o) + \theta_j\pi_0^{-j} \geq 0$.

When $\Pi \geq \frac{1}{1-\theta_i}\left(\left(1+\frac{\theta_i\theta_j}{1-\theta_j}\right)\Pi^{-j} - \frac{\theta_i}{1-\theta_j}((1-\theta_j)\Pi^{-i} + \theta_j\Pi^{-i}(q_j^o) - \tilde{\pi}_j)\right)$, we relax all constraints except (11). The optimal solution of the relaxed problem is binding at $\pi_0^{-j} \leq \Pi^{-j}$ and satisfies all other relaxed constraints. Thus, it is an optimal solution to the original problem. At optimality, $\pi_i = \frac{\theta_i((1-\theta_j)(\Pi-\Pi^{-i}) + \tilde{\pi}_j - \theta_j\Pi^{-i}(q_j^o) + \theta_j\Pi^{-j})}{1-\theta_j}$, $\pi_j = \theta_j(\Pi - \pi_i - \pi_0^{-j}) = \theta_j(\Pi - \Pi^{-j} - \frac{\theta_i((1-\theta_j)(\Pi-\Pi^{-i}) + \tilde{\pi}_j - \theta_j\Pi^{-i}(q_j^o) + \theta_j\Pi^{-j})}{1-\theta_j}) = \frac{\theta_j}{1-\theta_j}((1-\theta_i)(1-\theta_j)\Pi - (1-\theta_j+\theta_i\theta_j)\Pi^{-j} + \theta_i((1-\theta_j)\Pi^{-i} + \theta_j\Pi^{-i}(q_j^o) - \tilde{\pi}_j))$, $\pi_0 = \Pi - \pi_i - \pi_j = (1-\theta_i)(1-\theta_j)\Pi + \theta_j(1-\theta_i)\Pi^{-j} + \theta_i((1-\theta_j)\Pi^{-i} + \theta_j\Pi^{-i}(q_j^o) - \tilde{\pi}_j)$.

When $\Pi \leq \frac{1}{1-\theta_i}\left(\Pi^{-j} + \theta_i(\tilde{\pi}_j - \theta_j\Pi^{-i}(q_j^o) - (1-\theta_j)\Pi^{-i})\right)$, we relax all constraints except (12). The optimal solution of the relaxed problem is binding at $\Pi - \pi_i - \pi_0^{-j} \geq 0$ and satisfies all other relaxed constraints. Thus, it is an optimal solution to the original problem. At optimality, $\pi_i = \theta_i(\Pi - D_0) = \theta_i(\Pi + \tilde{\pi}_j - \theta_j \Pi^{-i}(q_j^o) - (1-\theta_j)\Pi^{-i})$, $\pi_j = 0$, $\pi_0 = \Pi - \pi_i - \pi_j = (1-\theta_i)\Pi + \theta_i((1-\theta_j)\Pi^{-i} + \theta_j \Pi^{-i}(q_j^o) - \tilde{\pi}_j)$.

When $\Pi \in \left(\frac{1}{1-\theta_i}\left(\Pi^{-j} + \theta_i(\tilde{\pi}_j - \theta_j \Pi^{-i}(q_j^o) - (1-\theta_j)\Pi^{-i})\right), \frac{1}{1-\theta_i}\left(\left(1 + \frac{\theta_i \theta_j}{1-\theta_j}\right)\Pi^{-j} - \frac{\theta_i}{1-\theta_j}((1-\theta_j)\Pi^{-i} + \theta_j \Pi^{-i}(q\right.\right.$ the optimal solution of the relaxed problem will violate (12) if we only keep constraint (11). If we only keep constraint (12), the optimal solution of the relaxed problem will violate (11). When we impose both (11) and (12), the optimal solution of the relaxed problem, satisfying (9) and (10), are binding at both $\pi_0^{-j} \le \Pi^{-j}$ and $\Pi - \pi_i - \pi_0^{-j} \ge 0$. Thus, it is an optimal solution to the original problem, (i.e., $\pi_i = \Pi - \Pi^{-j}$, $\pi_j = 0$, $\pi_0 = \Pi^{-j}$).

The above three subcases are mutually exclusive and collectively exhaustive because $\frac{1}{1-\theta_i}\left(\left(1 + \frac{\theta_i\theta_j}{1-\theta_j}\right)\Pi^{-j} - \frac{\theta_i}{1-\theta_j}((1-\theta_j)\Pi^{-i} + \theta_j \Pi^{-i}(q_j^o) - \tilde{\pi}_j)\right) \le \Pi \le \frac{1}{1-\theta_i}\left(\Pi^{-j} + \theta_i(\tilde{\pi}_j - \theta_j \Pi^{-i}(q_j^o) - (1-\theta_j)\Pi^{-i})\right)$ is impossible. If the inequality holds, $\Pi^{-j} < \Pi \le \frac{1}{1-\theta_i}\left(\Pi^{-j} + \theta_i(\tilde{\pi}_j - \theta_j \Pi^{-i}(q_j^o) - (1-\theta_j)\Pi^{-i})\right)$ leads to $\Pi^{-j} + \tilde{\pi}_j - \theta_j \Pi^{-i}(q_j^o) - (1-\theta_j)\Pi^{-i} > 0$. We thus have $\frac{1}{1-\theta_i}\left(\Pi^{-j} + \theta_i(\tilde{\pi}_j - \theta_j \Pi^{-i}(q_j^o) - (1-\theta_j)\Pi^{-i})\right) - \frac{1}{1-\theta_i}\left(\left(1 + \frac{\theta_i\theta_j}{1-\theta_j}\right)\Pi^{-j} - \frac{\theta_i}{1-\theta_j}((1-\theta_j)\Pi^{-i} + \theta_j \Pi^{-i}(q_j^o) - \tilde{\pi}_j)\right) = -\frac{\theta_i\theta_j}{(1-\theta_i)(1-\theta_j)}(\Pi^{-j} + \tilde{\pi}_j - \theta_j \Pi^{-i}(q_j^o) - (1-\theta_j)\Pi^{-i}) < 0$, a contradiction.

By checking the boundaries, it is easy to verify that both supplier j and the buyer's profit is continuous (weakly) decreasing function of $\tilde{\pi}_j$ and independent of $\tilde{\pi}_i$. $\square$

Lemma 4. *Assuming that at optimality the buyer first accepts the offer $\tilde{\pi}_i$ from supplier i, Lemma 2 provides the correct profit expressions for formulation [AI]. Furthermore, it is optimal for the buyer to accept supplier j's offer $\tilde{\pi}_j$ if and only if $\tilde{\pi}_j \le \theta_j(\Pi - \tilde{\pi}_i - \Pi^{-j})^+$ ($\{i,j\} = \{1,2\}$).*

Proof of Lemma 4. The outcome of the renegotiated contract between the buyer and supplier i solves

$$(\pi_i, \pi_0^{-j}) = \arg\max (\pi_i - \tilde{\pi}_i)^{\theta_i} (\Pi - \pi_i - \min\{\tilde{\pi}_j, \pi_j(\pi_i, \pi_0^{-j})\} - D_0)^{1-\theta_i}$$

$$s.t.\ \pi_i \ge \tilde{\pi}_i, \Pi - \pi_i - \min\{\tilde{\pi}_j, \pi_j(\pi_i, \pi_0^{-j})\} \ge D_0, \pi_0^{-j} \le \Pi^{-j}, \Pi - \pi_i - \pi_0^{-j} \ge 0,$$

where $D_0 = \max\{\Pi - \tilde{\pi}_i - \tilde{\pi}_j, \Pi - \tilde{\pi}_i - \pi_j(\tilde{\pi}_i, \Pi^{-j}(q_i^o) - \tilde{\pi}_i)\}$.

If $\tilde{\pi}_j \le \theta_j(\Pi - \tilde{\pi}_i - \Pi^{-j})^+$, we show that at optimality the buyer accepts supplier j's offer, (i.e., $\pi_j = \tilde{\pi}_j$). Because $\pi_0^{-j} \le \Pi^{-j}$ and $\Pi - \pi_i - \pi_0^{-j} \ge 0$, $\pi_j(\pi_i, \pi_0^{-j}) = \theta_j(\Pi - \pi_i - \pi_0^{-j}) \ge \theta_j(\Pi - \pi_i - \Pi^{-j})^+$. The buyer's profit with A_iR_j equals $\Pi - \pi_i - \min\{\tilde{\pi}_j, \theta_j(\Pi - \pi_i - \pi_0^{-j})\} \le \Pi - \pi_i - \min\{\tilde{\pi}_j, \theta_j(\Pi - \pi_i - \Pi^{-j})^+\}$. Because $\Pi - \pi_i - \min\{\tilde{\pi}_j, \theta_j(\Pi - \pi_i - \Pi^{-j})^+\}$ is decreasing in π_i and $\pi_i \ge \tilde{\pi}_i$, the buyer's profit with A_iR_j is no more than $\Pi - \tilde{\pi}_i - \min\{\tilde{\pi}_j, \theta_j(\Pi - \tilde{\pi}_i - \Pi^{-j})^+\} = \Pi - \tilde{\pi}_i - \tilde{\pi}_j$, which is the the buyer's profit when the buyer accepts both suppliers' offer. Thus, the buyer accepts supplier j's offer at optimality.

If $\tilde{\pi}_j > \theta_j(\Pi - \tilde{\pi}_i - \Pi^{-j})^+$, we show that at optimality, $\pi_i > \tilde{\pi}_i$, $\pi_0 > \Pi - \tilde{\pi}_i - \tilde{\pi}_j$, and $\pi_j < \tilde{\pi}_j$. That is, it is optimal to reject supplier j's offer. To show that, we only need to find a feasible solution so that (3), the objective value when accepting supplier i's offer first, is strictly positive. That is, we need to find $\hat{\pi}_i$ such that $\hat{\pi}_i > \tilde{\pi}_i$ and $\Pi - \hat{\pi}_i - \theta_j(\Pi - \hat{\pi}_i - \min\{\Pi^{-j}, \Pi - \hat{\pi}_i\}) = \Pi - \hat{\pi}_i - \theta_j(\Pi - \hat{\pi}_i - \Pi^{-j})^+ > D_0$. Due to continuity, we only need to show that $\Pi - \tilde{\pi}_i - \theta_j(\Pi - \tilde{\pi}_i - \Pi^{-j})^+ > D_0$.

Note that $D_0 = \max\{\Pi - \tilde{\pi}_i - \tilde{\pi}_j, \Pi - \tilde{\pi}_i - \theta_j(\Pi - \tilde{\pi}_i - (\Pi^{-j}(q_i^o) - \tilde{\pi}_i))\}$. Because $\tilde{\pi}_j > \theta_j(\Pi - \tilde{\pi}_i - \Pi^{-j})^+$, we have $\Pi - \tilde{\pi}_i - \theta_j(\Pi - \tilde{\pi}_i - \Pi^{-j})^+ > \Pi - \tilde{\pi}_i - \tilde{\pi}_j$. Because $\Pi^{-j} > \Pi^{-j}(q_i^o)$, we have $\Pi - \tilde{\pi}_i - \theta_j(\Pi - \tilde{\pi}_i - \Pi^{-j})^+ > \Pi - \tilde{\pi}_i - \theta_j(\Pi - \tilde{\pi}_i - (\Pi^{-j}(q_i^o) - \tilde{\pi}_i)) = \Pi - \tilde{\pi}_i - \theta_j(\Pi - \Pi^{-j}(q_i^o))$. Therefore, $\Pi - \tilde{\pi}_i - \theta_j(\Pi - \tilde{\pi}_i - \Pi^{-j})^+ > D_0$.

For both cases, both the buyer and supplier i know the functional form of the buyer's profit and Lemma 2 provides the correct profit expressions. $\square$

LEMMA 5. *Assuming that at optimality the buyer first rejects the offer $\tilde{\pi}_i$ from supplier i, Lemma 3 provides the correct profit expressions for formulation [RI]. Furthermore, it is optimal for the buyer to reject supplier j's offer ($\{i, j\} = \{1, 2\}$).*

Proof of Lemma 5. The outcome of the negotiated contract between the buyer and supplier i solves

$$(\pi_i, \pi_0^{-j}) = \arg\max(\pi_i - 0)^{\theta_i}(\Pi - \pi_i - \min\{\tilde{\pi}_j, \pi_j(\pi_i, \pi_0^{-j})\} - D_0)^{1-\theta_i}$$
$$s.t.\ \pi_i \geq 0, \Pi - \pi_i - \min\{\tilde{\pi}_j, \pi_j(\pi_i, \pi_0^{-j})\} \geq D_0, \pi_0^{-j} \leq \Pi^{-j}, \Pi - \pi_i - \pi_0^{-j} \geq 0,$$

where $D_0 = \max\{(1-\theta_j)\Pi^{-i}, (1-\theta_j)\Pi^{-i} + \theta_j \Pi^{-i}(q_j^o) - \tilde{\pi}_j\}$.

We first show that when at optimality the buyer first rejects the offer $\tilde{\pi}_i$ from supplier i, Lemma 3 provides the correct profit expressions.

When $\tilde{\pi}_j = 0$, the buyer accepts supplier j's offer, and Lemma 3 provides the correct profit expressions.

When $\tilde{\pi}_j > 0$, the buyer's acceptance or rejection decision is based on the comparison between $\tilde{\pi}_j$ and $\theta_j(\Pi - \pi_i - \Pi^{-j})^+$. Because $\tilde{\pi}_j > 0$, $\tilde{\pi}_j \geq \theta_j(\Pi - \pi_i - \Pi^{-j})^+$ if and only if $\tilde{\pi}_j \geq \theta_j(\Pi - \pi_i - \Pi^{-j})$. The buyer accepts $\tilde{\pi}_j$ (so $\pi_j = \tilde{\pi}_j$) if $\pi_i \in [0, \Pi - \Pi^{-j} - \tilde{\pi}_j/\theta_j]$ and rejects $\tilde{\pi}_j$ (so $\pi_j = \theta_j(\Pi - \pi_i - \Pi^{-j})$) if $\pi_i \in (\Pi - \Pi^{-j} - \tilde{\pi}_j/\theta_j, \infty)$. By Lemma 3, the objective function of formulation [RI] has a single local maximum for different forms of π_j, either $\pi_j = \tilde{\pi}_j$ or $\pi_j = \theta_j(\Pi - \pi_i - \Pi^{-j})$. The solutions specified in Lemma 3 could fall into the following three cases. (1) If the solutions belong to their respective regions (i.e., when assuming accepting (or rejecting) supplier j, the solution of π_i belongs to $[0, \Pi - \Pi^{-j} - \tilde{\pi}_j/\theta_j]$ (or $(\Pi - \Pi^{-j} - \tilde{\pi}_j/\theta_j, \infty)$), the global maximum is the larger of the two objective values. (2) If only one solution belongs to its respective region, it must be the case that the objective value is monotone in the other region and the global maximum is this solution. (3) If both solutions identified in Lemma 3 do not belong to their regions, the global maximum is obtained at the boundary $\pi_i = \Pi - \Pi^{-j} - \tilde{\pi}_j/\theta_j$. We next show that the last case is impossible, and Lemma 3 provides the correct characterization for the profit expressions.

When $\tilde{\pi}_j \geq \theta_j \Pi^{-i}(q_j^o)$, $D_0 = (1-\theta_j)\Pi^{-i}$. By Lemma 3, supplier i's profit when accepting supplier j's offer is $\pi_i = \theta_i(\Pi - \tilde{\pi}_j - (1-\theta_j)\Pi^{-i})$. Supplier i's profit when rejecting supplier j's offer is

$$\pi_i = \begin{cases} \theta_i(\Pi - \Pi^{-i}) + \frac{\theta_i\theta_j}{1-\theta_j}\Pi^{-j}, & \text{if } \Pi \geq \frac{1}{1-\theta_i}\left(-\theta_i\Pi^{-i} + \left(1 + \frac{\theta_i\theta_j}{1-\theta_j}\right)\Pi^{-j}\right); \\ \theta_i(\Pi - (1-\theta_j)\Pi^{-i}), & \text{if } \Pi \leq \frac{1}{1-\theta_i}\left(\Pi^{-j} - \theta_i(1-\theta_j)\Pi^{-i}\right); \\ \Pi - \Pi^{-j}, & \text{otherwise.} \end{cases}$$

We will show that if supplier i's profit belongs to $[0, \Pi - \Pi^{-j} - \tilde{\pi}_j/\theta_j]$ when *rejecting* supplier j's offer, then supplier i's profit belongs to $[0, \Pi - \Pi^{-j} - \tilde{\pi}_j/\theta_j]$ when *accepting* supplier j's offer.

If $\Pi \geq \frac{1}{1-\theta_i}\left(-\theta_i\Pi^{-i} + \left(1 + \frac{\theta_i\theta_j}{1-\theta_j}\right)\Pi^{-j}\right)$ and $\theta_i(\Pi - \Pi^{-i}) + \frac{\theta_i\theta_j}{1-\theta_j}\Pi^{-j} \leq \Pi - \Pi^{-j} - \tilde{\pi}_j/\theta_j$, we have $\theta_i(\Pi - \tilde{\pi}_j - (1-\theta_j)\Pi^{-i}) \leq \frac{\theta_i\theta_j}{1-\theta_j}((1-\theta_i)(\Pi - \Pi^{-i}) + \theta_j(1-\theta_i)\Pi^{-j}) + \theta_i(\Pi - \tilde{\pi}_j - (1-\theta_j)\Pi^{-i}) = \theta_i\theta_j(\Pi - \Pi^{-j} - \tilde{\pi}_j/\theta_j) + (1-\theta_i\theta_j)\left(\theta_i(\Pi - \Pi^{-i}) + \frac{\theta_i\theta_j}{1-\theta_j}\Pi^{-j}\right) \leq \theta_i\theta_j(\Pi - \Pi^{-j} - \tilde{\pi}_j/\theta_j) + (1-\theta_i\theta_j)(\Pi - \Pi^{-j} - \tilde{\pi}_j/\theta_j) = \Pi - \Pi^{-j} - \tilde{\pi}_j/\theta_j$.

If $\Pi < \frac{1}{1-\theta_i}\left(\Pi^{-j} - \theta_i(1-\theta_j)\Pi^{-i}\right)$ and $\theta_i(\Pi - (1-\theta_j)\Pi^{-i}) \leq \Pi - \Pi^{-j} - \tilde{\pi}_j/\theta_j$, we have $\theta_i(\Pi - \tilde{\pi}_j - (1-\theta_j)\Pi^{-i}) < \theta_i(\Pi - (1-\theta_j)\Pi^{-i}) \leq \Pi - \Pi^{-j} - \tilde{\pi}_j/\theta_j$.

If $\Pi \in \left(\frac{1}{1-\theta_i}\left(\Pi^{-j} - \theta_i(1-\theta_j)\Pi^{-i}\right), \frac{1}{1-\theta_i}\left(-\theta_i\Pi^{-i} + \left(1 + \frac{\theta_i\theta_j}{1-\theta_j}\right)\Pi^{-j}\right)\right)$, then $\Pi - \Pi^{-j} \leq \Pi - \Pi^{-j} - \tilde{\pi}_j/\theta_j$ leads to a contradiction because $\tilde{\pi}_j > 0$.

When $\tilde{\pi}_j < \theta_j \Pi^{-i}(q_j^o)$, $D_0 = (1-\theta_j)\Pi^{-i} + \theta_j\Pi^{-i}(q_j^o) - \tilde{\pi}_j$. By Lemma 3, supplier i's profit when accepting supplier j's offer is $\pi_i = \theta_i(\Pi - \theta_j\Pi^{-i}(q_j^o) - (1-\theta_j)\Pi^{-i})$. Supplier i's profit when rejecting supplier j's offer is

$$\pi_i = \begin{cases} \frac{\theta_i((1-\theta_j)(\Pi-\Pi^{-i}) + \tilde{\pi}_j - \theta_j\Pi^{-i}(q_j^o) + \theta_j\Pi^{-j})}{1-\theta_j}, & \text{if } \Pi \geq \frac{1}{1-\theta_i}\left(\left(1 + \frac{\theta_i\theta_j}{1-\theta_j}\right)\Pi^{-j} - \frac{\theta_i}{1-\theta_j}((1-\theta_j)\Pi^{-i} + \theta_j\Pi^{-i}(q_j^o) - \tilde{\pi}_j)\right); \\ \theta_i(\Pi + \tilde{\pi}_j - \theta_j\Pi^{-i}(q_j^o) - (1-\theta_j)\Pi^{-i}), & \text{if } \Pi \leq \frac{1}{1-\theta_i}\left(\Pi^{-j} + \theta_i(\tilde{\pi}_j - \theta_j\Pi^{-i}(q_j^o) - (1-\theta_j)\Pi^{-i}\right); \\ \Pi - \Pi^{-j}, & \text{otherwise.} \end{cases}$$

We will show that if supplier i's profit belongs to $[0, \Pi - \Pi^{-j} - \tilde{\pi}_j/\theta_j]$ when *rejecting* supplier j's offer, then supplier i's profit belongs to $[0, \Pi - \Pi^{-j} - \tilde{\pi}_j/\theta_j]$ when *accepting* supplier j's offer.

If $\Pi \geq \frac{1}{1-\theta_i}\left(\left(1 + \frac{\theta_i\theta_j}{1-\theta_j}\right)\Pi^{-j} - \frac{\theta_i}{1-\theta_j}((1-\theta_j)\Pi^{-i} + \theta_j\Pi^{-i}(q_j^o) - \tilde{\pi}_j)\right)$ and $\frac{\theta_i((1-\theta_j)(\Pi-\Pi^{-i}) + \tilde{\pi}_j - \theta_j\Pi^{-i}(q_j^o) + \theta_j\Pi^{-j})}{1-\theta_j} \leq \Pi - \Pi^{-j} - \tilde{\pi}_j/\theta_j$, we have $\theta_i(\Pi - \theta_j\Pi^{-i}(q_j^o) - (1-\theta_j)\Pi^{-i}) \leq \frac{\theta_i\theta_j(1-\theta_i)}{1-\theta_j}((1-\theta_j)(\Pi - \Pi^{-i}) + \theta_j\Pi^{-j} - (\theta_j\Pi^{-i}(q_j^o) - \tilde{\pi}_j)) + \theta_i(\Pi - \theta_j\Pi^{-i}(q_j^o) - (1-\theta_j)\Pi^{-i}) = \theta_i\theta_j(\Pi - \Pi^{-j} - \tilde{\pi}_j/\theta_j) + (1-\theta_i\theta_j)\frac{\theta_i((1-\theta_j)(\Pi-\Pi^{-i}) + \tilde{\pi}_j - \theta_j\Pi^{-i}(q_j^o) + \theta_j\Pi^{-j})}{1-\theta_j} \leq \Pi - \Pi^{-j} - \tilde{\pi}_j/\theta_j$. The first inequality is due to the feasibility constraint shown in Lemma 3.

If $\Pi \leq \frac{1}{1-\theta_i}\left(\Pi^{-j} + \theta_i(\tilde{\pi}_j - \theta_j\Pi^{-i}(q_j^o) - (1-\theta_j)\Pi^{-i})\right)$ and $\theta_i(\Pi + \tilde{\pi}_j - \theta_j\Pi^{-i}(q_j^o) - (1-\theta_j)\Pi^{-i}) \leq \Pi - \Pi^{-j} - \tilde{\pi}_j/\theta_j$, we have $\theta_i(\Pi - \theta_j\Pi^{-i}(q_j^o) - (1-\theta_j)\Pi^{-i}) < \theta_i(\Pi + \tilde{\pi}_j - \theta_j\Pi^{-i}(q_j^o) - (1-\theta_j)\Pi^{-i}) \leq \Pi - \Pi^{-j} - \tilde{\pi}_j/\theta_j$.

If $\Pi \in \left(\frac{1}{1-\theta_i}\left(\Pi^{-j}+\theta_i(\tilde{\pi}_j-\theta_j\Pi^{-i}(q_j^o)-(1-\theta_j)\Pi^{-i})\right), \frac{1}{1-\theta_i}\left(\left(1+\frac{\theta_i\theta_j}{1-\theta_j}\right)\Pi^{-j}-\frac{\theta_i}{1-\theta_j}((1-\theta_j)\Pi^{-i}+\theta_j\Pi^{-i}(q_j^o)-\right.\right.$ then $\Pi-\Pi^{-j}\le\Pi-\Pi^{-j}-\tilde{\pi}_j/\theta_j$ leads to a contradiction because $\tilde{\pi}_j>0$.

Therefore, case (3) is impossible and Lemma 3 provides the correct profit expressions.

We now show that R_iA_j is suboptimal.

When $\tilde{\pi}_j\le\theta_j\Pi^{-i}(q_j^o)$, we show that the buyer's profit with R_iA_j is no higher than her profit with A_jR_i. Therefore, by the tie-breaking rule, R_iA_j is suboptimal. The buyer's profit under R_iA_j is $\hat{\pi}_0=(1-\theta_i)\Pi+\theta_i((1-\theta_j)\Pi^{-i}+\theta_j\Pi^{-i}(q_j^o))-\tilde{\pi}_j$. When $\tilde{\pi}_i\ge\max(\theta_i(\Pi-\tilde{\pi}_j-\Pi^{-i}),\pi_i(\tilde{\pi}_j,(\Pi^{-i}(q_j^0)-\tilde{\pi}_j)))$, $D_0=\Pi-\tilde{\pi}_j-\pi_i(\tilde{\pi}_j,(\Pi^{-i}(q_j^0)-\tilde{\pi}_j))$. By the proof of Lemma 2, the buyer's profit with A_jR_i is

$$\pi_0=\begin{cases}(1-\theta_i)(\Pi-\tilde{\pi}_j)+\theta_i(1-\theta_j)\Pi^{-i}+\theta_i\theta_j(\Pi^{-i}(q_j^o)-\tilde{\pi}_j), & \text{if } \Pi\ge\tilde{\pi}_j+\Pi^{-i}+\frac{\theta_i\theta_j}{1-\theta_i}(\Pi^{-i}-(\Pi^{-i}(q_j^o)-\tilde{\pi}_j));\\ (1-\theta_i\theta_j)(\Pi-\tilde{\pi}_j)+\theta_i\theta_j(\Pi^{-i}(q_j^o)-\tilde{\pi}_j), & \text{if } \Pi\le\tilde{\pi}_j+\frac{1}{1-\theta_i\theta_j}(\Pi^{-i}-\theta_i\theta_j(\Pi^{-i}(q_j^o)-\tilde{\pi}_j));\\ \Pi^{-i}, & \text{otherwise.}\end{cases}$$

When $\Pi\ge\tilde{\pi}_j+\Pi^{-i}+\frac{\theta_i\theta_j}{1-\theta_i}(\Pi^{-i}-(\Pi^{-i}(q_j^o)-\tilde{\pi}_j))$, $\pi_0-\hat{\pi}_0=\theta_i(1-\theta_j)\tilde{\pi}_j\ge0$. When $\Pi\le\tilde{\pi}_j+\frac{1}{1-\theta_i\theta_j}(\Pi^{-i}-\theta_i\theta_j(\Pi^{-i}(q_j^o)-\tilde{\pi}_j))$, $\pi_0-\hat{\pi}_0=\theta_i(1-\theta_j)(\Pi-\Pi^{-i})>0$. When $\Pi\in(\tilde{\pi}_j+\frac{1}{1-\theta_i\theta_j}(\Pi^{-i}-\theta_i\theta_j(\Pi^{-i}(q_j^o)-\tilde{\pi}_j)),\tilde{\pi}_j+\Pi^{-i}+\frac{\theta_i\theta_j}{1-\theta_i}(\Pi^{-i}-(\Pi^{-i}(q_j^o)-\tilde{\pi}_j)))$, $\pi_0-\hat{\pi}_0=\Pi^{-i}-\hat{\pi}_0=(1-\theta_i+\theta_i\theta_j)\Pi^{-i}-(1-\theta_i)(\Pi-\tilde{\pi}_j)+\theta_i(\tilde{\pi}_j-\theta_j\Pi^{-i}(q_j^0))\ge\theta_i(1-\theta_j)\tilde{\pi}_j\ge0$. We thus see that the buyer is better off under A_jR_i. Because the buyer's profit of accepting supplier j's offer first is weakly decreasing in $\tilde{\pi}_i$, $\pi_0-\hat{\pi}_0\ge0$ when $\tilde{\pi}_i\le\max(\theta_i(\Pi-\tilde{\pi}_j-\Pi^{-i}),\pi_i(\tilde{\pi}_j,(\Pi^{-i}(q_j^o)-\tilde{\pi}_i)))$. Thus, we reach a contradiction and it is optimal to reject supplier j's offer if the buyer rejects supplier i's offer first.

When $\tilde{\pi}_j\ge\theta_j\Pi^{-i}(q_j^o)$ and suppose that the buyer obtains the optimal profit with R_iA_j. The optimality of the acceptance implies that $\tilde{\pi}_j\le\theta_j(\Pi-\tilde{\pi}_i-\Pi^{-j})^+\le\theta_j(\Pi-\Pi^{-j})<\theta_j(\Pi-\Pi^{-j}(q_i^o))$, which is no more than $\theta_j\Pi^{-i}(q_j^o)$ by submodularity ($\Pi\le\Pi^{-i}(q_j^o)+\Pi^{-j}(q_i^o)$). Therefore, $\tilde{\pi}_j<\theta_j\Pi^{-i}(q_j^o)$ and we reach a contradiction.

Therefore, it is optimal to reject supplier j's offer if the buyer rejects supplier i's offer first. □

LEMMA 6. *If $\tilde{\pi}_i=0$, the buyer should accept supplier i's offer immediately* ($i=1,2$).

Proof of Lemma 6. We first show that the buyer will not bargain with supplier j first if $\tilde{\pi}_i=0$. If the buyer accepts supplier j's offer first, it is optimal for the buyer to accept both offers by Lemma 4. If the buyer rejects supplier j's offer first, R_jA_i is optimal due to $\tilde{\pi}_i=0$, which is a contradiction by Lemma 5.

We then show that the buyer prefers to first accepting supplier i's offer (A_iA_j or A_iR_j) over first rejects supplier i's offer at $\tilde{\pi}_i=0$ (i.e., R_iR_j by Lemma 5).

When $\tilde{\pi}_j\le\theta_j(\Pi-\Pi^{-j})$, if the buyer first accepts supplier i's offer, she chooses A_iA_j by Lemma 4 and obtains profit $\Pi-\tilde{\pi}_j$. If the buyer first rejects supplier i's offer, she chooses R_iR_j by Lemma 5 and obtains profit $\Pi-\pi_i-\theta_j(\Pi-\pi_i-\Pi^{-j})^+$ with $\pi_i>0$ by Lemma 3. The buyer's profit with

R_iR_j is strictly less than $\Pi - \tilde{\pi}_j$ (the buyer's profit with A_iA_j) because $\pi_i + \theta_j(\Pi - \pi_i - \Pi^{-j})^+ > \theta_j(\Pi - \Pi^{-j}) \geq \tilde{\pi}_j$ for $\pi_i > 0$.

When $\tilde{\pi}_j > \theta_j(\Pi - \Pi^{-j})$, if the buyer first accepts supplier i's offer, she chooses A_iR_j by Lemma 4. If the buyer first rejects supplier i's offer, she chooses R_iR_j by Lemma 5. For both A_iR_j and R_iR_j, the contract between the buyer and supplier i solves

$$(\pi_i, \pi_0^{-j}) = \arg\max(\pi_i - 0)^{\theta_i}(\Pi - \pi_i - \pi_j(\pi_i, \pi_0^{-j}) - D_0)^{1-\theta_i}$$

$$s.t.\, \pi_i \geq 0, \Pi - \pi_i - \pi_j(\pi_i, \pi_0^{-j}) \geq D_0, \pi_0^{-j} \leq \Pi^{-j}, \Pi - \pi_i - \pi_0^{-j} \geq 0,$$

where D_0 is the reservation utility. optimally, $\pi_i = \frac{\theta_i((1-\theta_j)\Pi + \theta_j\Pi^{-j} - D_0)}{1-\theta_j}$ when $D_0 > ((1-\theta_j + \theta_i\theta_j)\Pi^{-j} - (1-\theta_i)(1-\theta_j)\Pi)/\theta_i$; $\pi_i = \theta_i(\Pi - D_0)$ when $D_0 < (\Pi^{-j} - (1-\theta_i)\Pi)/\theta_i$; and $\pi_i = \Pi - \Pi^{-j}$ otherwise. π_i is weakly decreasing in D_0, and the buyer's profit $\pi_0 = \Pi - \pi_i - \theta_j(\Pi - \pi_i - \Pi^{-j})^+$ is weakly increasing in D_0. Notice that at $\tilde{\pi}_i = 0$, the reservation utility of first accepting supplier i's offer is $D_0^A(\tilde{\pi}_j) \equiv \max\{\Pi - \tilde{\pi}_j, \Pi - \pi_j(0, \Pi^{-j}(q_i^o))\}$, while the reservation utility of first rejecting supplier i's offer is $D_0^R(\tilde{\pi}_j) \equiv \max\{(1-\theta_j)\Pi^{-i}, (1-\theta_j)\Pi^{-i} + \theta_j\Pi^{-i}(q_j^o) - \tilde{\pi}_j\}$. Furthermore, $\Pi - \tilde{\pi}_j > (1-\theta_j)\Pi^{-i} + \theta_j\Pi^{-i}(q_j^o) - \tilde{\pi}_j$ and $\Pi - \pi_j(0, \Pi^{-j}(q_i^o)) = \Pi - \theta_j(\Pi - \Pi^{-j}(q_i^o)) > (1-\theta_j)\Pi^{-i}$. $D_0^A(\tilde{\pi}_j) > D_0^R(\tilde{\pi}_j)$, and the higher reservation utility implies a (weakly) higher buyer's profit.

Therefore, if $\tilde{\pi}_i = 0$, the buyer should accept supplier i's offer immediately. □

LEMMA 7. *The buyer should accept both offers without renegotiation if and only if* $\tilde{\pi}_j \leq \theta_j(\Pi - \tilde{\pi}_i - \Pi^{-j})^+$ *for* $\{i,j\} = \{1,2\}$.

Proof of Lemma 7. By Lemma 4, the buyer should accept both offers only if $\tilde{\pi}_j \leq \theta_j(\Pi - \tilde{\pi}_i - \Pi^{-j})^+$ for $\{i,j\} = \{1,2\}$. Now, we prove the "if" part of the statement.

Suppose that $\tilde{\pi}_j \leq \theta_j(\Pi - \tilde{\pi}_i - \Pi^{-j})^+$ for $\{i,j\} = \{1,2\}$, if the buyer accepts either offer first, it is optimal for the buyer to accept both offers by Lemma 4. Furthermore, if either $\tilde{\pi}_i$ or $\tilde{\pi}_j$ is 0, it is optimal to accept both offers by Lemma 6. It suffices to show that accepting both offers dominates the choice that first rejects an offer when $\tilde{\pi}_j \leq \theta_j(\Pi - \tilde{\pi}_i - \Pi^{-j})$ for $\{i,j\} = \{1,2\}$.

WLOG, we assume that rejects supplier i's offer first, the buyer will then reject supplier j's offer by Lemma 5. We examine both the degenerate case and the regular case.

In the degenerate case, the minimal buyer's profit equals to $\Pi - \min\{\theta_i(\Pi - \Pi^{-i}), \theta_j(\Pi - \Pi^{-j})\}$ when the buyer accepts both offers and $0 \leq \tilde{\pi}_j \leq \theta_j(\Pi - \tilde{\pi}_i - \Pi^{-j})$ for $\{i,j\} = \{1,2\}$. Notice that with R_iR_j, the suppliers' aggregate profit $\pi_i + \pi_j = \pi_i + \theta_j(\Pi - \pi_i - \Pi^{-j})^+$, which is a monotone increasing function of π_i. Therefore, $\pi_i + \pi_j \geq \theta_j(\Pi - \Pi^{-j})$ when the buyer chooses R_iR_j, and the buyer should choose A_iA_j and accept both offers.

In the regular case, the minimal buyer's profit equals to $\Pi - \pi_i' - \theta_j(\Pi - \pi_i' - \Pi^{-j})$ with $\pi_i' = \theta_i \frac{(1-\theta_j)\Pi + \theta_j \Pi^{-j} - \Pi^{-i}}{1-\theta_i\theta_j}$ when the buyer accepts both offers and $0 \le \tilde{\pi}_j \le \theta_j(\Pi - \tilde{\pi}_i - \Pi^{-j})$ for $\{i,j\} = \{1,2\}$. By the monotonicity of $\pi_i + \theta_j(\Pi - \pi_i - \Pi^{-j})^+$ with respect to π_i, it suffices to show that when $\tilde{\pi}_i \le \theta_i(\Pi - \Pi^{-i})$ and the buyer prefers R_iR_j over A_iR_j, supplier i's payoff with R_iR_j is at least $\theta_i(\Pi - \Pi^{-i})$, which is greater than $\pi_i' = \theta_i \frac{(1-\theta_j)\Pi + \theta_j \Pi^{-j} - \Pi^{-i}}{1-\theta_i\theta_j}$ in the regular case.

By Lemma 3, when $\tilde{\pi}_j \ge \theta_j \Pi^{-i}(q_i^o)$, supplier i's payoff π_i with R_iR_j is higher than $\theta_i(\Pi - \Pi^{-i})$ with $\frac{\partial \pi_i}{\partial \Pi} \ge \theta_i$. Now suppose that $\tilde{\pi}_j \le \theta_j \Pi^{-i}(q_i^o)$. When $\frac{1}{1-\theta_i}\left(\left(1 + \frac{\theta_i\theta_j}{1-\theta_j}\right)\Pi^{-j} - \frac{\theta_i}{1-\theta_j}((1-\theta_j)\Pi^{-i} + \theta_j\Pi^{-i}(q_j^o) - \tilde{\pi}_j)\right) > \frac{1}{1-\theta_i}\left(\Pi^{-j} + \theta_i(\tilde{\pi}_j - \theta_j\Pi^{-i}(q_j^o) - (1-\theta_j)\Pi^{-i})\right)$, supplier i's payoff π_i is higher than $\theta_i(\Pi - \Pi^{-i})$ with $\frac{\partial \pi_i}{\partial \Pi} \ge \theta_i$. The proof of Lemma 3 rules out the possibility of $\frac{1}{1-\theta_i}\left(\left(1 + \frac{\theta_i\theta_j}{1-\theta_j}\right)\Pi^{-j} - \frac{\theta_i}{1-\theta_j}((1-\theta_j)\Pi^{-i} + \theta_j\Pi^{-i}(q_j^o) - \tilde{\pi}_j)\right) \le \Pi \le \frac{1}{1-\theta_i}\left(\Pi^{-j} + \theta_i(\tilde{\pi}_j - \theta_j\Pi^{-i}(q_j^o) - (1-\theta_j)\Pi^{-i})\right)$. Therefore, when $\frac{1}{1-\theta_i}\left(\left(1 + \frac{\theta_i\theta_j}{1-\theta_j}\right)\Pi^{-j} - \frac{\theta_i}{1-\theta_j}((1-\theta_j)\Pi^{-i} + \theta_j\Pi^{-i}(q_j^o) - \tilde{\pi}_j)\right) \le \frac{1}{1-\theta_i}\left(\Pi^{-j} + \theta_i(\tilde{\pi}_j - \theta_j\Pi^{-i}(q_j^o) - (1-\theta_j)\Pi^{-i})\right)$, either $\Pi < \frac{1}{1-\theta_i}\left(\Pi^{-j} + \theta_i(\tilde{\pi}_j - \theta_j\Pi^{-i}(q_j^o) - (1-\theta_j)\Pi^{-i})\right)$ and supplier i's payoff is higher than $\theta_i(\Pi - \Pi^{-i})$, or $\Pi > \frac{1}{1-\theta_i}\left(\left(1 + \frac{\theta_i\theta_j}{1-\theta_j}\right)\Pi^{-j} - \frac{\theta_i}{1-\theta_j}((1-\theta_j)\Pi^{-i} + \theta_j\Pi^{-i}(q_j^o) - \tilde{\pi}_j)\right)$. We show that under the latter case, the buyer would prefer A_iR_j over R_iR_j when $\tilde{\pi}_i \le \theta_i(\Pi - \Pi^{-i})$; thus, when the buyer prefers R_iR_j over A_iR_j, supplier i's payoff with R_iR_j is at least $\theta_i(\Pi - \Pi^{-i})$.

Because the buyer profit is $\Pi - \pi_i - \theta_j(\Pi - \pi_i - \Pi^{-j})^+$, which is decreasing in π_i, it suffices to show that supplier i's profit is lower with A_iR_j than it is with R_iR_j when $\tilde{\pi}_i \le \theta_i(\Pi - \Pi^{-i})$, $\tilde{\pi}_j \le \theta_j\Pi^{-i}(q_i^o)$, and $\Pi > \frac{1}{1-\theta_i}\left(\left(1 + \frac{\theta_i\theta_j}{1-\theta_j}\right)\Pi^{-j} - \frac{\theta_i}{1-\theta_j}((1-\theta_j)\Pi^{-i} + \theta_j\Pi^{-i}(q_j^o) - \tilde{\pi}_j)\right)$. It suffices to show that the statement is true when $\tilde{\pi}_j \le \theta_j(\Pi - \Pi^{-j}(q_i^o))$, because supplier i's profit with A_iR_j is a constant in $\tilde{\pi}_j$ when $\tilde{\pi}_j \ge \theta_j(\Pi - \Pi^{-j}(q_i^o))$ by Lemma 2, and supplier i's profit with R_iR_j is a weakly increasing function of $\tilde{\pi}_j$ by Lemma 3. Now, we prove by contradiction. Suppose that supplier i's profit is higher with A_iR_j than it is with R_iR_j for some $\tilde{\pi}_i \le \theta_i(\Pi - \Pi^{-i})$ and $\tilde{\pi}_j \le \min\{\theta_j\Pi^{-i}(q_i^o), \theta_j(\Pi - \Pi^{-j}(q_i^o))\}$ when $\Pi > \frac{1}{1-\theta_i}\left(\left(1 + \frac{\theta_i\theta_j}{1-\theta_j}\right)\Pi^{-j} - \frac{\theta_i}{1-\theta_j}((1-\theta_j)\Pi^{-i} + \theta_j\Pi^{-i}(q_j^o) - \tilde{\pi}_j)\right)$. The proof of Lemma 6 shows that supplier i's profit is lower with A_iR_j than it is with R_iR_j at $\tilde{\pi}_i = 0$. By the continuity of the profit functions with respect to $\tilde{\pi}_i$ with A_iR_j and R_iR_j, there exists $\tilde{\pi}_i \le \theta_i(\Pi - \Pi^{-i})$ such that supplier i's profit is the same under both A_iR_j and R_iR_j. When $\tilde{\pi}_j \le \theta_j\Pi^{-i}(q_i^o)$ and $\Pi > \frac{1}{1-\theta_i}\left(\left(1 + \frac{\theta_i\theta_j}{1-\theta_j}\right)\Pi^{-j} - \frac{\theta_i}{1-\theta_j}((1-\theta_j)\Pi^{-i} + \theta_j\Pi^{-i}(q_j^o) - \tilde{\pi}_j)\right)$, supplier i's profit with R_iR_j is less than $\Pi - \Pi^{-j}$. Because supplier i's profit is the same under both A_iR_j and R_iR_j, this implies that supplier i's profit expression with A_iR_j follows the subcase $\tilde{\pi}_j \le \theta_j(\Pi - \Pi^{-j}(q_i^o))$ and $\Pi > \Pi^{-j} + \tilde{\pi}_i + \frac{\theta_i}{1-\theta_j+\theta_i\theta_j}\tilde{\pi}_j$. Supplier i's profit with R_iR_j is then $\frac{\theta_i((1-\theta_j)(\Pi-\Pi^{-i}) + \tilde{\pi}_j - \theta_j\Pi^{-i}(q_j^o) + \theta_j\Pi^{-j})}{1-\theta_j}$, which is greater than $\frac{\theta_i(\tilde{\pi}_j - \theta_j(\Pi-\Pi^{-j})) + \theta_i(\Pi-\Pi^{-i})}{1-\theta_j} > \frac{\theta_i(\tilde{\pi}_j - \theta_j(\Pi-\Pi^{-j})) + (1-\theta_j+\theta_i\theta_j)\tilde{\pi}_i}{1-\theta_j} = \tilde{\pi}_i + \frac{\theta_i(\tilde{\pi}_j - \theta_j(\Pi - \tilde{\pi}_i - \Pi^{-j}))}{1-\theta_j}$, supplier i's profit with A_iR_j when $\tilde{\pi}_i \le \theta_i(\Pi - \Pi^{-i})$. Thus, we reach a contradiction.

Therefore, the buyer should choose A_iA_j and accept both offers in the regular case as well. □

LEMMA 8. *In the regular case, there exists $\delta > 0$ such that when supplier i quotes $\tilde{\pi}_i < \delta$, his offer is accepted immediately* $(i = 1, 2)$.

Proof of Lemma 8. We first show that for given $\tilde{\pi}_j^o (> 0)$, there exist $\epsilon, \delta > 0$ such that when $\tilde{\pi}_j \in (\tilde{\pi}_j^o - \epsilon, \tilde{\pi}_j^o + \epsilon)$ and $\tilde{\pi}_i < \delta$, the buyer accepts supplier i's offer immediately in the regular case. It suffices to show that there exist $\epsilon, \delta > 0$ such that when $\tilde{\pi}_j \in (\tilde{\pi}_j^o - \epsilon, \tilde{\pi}_j^o + \epsilon)$ and $\tilde{\pi}_i < \delta$, the buyer will not first reject neither supplier j's offer nor supplier i's offer, nor first accept supplier j's offer and then reject supplier i's offer.

We consider the case of first rejecting supplier j's offer. At $\tilde{\pi}_i = 0$, after rejecting supplier j's offer, the buyer's optimal choice is $R_j A_i$, which yields a strictly suboptimal buyer's profit by the proof of Lemma 5. By the continuity of the profit functions under all the negotiation strategies, there exist $\epsilon, \delta > 0$ such that when $\tilde{\pi}_j \in (\tilde{\pi}_j^o - \epsilon, \tilde{\pi}_j^o + \epsilon)$ and $\tilde{\pi}_i < \delta$, the buyer will not first reject supplier j's offer.

We then consider the case of first rejecting supplier i's offer. By the proof of Lemma 6, when $\tilde{\pi}_j = \tilde{\pi}_j^o \leq \theta_j(\Pi - \Pi^{-j})$, the buyer's profit when first rejecting supplier i is strictly suboptimal at $\tilde{\pi}_i = 0$. By the continuity of the profit functions under all the negotiation strategies, there exist $\epsilon, \delta > 0$ such that when $\tilde{\pi}_j \in (\tilde{\pi}_j^o - \epsilon, \tilde{\pi}_j^o + \epsilon)$ and $\tilde{\pi}_i < \delta$, the buyer will not first reject supplier i's offer. When $\tilde{\pi}_j = \tilde{\pi}_j^o > \theta_j(\Pi - \Pi^{-j})$ and $\tilde{\pi}_i$ is small, we only need to show that buyer's profit π_0 in $A_i R_j$ dominates her profit in $R_i R_j$ in a neighborhood by Lemma 5. For both $A_i R_j$ and $R_i R_j$, at $\tilde{\pi}_i = 0$, the contract between the buyer and supplier i solves

$$(\pi_i, \pi_0^{-j}) = \arg\max (\pi_i - 0)^{\theta_i} (\Pi - \pi_i - \pi_j(\pi_i, \pi_0^{-j}) - D_0)^{1-\theta_i}$$

$$s.t.\ \pi_i \geq 0, \Pi - \pi_i - \pi_j(\pi_i, \pi_0^{-j}) \geq D_0, \pi_0^{-j} \leq \Pi^{-j}, \Pi - \pi_i - \pi_0^{-j} \geq 0.$$

Recall from the proof of Lemma 6, π_i is strictly decreasing and π_0 is strictly increasing in the reservation utility D_0 when $D_0 > ((1 - \theta_j + \theta_i\theta_j)\Pi^{-j} - (1 - \theta_i)(1 - \theta_j)\Pi)/\theta_i$ or $D_0 < (\Pi^{-j} - (1 - \theta_i)\Pi)/\theta_i$; and the buyer's reservation utility with $A_i R_j$, $D_0^A(\tilde{\pi}_j) = \max\{\Pi - \tilde{\pi}_j, \Pi - \pi_j(0, \Pi^{-j}(q_i^o))\}$, is strictly higher than the reservation utility with $R_i R_j$, $D_0^R(\tilde{\pi}_j) = \max\{(1 - \theta_j)\Pi^{-i}, (1 - \theta_j)\Pi^{-i} + \theta_j \Pi^{-i}(q_j^o) - \tilde{\pi}_j\}$. Therefore, if either $D_0^R(\tilde{\pi}_j) \geq ((1 - \theta_j + \theta_i\theta_j)\Pi^{-j} - (1 - \theta_i)(1 - \theta_j)\Pi)/\theta_i)$ or $D_0^A(\tilde{\pi}_j) \leq (\Pi^{-j} - (1 - \theta_i)\Pi)/\theta_i$, the buyer's profit with $R_i R_j$ is strictly suboptimal. By the continuity of the profit functions under all the negotiation strategies, there exist $\epsilon, \delta > 0$ such that when $\tilde{\pi}_j \in (\tilde{\pi}_j^o - \epsilon, \tilde{\pi}_j^o + \epsilon)$ and $\tilde{\pi}_i < \delta$, the buyer chooses not to first reject supplier i's offer. When $D_0^R(\tilde{\pi}_j) < ((1 - \theta_j + \theta_i\theta_j)\Pi^{-j} - (1 - \theta_i)(1 - \theta_j)\Pi)/\theta_i)$, by the boundary conditions in Lemma 3, we can find a neighborhood around $(\tilde{\pi}_i, \tilde{\pi}_j) = (0, \tilde{\pi}_j^o)$ such that the buyer's profit with $R_i R_j$ is no more than Π^{-j}. Similarly, when $D_0^A(\tilde{\pi}_j) > (\Pi^{-j} - (1 - \theta_i)\Pi)/\theta_i$, by the boundary conditions in Lemma 2, we can find a neighborhood around $(\tilde{\pi}_i, \tilde{\pi}_j) = (0, \tilde{\pi}_j^o)$ such that the buyer's profit with $A_i R_j$ is no less than Π^{-j}. Therefore, there exist $\epsilon, \delta > 0$ such that when $\tilde{\pi}_j \in (\tilde{\pi}_j^o - \epsilon, \tilde{\pi}_j^o + \epsilon)$ and $\tilde{\pi}_i < \delta$, the buyer chooses not to first reject supplier i's offer.

Last we consider the case of first accepting supplier j's offer and then rejecting supplier i's offer. Because $\theta_j(\Pi - \Pi^{-j}) < \Pi - \Pi^{-i}$ in the regular case, there exists $\delta' \in (0, \Pi - \Pi^{-i} - \theta_j(\Pi - \Pi^{-j}))$. If $\tilde{\pi}_j \le \Pi - \Pi^{-i} - \delta'$, we show that when $\tilde{\pi}_i < \theta_i \delta'$, $A_j R_i$ is suboptimal. Notice that $\tilde{\pi}_i < \theta_i(\Pi - \Pi^{-i} - \tilde{\pi}_j) \le \theta_i(\Pi - \Pi^{-i} - \tilde{\pi}_j)^+$. If the buyer first accepts supplier j's offer, she will also accept supplier i's offer by Lemma 4, that is, the buyer prefers $A_j A_i$ over $A_j R_i$. If $\tilde{\pi}_j > \Pi - \Pi^{-i} - \delta' > \theta_j(\Pi - \Pi^{-j})$, we show that the buyer's profit of first accepting supplier i's offer is strictly higher than the profit of first accepting supplier j's offer at $\tilde{\pi}_i = 0$. If the buyer first accepts supplier j's offer, the buyer also immediately accepts supplier i's offer by Lemma 4. Nevertheless, accepting both offers is not optimal by Lemma 7. Therefore, accepting supplier j's offer first yields a strictly suboptimal profit at $\tilde{\pi}_i = 0$. By the continuity of the profit functions under all the negotiation strategies, there exist $\epsilon, \delta > 0$ such that when $\tilde{\pi}_j \in (\tilde{\pi}_j^o - \epsilon, \tilde{\pi}_j^o + \epsilon)$ and $\tilde{\pi}_i < \delta$, the buyer chooses not to first accepts supplier j's offer.

Therefore, for given $\tilde{\pi}_j^o > 0$, there exist $\epsilon, \delta > 0$ such that when $\tilde{\pi}_j \in (\tilde{\pi}_j^o - \epsilon, \tilde{\pi}_j^o + \epsilon)$ and $\tilde{\pi}_i < \delta$, the buyer accepts supplier i's offer immediately in the regular case. Notice that when $\tilde{\pi}_j^o$ is sufficiently large, the buyer would accept supplier i's offer if $\tilde{\pi}_i < \delta$ for some constant $\delta > 0$; when $\tilde{\pi}_j^o = 0$, we may choose $\epsilon = \delta = \min\{\theta_i(\Pi - \Pi^{-i}), \theta_j(\Pi - \Pi^{-j})\}/2 > 0$ to ensure that the buyer accepts both offers immediately when $\tilde{\pi}_j \in [0, \tilde{\pi}_j^o + \epsilon)$ and $\tilde{\pi}_i < \delta$. As a result, we can view $\tilde{\pi}_j^o$ is on a compact set with an open cover. Because every open cover has a finite subcover, there exists $\delta > 0$ such that supplier i can assure a positive profit by submitting $\tilde{\pi}_i \le \delta$ and getting accepted immediately in the regular case. $\square$

LEMMA 9. *Suppose that the buyer's optimal acceptance/rejection strategy to $(\tilde{\pi}_1, \tilde{\pi}_2)$ is to first contract with supplier i, then reject supplier j's offer and negotiate with him, and supplier j's resulting profit is positive. Under the iterative quotation process, supplier i's optimal response is to keep his current offer price and supplier j can increase his final profit by reducing his offer price.*

Proof of Lemma 9. We consider the two possible optimal bargaining and acceptance/rejection choices for the buyer: $A_i R_j$ and $R_i R_j$ ($\{i, j\} = \{1, 2\}$) as $R_i A_j$ is suboptimal by Lemma 5.

By Lemmas 4 and 5, given the current offers $(\tilde{\pi}_1, \tilde{\pi}_2)$, supplier j's profit is $\pi_j = \theta_j(\Pi - \pi_i - \Pi^{-j})^+$ under both $A_i R_j$ and $R_i R_j$. It is ready to verify that under both $A_i R_j$ and $R_i R_j$, $\pi_i > 0$ and $\pi_j < \theta_j(\Pi - \Pi^{-j})$. When $\pi_j > 0$, $\theta_j \in (0, 1)$ implies that the buyer's profit would be higher if and only if supplier j's profit is higher, and both prefer a lower π_i (with $A_i R_j$ and $R_i R_j$).

We first show that supplier j can increase his final profit by reducing his offer price and thus he can continue the quotation process. Suppose that supplier j gradually reduces his current offer. If the buyer's optimal choice (over A_iR_j/R_iR_j) remains the same, supplier j's profit and the buyer's profit are continuous (weakly) decreasing in $\tilde{\pi}_i$ and $\tilde{\pi}_j$ by Lemmas 4 and 5. If the buyer's optimal choice switches between A_iR_j and R_iR_j, the buyer's profit is same with A_iR_j and R_iR_j due to continuity at the switching point, and supplier j's profit is the same under either choice.

Lemma 8 shows that when $\tilde{\pi}_j$ becomes a sufficiently small positive number, the buyer's optimal choice is to accept supplier j's offer immediately. Given supplier i's quotation $\tilde{\pi}_i$, let $\tilde{\pi}'_j$ denote the supremum of supplier j's offer $(\leq \min\{\tilde{\pi}_j, \theta_j(\Pi - \Pi^{-j})\})$ such that the buyer's optimal choice is to accept supplier j's offer immediately.

We now consider the two probabilities: (i) the buyer chooses not to accept supplier j's offer immediately when supplier j's offer is $\tilde{\pi}'_j$, and (ii) the buyer chooses to accept supplier j's offer immediately when supplier j's offer is $\tilde{\pi}'_j$.

If the buyer chooses not to accept supplier j's offer immediately when supplier j's offer is $\tilde{\pi}'_j$, the buyer obtains the same optimal profit whether he accepts supplier j's offer immediately or not when supplier j's offer is $\tilde{\pi}'_j$ due to the continuity of the buyer's profit in $\tilde{\pi}_j$. Let π'_j denote supplier j's profit when supplier j's offer is $\tilde{\pi}'_j$ and the buyer does not accept supplier j's offer immediately. Because that optimal profit can be obtained by rejecting supplier j's offer, $\tilde{\pi}'_j \geq \pi'_j$. Furthermore, $\pi'_j \geq \pi_j$ because supplier j's profit is weakly decreasing in $\tilde{\pi}_j$ with A_iR_j and R_iR_j.

Now we consider the two probabilities $\pi'_j = \tilde{\pi}'_j$ and $\pi'_j < \tilde{\pi}'_j$ and show that supplier j can improve his profit under either case.

If $\pi'_j = \tilde{\pi}'_j$, we show that the buyer accepts both offers immediately when supplier j reduces his offer to $\tilde{\pi}'_j$. At this reduced offer, the buyer can obtain the optimal profit by either first accepting supplier j's offer or rejecting supplier j's offer after contracting with supplier i. Furthermore, if the buyer obtains the optimal profit and rejecting supplier j's offer after contracting with supplier i, supplier j's profit is his offer. Notice that $\tilde{\pi}'_j > 0$ by Lemma 8 and R_iA_j yields a strictly suboptimal profit when supplier j's offer is positive by the proof of Lemma 5, this implies that the buyer can obtain the optimal profit by either A_iA_j and A_iR_j. Therefore, A_iA_j is the buyer's optimal choice, and we reaches a contradiction by assuming that the buyer chooses not to accept supplier j's offer immediately and $\pi'_j = \tilde{\pi}'_j$.

If $\pi'_j < \tilde{\pi}'_j$, by the definition of $\tilde{\pi}'_j$, there exist $\ddot{\pi}_j \in (\pi'_j, \tilde{\pi}'_j)$ such that the buyer's optimal choice is to accept supplier j's offer immediately if supplier j reduces his offer to $\ddot{\pi}_j$. If supplier i does not reduce his offer price after supplier j's price reduction, the buyer accepts supplier j's offer immediately and supplier j's profit is at least $\ddot{\pi}_j$, which is greater than $\pi'_j \geq \pi_j$. Therefore, supplier j's profit would increase. Suppose that after supplier i's responds to the reduction of $\tilde{\pi}_j$ by reducing his offer price. If supplier i reduces his offer price, either the buyer continues to accept supplier j's offer immediately, or he first rejects supplier j's offer, or he rejects supplier j's offer after contracting with supplier i (i.e., A_iR_j or R_iR_j). If the buyer accepts supplier j's offer immediately, supplier j's profit is at least $\ddot{\pi}_j$, which implies a profit increase. If the buyer first rejects supplier j's offer, it is optimal for the buyer to choose R_jR_i by Lemma 5. By the proof of Lemma 7, when $\ddot{\pi}_j \le \theta_j(\Pi - \Pi^{-j})$ and the buyer prefers R_jR_i over A_jR_i, supplier j's payoff with R_jR_i is at least $\theta_j(\Pi - \Pi^{-j})$. Recall that $\pi_j < \theta_j(\Pi - \Pi^{-j})$, supplier j also enjoys a profit increase. When the buyer chooses A_iR_j/R_iR_j, supplier j's profit is no less than π_j because both the buyer and supplier j's profit is weakly decreasing functions of the offers when supplier j's offer is rejected after the buyer contracts with supplier i.

Now we show that if supplier j's profit remains the same, supplier j can increase his profit by repeating the above process. Note that the buyer profit is weakly decreasing when both suppliers lower their offers. If supplier j's profit remains the same after two iterations, the buyer's profit must remain the same after four swaps of the negotiation sequences, which implies that the sequence swaps are purely due to the tie-breaking rule. Therefore, after each supplier lowers the offer twice, either the buyer and supplier j's profit increases strictly, or the buyer accepts both offers and supplier j's profit also increases strictly.

If the buyer chooses to accept supplier j's offer immediately when supplier j's offer is $\tilde{\pi}'_j$, the reasoning is identical to aforementioned discussion for $\pi'_j < \tilde{\pi}'_j$ by setting $\ddot{\pi}_j = \tilde{\pi}'_j$.

Therefore, when the buyer chooses A_iR_j or R_iR_j, supplier j can increase his final profit by reducing his offer price.

We now show that supplier i's optimal response is to keep his current offer price to end the quotation process with the final quotation of $(\tilde{\pi}_1, \tilde{\pi}_2)$ by contradiction. If supplier i lowers his offer price, let $\tilde{\pi}'_i (< \tilde{\pi}_i)$ denote his final offer and $\tilde{\pi}'_j (\le \tilde{\pi}_j)$ denote supplier j' final offer. We show that supplier i obtains a (weakly) lower profit compared with that of keeping his current offer price. Notice that the buyer's optimal strategy to the final quotation cannot be A_jR_i or R_jR_i, because supplier i as the Stackelberg follower would have incentives to increase his profit by lowering his quotation. Furthermore, if the buyer's optimal strategy to the final quotation is A_iA_j, supplier i will obtain a lower profit because $\tilde{\pi}'_i < \tilde{\pi}_i$ when the buyer chooses A_iR_j to quotation $(\tilde{\pi}_1, \tilde{\pi}_2)$. When the buyer chooses R_iR_j to quotation $(\tilde{\pi}_1, \tilde{\pi}_2)$ and A_iA_j to $(\tilde{\pi}'_1, \tilde{\pi}'_2)$, it is readily verified that by Lemmas 3 and 5, supplier i's profit is greater than $\theta_i(\Pi - \Pi^{-i})$, which is no less than $\tilde{\pi}'_i$ by Lemma 7. Therefore, $(\tilde{\pi}'_i, \tilde{\pi}'_j)$ must in the region of either A_iR_j or R_iR_j. We now suppose that the suppliers gradually reduce their offers from $(\tilde{\pi}_1, \tilde{\pi}_2)$ to $(\tilde{\pi}'_i, \tilde{\pi}'_j)$. If the buyer's optimal strategies are the same for both $(\tilde{\pi}'_i, \tilde{\pi}'_j)$ and $(\tilde{\pi}_1, \tilde{\pi}_2)$, supplier i's profit is continuously (weakly) decreasing in the offer prices by Lemmas 4 and 5. If the buyer's optimal choice switches between A_iR_j and R_iR_j, the buyer's profit is the same with A_iR_j and R_iR_j due to continuity at the switching point, and both suppliers' profits are the same under either choice. By continuity of the profit function of supplier i, and that supplier i's profit is continuous (weakly) decreasing in the offer prices by Lemmas 4 and 5, supplier i will obtain a (weakly) lower profit at $(\tilde{\pi}'_i, \tilde{\pi}'_j)$.

Therefore, supplier i's optimal response is to keep his current offer price and end the quotation process. □

THEOREM 4. *In equilibrium, the buyer accepts both offers in the regular case.*

Proof of Theorem 4. In equilibrium, both suppliers receive positive profit, otherwise one of the suppliers can increase his final profit by reducing his offer price by Lemma 8. If the buyer does not accept both offers, Lemma 9 shows that the current quotation is not an equilibrium because one of the suppliers can increase his profit by reducing his offer price. Therefore, the buyer would accept both offers in equilibrium. □

THEOREM 5. *In equilibrium, the buyer's profit with the quotation process is no less than the profit of the most favorable bargaining outcome in the regular case.*

Proof of Theorem 5. By Theorem 4, the buyer's profit is $\Pi - \tilde{\pi}_i - \tilde{\pi}_j$ in equilibrium, while $\tilde{\pi}_j \le \theta_j(\Pi - \tilde{\pi}_i - \Pi^{-j})^+$ for $\{i,j\} = \{1,2\}$ by Lemma 7. This implies that $\tilde{\pi}_i \le \theta_i(\Pi - \Pi^{-i}) < \Pi - \Pi^{-j}$ in the regular case. Therefore, in equilibrium, $\tilde{\pi}_j \le \theta_j(\Pi - \tilde{\pi}_i - \Pi^{-j})$ for $\{i,j\} = \{1,2\}$.

The buyer's minimum profit can be solved by a linear program and it is ready to verify that the minimum is obtained when $\tilde{\pi}_j = \theta_j(\Pi - \tilde{\pi}_i - \Pi^{-j})$. Recall that Propositions 1 and 2 show that in the regular case, the most favorable bargaining outcome is $\pi_i = \theta_i \frac{(1-\theta_j)\Pi - \Pi^{-i} + \theta_j \Pi^{-j}}{1-\theta_i\theta_j}$ and satisfy $\pi_i = \theta_i(\Pi - \pi_j - \Pi^{-i})$.

Therefore, the buyer's profit under the iterative quotation process is no less than the profit of the most favorable bargaining outcome in the regular case. □

THEOREM 6. *If an equilibrium exists under the one-shot simultaneous RFQ process in the regular case, the buyer's equilibrium profit is the profit of the most favorable bargaining outcome.*

Proof of Theorem 6. By the proof of Theorem 5, in equilibrium, $\tilde{\pi}_j \le \theta_j(\Pi - \tilde{\pi}_i - \Pi^{-j})$ for $\{i,j\} = \{1,2\}$.

When $\tilde{\pi}_j = \theta_j(\Pi - \tilde{\pi}_i - \Pi^{-j})$ for $\{i,j\} = \{1,2\}$, quotations $(\tilde{\pi}_1, \tilde{\pi}_2)$ correspond to the suppliers' profit in the most favorable bargaining outcome by Propositions 1 and 2.

When $\tilde{\pi}_j < \theta_j(\Pi - \tilde{\pi}_i - \Pi^{-j})$ for $\{i,j\} = \{1,2\}$, quotations $(\tilde{\pi}_1, \tilde{\pi}_2)$ is not an equilibrium because either supplier can increase his profit by increasing his quotation while ensuring the buyer accepts both offers by Lemma 7.

To prove the theorem, it suffices to show that when $\tilde{\pi}_1 < \theta_1(\Pi - \tilde{\pi}_2 - \Pi^{-1})$ and $\tilde{\pi}_2 = \theta_2(\Pi - \tilde{\pi}_1 - \Pi^{-2})$, quotations $(\tilde{\pi}_1, \tilde{\pi}_2)$ is not an equilibrium because supplier 1 can increase his profit by increasing his quotation to $\tilde{\pi}_1 + d\tilde{\pi}_1 \leq \theta_1(\Pi - \tilde{\pi}_2 - \Pi^{-1})$.

Given the quotation $(\tilde{\pi}_1 + d\tilde{\pi}_1, \tilde{\pi}_2)$, if it is optimal for the buyer to accept supplier 1's offer immediately, supplier 1 would secure higher profit and $(\tilde{\pi}_1, \tilde{\pi}_2)$ is not an equilibrium. Now, we consider the remaining acceptance/rejection strategies for the buyer, namely, $A2R1$, $R1A2$, $R1R2$, and $R2R1$.

Because $\tilde{\pi}_1 + d\tilde{\pi}_1 \leq \theta_1(\Pi - \tilde{\pi}_2 - \Pi^{-1})$, Lemma 4 implies that $A2R1$ cannot be optimal. Furthermore, Lemma 5 shows that $R1A2$ cannot be optimal.

Now, we show that if the buyer's optimal response strategy is $R1R2$, supplier 1 obtains higher profit by deviating from $\tilde{\pi}_1$ to $\tilde{\pi}_1 + d\tilde{\pi}_1$. First, when $\tilde{\pi}_2 > \theta_2\Pi^{-1}(q_2^o)$: if $\Pi \leq \frac{1}{1-\theta_1}(\Pi^{-2} - \theta_1(1-\theta_2)\Pi^{-1})$, $\pi_1 = \theta_1(\Pi - (1-\theta_2)\Pi^{-1}) > \theta_1(\Pi - \tilde{\pi}_2 - \Pi^{-1}) > \tilde{\pi}_1$ by Lemma 3; because π_1 is increasing in Π with a rate no less than θ_1, $\pi_1 > \tilde{\pi}_1$ continue to hold when Π increases. Second, when $\tilde{\pi}_2 \leq \theta_2\Pi^{-1}(q_2^o)$: if $\Pi \leq \frac{1}{1-\theta_1}(\Pi^{-2} + \theta_1(\tilde{\pi}_2 - \theta_2\Pi^{-1}(q_2^o) - (1-\theta_2)\Pi^{-1}))$, $\pi_1 = \theta_1(\Pi + \tilde{\pi}_2 - \theta_2\Pi^{-1}(q_2^o) - (1-\theta_2)\Pi^{-1}) > \theta_1(\Pi - \Pi^{-1}) > \tilde{\pi}_1$ by Lemma 3; because π_1 is increasing in Π with a rate no less than θ_1, $\pi_1 > \tilde{\pi}_1$ continue to hold when Π increases. Therefore, if the buyer's optimal response strategy is $R1R2$, supplier 1 obtains higher profit by deviating from $\tilde{\pi}_1$ to $\tilde{\pi}_1 + d\tilde{\pi}_1$.

Last, we show that $R2R1$ cannot be optimal. By the argument in the previous paragraph, we can establish that supplier 2's profit $\pi_2 > \tilde{\pi}_2$ under $R2R1$ and the buyer's profit is $\Pi - \pi_2 - \theta_1(\Pi - \pi_2 - \Pi^{-1})$ by Lemma 3. Nevertheless, Lemma 2, the buyer's profit under $A2R1$ is $\Pi - \tilde{\pi}_2 - \theta_1(\Pi - \tilde{\pi}_2 - \Pi^{-1})$, which is greater than $\Pi - \pi_2 - \theta_1(\Pi - \pi_2 - \Pi^{-1})$. Therefore, $R2R1$ cannot be optimal.

That is, when $\tilde{\pi}_1 < \theta_1(\Pi - \tilde{\pi}_2 - \Pi^{-1})$ and $\tilde{\pi}_2 = \theta_2(\Pi - \tilde{\pi}_1 - \Pi^{-2})$, quotations $(\tilde{\pi}_1, \tilde{\pi}_2)$ is not an equilibrium because supplier 1 can increase his profit by increasing his quotation. $\square$

PROPOSITION 4. *The equilibrium solution* $\boldsymbol{\pi}^* \equiv \boldsymbol{\pi}(\min\{\Pi^{-1}, (1-\theta_2)\Pi + \theta_2\Pi^{-2}\}, \min\{\Pi^{-2}, (1-\theta_1)\Pi + \theta_1\Pi^{-1}\})$ *offers the buyer the maximum profit among all the simultaneous bilateral bargaining equilibria, where* $\Pi \equiv E_\xi[\max_{q_1,q_2\geq 0}\Pi(q_1,q_2|\xi)]$ *and* $\Pi^{-i} \equiv E_\xi[\max_{q_j}\Pi^{-i}(q_j)]$ *for* $\{i,j\} = \{1,2\}$.

Proof of Proposition 4. It is ready to verify that a CPFF contract is capable of implementing the equilibrium solution $\boldsymbol{\pi}^* \equiv \boldsymbol{\pi}(\min\{\Pi^{-1}, (1-\theta_2)\Pi + \theta_2\Pi^{-2}\}, \min\{\Pi^{-2}, (1-\theta_1)\Pi + \theta_1\Pi^{-1}\})$, where $\Pi = E_\xi[\max_{q_1,q_2\geq 0}\Pi(q_1,q_2|\xi)]$ and $\Pi^{-i} = E_\xi[\max_{q_j}\Pi^{-i}(q_j)]$ for $\{i,j\} = \{1,2\}$.

Furthermore, this equilibrium solution offers the buyer the maximum profit among all the simultaneous bilateral bargaining equilibria because Proposition 2 shows that the buyer's expected profit is monotone increasing in the expected supply chain profit and the expected single-sourcing supply chain profit in equilibrium and the expected equilibrium supply chain profit is bounded by $E_\xi[\max_{q_1,q_2\geq 0}\Pi(q_1,q_2|\xi)]$ and the expected single-sourcing supply chain profit is bounded by $E_\xi[\max_{q_j}\Pi^{-i}(q_j)]$ for $\{i,j\}=\{1,2\}$. $\square$