Responsibility Centers, Decision Rights, and Synergies*

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Abstract

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We consider the optimal allocation of decision rights in an incomplete contracting setting where business unit managers choose inputs that enhance the efficiency of "joint projects" (projects that benefit their own and other divisions). With scalable project inputs, decision rights should be *bundled* in the hands of one division manager. Which of the managers to designate the investment center manager—that facing the more volatile or the more stable environment—depends on whether the project input is a monetary investment or personally-costly effort. With discrete project-specific inputs, on the other hand, it is always optimal to *split* decision rights symmetrically between the managers provided they face comparable levels of operating volatility. The model also generates empirical predictions about the effect of contractual incompleteness on the managers' relative incentive strength: bundling of decision rights results in PPS divergence across divisions; the symmetric regime results in PPS convergence.

Keywords: responsibility centers, task allocation, pay-performance sensitivity, investments, risk

JEL codes: M41, D23, D86

1 Introduction

How should decision rights over investments that affect more than one divisions (synergies) within a firm be assigned to the units? Should they be bundled in the hands of a single division manager, making that division an investment center and the other(s) mere profit centers, or should the decision rights be distributed more evenly? The management literature invokes information and coordination needs as key determinants of this design choice (e.g., Sosa and Mihm 2011): if dispersed information is central for the investment choice, and such information cannot easily be shared, then decision rights should be decentralized; otherwise, they should be bundled to improve coordination. By contrast, we argue in this paper that even in the absence of informational frictions among the business units involved in a "joint project," splitting decision rights between the managers can be advantageous.

We study a setting with two business units that are engaged in stand-alone (general) operations and collaborate on a project. The efficiency of the project can be enhanced by upfront specific investments. For example, in a supply chain an upstream investment (e.g., in product design) may reduce the marginal production cost; a downstream investment (e.g., in marketing) may increase the customers' willingness to pay. We adopt an incomplete contracting approach by assuming both *ex-ante* investments and *ex-post* project proceeds are non-contractible. However, we allow for the principal to assign decision rights over investments to the business units, and we ask whether each division manager should be in charge of one investment decision (the *symmetric* regime), or whether one manager should choose both (*bundling*)—and if the latter, which manager.

Non-verifiability of investments has two consequences: the manager in charge of making an investment will select it in his own best interest given his compensation contract, and he will have the attendant fixed cost charged against his divisional income measure on which his compensation is based.[1] With ex-post

[1] There is no inconsistency in assuming nonverifiable investments and, at the same time,

project returns non-contractible, due to an unverifiable state realization that is symmetrically observed by the division managers but not by the principal, we assume the managers split the surplus equally at the margin. To avoid trivial arguments favoring a particular allocation of decision rights, we assume these rights can be transferred across divisions at no cost. The managers in our model are risk averse, and they are identical except for the fact that their divisions face differential levels of uncertainty in their operations.

We show that scalable (continuous) investments should be bundled in the hands of the manager facing the more volatile operating environment. This result builds on the observation in Baldenius and Michaeli (2017) that state uncertainty translates into outcome risk and thus into compensation risk, and that specific investments add to this risk. Therefore, an increase in a manager's pay-performance sensitivity (PPS) elicits greater general-purpose effort but depresses investment incentives by making his compensation more sensitive to the incremental risk.[2] The manager facing greater volatility has muted PPS to begin with and thus greater *induced risk tolerance*, at the margin. He is more willing to invest, which mitigates the hold-up problem associated with surplus splitting (Williamson, 1975).

If project investments are lumpy (binary), on the other hand, decision rights should be split symmetrically between the managers if their operating volatility levels are not too different. The reason is that for lumpy investments an additional strategic effect comes into play, related to the coordination motif from the opening paragraph, but running counter to conventional wisdom. Specific investments tend to be strategic complements: by improving the efficiency of the project at the margin, an investment increases the optimal project scale, which in

fixed cost charges being applied to divisional P&Ls. In practice, divisions invest all the time to support their various activities. While total divisional investments may well be verifiable, disaggregating and assigning them to particular projects is typically infeasible.

[2] While the PPS also scales the project-related cash flows, it does so equally for the cash proceeds and fixed costs. Therefore, the only first-order effect the PPS has on investment incentives is through the marginal risk premium.

turn raises the marginal return to the other investment; and vice versa. Because complementarities are externalities, the standard view is that they are best dealt with by bundling decision rights—e.g., Brynjolfsson and Milgrom (2012, p.13, emphasis added): "When many complementarities among practices exist ... the transition will be difficult, *especially when decisions are decentralized*."

On the contrary, we show that, abstracting from risk considerations, complementary lumpy investments are elicited more cheaply as a (Pareto-dominant) Nash equilibrium of a two-player game (the symmetric regime) than from a single decision maker (bundling). The symmetric regime only requires investing be each player's best response to the other player investing. Under bundling, the investment center manager must prefer investing two units (and paying the attendant fixed costs) to not investing at all. By strategic complementarity, this is a more demanding condition than the Nash condition under the symmetric regime, all else equal. Even taking into account the induced risk tolerance argument, the symmetric regime remains optimal as long as the respective operating environments are sufficiently similar.

We also consider project-specific inputs that are *personally* costly to the manager who chooses them, and label them "project efforts." Changing the nature of specific inputs flips the relation between PPS and equilibrium input levels: standard moral hazard arguments apply as the PPS no longer scales the input cost. Each manager's project effort now is increasing in his PPS.[3] There is no longer any tradeoff between eliciting general-purpose efforts and project-specific inputs. Decision rights over *scalable* investments should again be bundled, but now in the hands of the manager with a more stable environment: facing higher-power PPS, he will exert greater project effort. With *lumpy* project efforts, for sufficiently similar volatility levels, decision rights should again be split symmetrically to

[3] The investment-risk link—or more generally, input-risk link—is present also for personally-costly project efforts. But this second-moment effect is outweighed by the first-moment effect, as in standard moral hazard models, that the PPS now scales up the manager's internalized share of the project proceeds, without affecting how he internalizes the input (effort) costs.

better leverage the strategic complementarity.

Our analysis also sheds light on the effect of contractual incompleteness on the managers' PPS: bundling of decision rights results in PPS *divergence* across divisions; the symmetric regime results in PPS *convergence*. Consider monetary investments. Under bundling, the high-volatility/low-PPS manager is given investment authority; to stimulate investment, the principal will lower his PPS even further. Under the symmetric regime, it is the low-volatility/high-PPS manager who is the bottleneck and whose PPS has to be lowered first to provide investment incentives. These arguments reverse in direction for personally costly project efforts, leaving intact however the prediction of PPS divergence (convergence) under bundling (under the symmetric regime).

Our paper contributes to the literature on task allocation. Darrough and Melumad (1995) and Baiman et al. (1995) study how the organization structure is affected by the relative importance of the business units to the performance of the firm. In Bushman et al. (1995), an agent's action affects the performance of other agents. Holmstrom and Milgrom (1991), Feltham and Xie (1994), Zhang (2003), and Hughes et al. (2005) consider task allocation in multi-tasking settings.[4] In Reichmann and Rohlfing-Bastian (2013) and Hofmann and Indjijekian (2017), the allocation of tasks or contracting power is delegated to lower hierarchical levels. Liang and Nan (2014) and Friedman (2014, 2016) consider models in which agents' actions directly affect the variance of performance measures. Our finding on optimal bundling of scalable personally-costly project efforts is related to this multitasking literature, in that high-powered PPS elicits different dimensions of efforts—some yielding only local benefits, others with externalities across divisions—in lockstep.[5]

Our finding that scalable monetary investments should be bundled in the

[4] Autrey et al. (2010) study the determinants of agency costs due to aggregation in a multi-task setting. Heinle et al. (2012) discuss behavioral incentives in a multitask setting.

[5] A different but related strand of literature deals with the interaction of divisionalized firms' structure and product market competition; e,g., Arya and Mittendorf (2010).

hands of the high-volatility manager builds on Baldenius and Michaeli (2017). The result that decision rights over lumpy (monetary or personally-costly) inputs should be allocated evenly among the managers contrasts with earlier calls for bundling of authority in the presence of complementarities, e.g., Brynjolfsson and Milgrom (2012). The reason is that in our model (i) the managers always split the gross project returns and (ii) the manager that chooses an input also has to "pay" for it. These assumptions seem natural in incomplete contracting settings. Surplus splitting sets our model apart from earlier agency papers that allow for more general contracts to divvy up the output, e.g., Zhang (2003), Hughes et al. (2005), while the linkage of decisions rights and cost charges sets our model apart from the literature on authority, e.g., Dessein et al. (2010).[6]

While our model assumes decision rights can be moved across divisions at no direct cost, this may not always be descriptive. Instead, the firm may be "stuck" with the symmetric regime at times. As noted above, the symmetric regime calls for PPS convergence across divisions. Our model may therefore shed new light on the puzzle of "corporate socialism."[7] In contrast, if tasks can be freely allocated, and they are scalable, our model predicts greater disparity in pay-performance sensitivity across business units as tasks will then be bundled.

The paper proceeds as follows. Section 2 describes the model and the benchmark case. Section 3 and Section 4 consider the optimal allocation of decision rights with scalable and lumpy monetary investments, respectively. Section 5 extends the results to personally-costly project-specific efforts. Section 6 concludes.

[6]For a survey of the authority literature see Bolton and Dewatripont (2012). Unlike this literature, in our model the investment center manager has no authority over the actions of the other manager: "[a]uthority is a supervisor's power to initiate projects and direct subordinates to take certain actions" (Bolton and Dewatripont, 2012, p. 343).

[7]E.g., Levine (1991), Zenger and Hesterly (1997), Shaw et al. (2002), Siegel and Hambrick (2005).

2 Model

Consider two division managers $i = A, B$. Each exerts general (operating) efforts, and together they implement a joint project. The setting builds on Baldenius and Michaeli (2017). The return to general effort, $a_i \in \mathbb{R}_+$, is normalized to one; it is exerted at personal disutility $\frac{v}{2}a_i^2,\ v > 0$. The joint project creates a (gross) surplus $M(q, \theta, \mathbf{k})$, which depends on the project scale, $q \in \mathbb{R}_+$, a random state of nature, $\theta \in \mathbb{R}_+$, and relationship-specific investments, $\mathbf{k} \equiv (k_A, k_B)$, where k_i is chosen from some set $\mathcal{K}_i$, with $\mathcal{K} = \mathcal{K}_A \times \mathcal{K}_B$. (We will consider both the case of scalable investments ($\mathcal{K}_i = \mathbb{R}_+$) and lumpy investments ($\mathcal{K}_i = \{0, 1\}$).) We assume $M(q, \theta, \mathbf{k}) = (\theta + k_A + k_B)q - \frac{q^2}{2}$, i.e., the project surplus is represented by a quadratic function, which in turn can be derived from a standard linear-quadratic supply chain setting.[8]

Before observing θ, the managers choose the investments. Investment k_i comes at a fixed cost of $F(k_i)$. Investments and the state θ are jointly observable to the managers but cannot be communicated to the principal.[9] Having observed θ, the managers implement the project under symmetric information and split the project surplus equally. Hence, they choose the ex-post efficient project scale, $q^*(\mathbf{k}, \theta) \in \arg\max_q M(q, \theta, \mathbf{k}) = \theta + \sum_i k_i$, resulting in a value function of $M(\theta, \mathbf{k}) \equiv M(q^*(\theta, \mathbf{k}), \theta, \mathbf{k}) = \frac{1}{2}(\theta + \sum_i k_i)^2$. With equal probability, the random state variable θ takes values $(\mu - \sqrt{\eta})$ or $(\mu + \sqrt{\eta})$, $\sqrt{\eta} < \mu$, so that $E(\theta) \equiv \mu$ and $Var(\theta) \equiv \eta$. The variance of the project surplus then simplifies to:

$$Var(\mathbf{k}) \equiv Var(M(\theta, \mathbf{k})) = (q^*(\mu, \mathbf{k}))^2\, \eta, \tag{1}$$

so that $Var(\mathbf{k})$ is increasing in each k_i (as $\frac{\partial}{\partial k_i} Var(\mathbf{k}) = 2q^*(\mu, \mathbf{k})\eta > 0$), with increasing differences in $\mathbf{k}$. Specific investments make the joint project more efficient *at the margin* and thereby increase the project scale pointwise, for any

[8] Suppose an upstream unit makes q units of an intermediate good at variable cost $C(q, \theta_A, k_A) = (c - \theta_A - k_A)q$. The downstream unit sells a final product at revenues $R(q, \theta_B, k_B) = \left(r - \frac{q}{2} + \theta_B + k_B\right) q$. Setting $\sum_i \theta_i = \theta$ and $r = c$ (with r sufficiently high to ensure non-negative costs and revenues) recoups the expression for $M(q, \theta, \mathbf{k})$ in the text.

[9] We ignore message games that elicit information from the managers.

θ-realization. *Ex ante*, however, each expected unit of the project is subject to the random shock θ. Hence, specific investment scales up the surplus variance. Baldenius and Michaeli (2017) refer to this as the *investment-risk link*.

The managers split the surplus $M(\cdot)$ equally resulting in divisional income of

$$\pi_i = a_i + \varepsilon_i + \frac{M(\theta, \mathbf{k})}{2} - FC_i(\mathbf{k}), \tag{2}$$

where ε_i is an additively separable random shock to the general environment of the division with mean zero and variance σ_i^2, and $FC_i(\mathbf{k})$ is division i's fixed cost, to be specified below. We confine attention to linear contracts and divisional performance evaluation, $s_i(\pi_i) = \alpha_i + \beta_i \pi_i$, where α_i is Manager i's fixed salary, and $\boldsymbol{\beta} \equiv (\beta_A, \beta_B) \in [0,1]^2$ is the vector of the managers' pay-performance sensitivities (PPS).[10,11] The managers are risk-averse with mean-variance preferences, $EU_i = E[s_i(\cdot)] - \frac{v}{2}a_i^2 - \frac{\rho}{2}Var(s_i(\cdot))$, where $\rho > 0$ is the managers' (common) coefficient of risk aversion. Manager i's expected utility is hence:

$$EU_i = \alpha_i + \beta_i \left(a_i + \frac{E[M(\theta, \mathbf{k})]}{2} - FC_i(\mathbf{k}) \right) - \frac{v}{2}a_i^2 - \frac{\rho}{2}\beta_i^2 \left(\sigma_i^2 + \frac{Var(\mathbf{k})}{4} \right). \tag{3}$$

Surplus splitting serves as a risk sharing instrument (note the scaling of $Var(\mathbf{k})$ in the risk premium term). We label σ_i^2 Division i's *general uncertainty*, and η the *project uncertainty*. Without loss of generality, we rank the general uncertainty levels such that $\sigma_A^2 > \sigma_B^2$, i.e., Division A faces a more volatile environment. All noise terms, θ and ε_i, $i = A, B$, are independent.

To ensure the managers' participation, we impose the individual rationality condition that, for any $i = A, B$,

$$EU_i \geq 0. \tag{4}$$

Moreover, the principal observes the general effort incentive constraints,

$$a_i(\beta_i) \in \arg\max_{a_i} \; EU_i[\cdot \mid \beta_i]. \tag{5}$$

[10] The lion share of managerial compensation in practice is based on divisional profits, see Merchant (1989), Bushman, et al. (1995), Keating (1997), Abernethy, et al. (2004).

[11] The assumption $\beta_i \in [0,1]$ ensures the principal has no incentives to destroy output and the managers have incentives to provide general efforts.

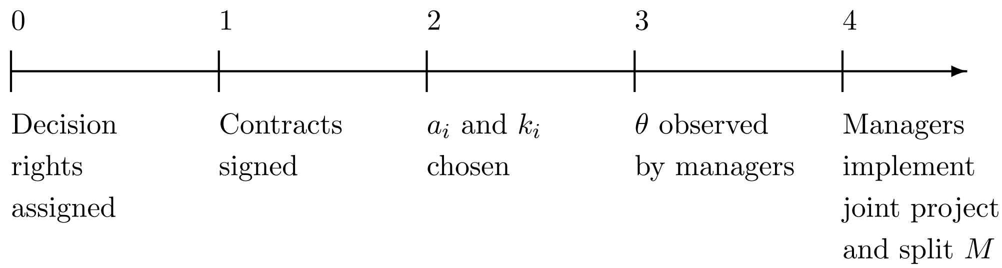


Figure 1: Timeline

Given the assumed technology, each manager's general effort choice is a function only of his own PPS—specifically, it does not depend on any investment choices. The fixed salary α_i will be set to extract all surplus (net of compensation for the managers' personal disutility and risk premium), i.e., to make (4) binding—*ex ante* the managers earn zero rents. The firmwide expected surplus is

$$\begin{aligned} W(\mathbf{k}, \boldsymbol{\beta}) \quad \equiv \quad & E[M(\theta, \mathbf{k})] + \sum_{i=A,B} \left[a_i(\beta_i) - \frac{v}{2}(a_i(\beta_i))^2 - FC_i(\mathbf{k}) \right] \\ & - \sum_{i=A,B} \frac{\rho}{2} \beta_i^2 \left(\sigma_i^2 + \frac{Var(\mathbf{k})}{4} \right). \end{aligned} \tag{6}$$

The timeline is given in Figure 1. At the outset, the principal assigns the investment decision rights and contracts with the managers. The managers choose their investment and general effort levels. The state of nature is realized, the project is implemented, and the payoffs are realized.

Consider as a benchmark the case of contractible investments. The principal instructs the managers as to the investment levels, and the managers subsequently implement the project and split the proceeds. Investments and PPS then are given by (superscript "*" indicates the benchmark):

$$(\mathbf{k}^*, \boldsymbol{\beta}^*) \in \arg \max_{\boldsymbol{\beta} \in [0,1]^2, \, \mathbf{k} \in \mathcal{K}} W(\mathbf{k}, \boldsymbol{\beta}). \tag{7}$$

To address a classic organization design issue, we now turn to non-contractible, delegated investments: how should the principal assign decision rights over specific investments to self-interested managers? We label the *symmetric* regime one in which decision rights are split between the managers: each division is organized as investment centers, and each manager chooses his own investment.[12] Under *bundling*, in contrast, the principal concentrates decision rights in the hands of Manager $\ell \in \{A, B\}$ who then chooses both k_A and k_B; the other manager has no investment authority whatsoever, his unit is organized as a profit center. Irrespective of the regime, both business units remain essential at the Date-4 project implementation stage (in the supply chain example of footnote 8, the upstream unit makes an intermediate good which the downstream unit further processes and sells), and the managers split the surplus $M(\cdot)$ equally.

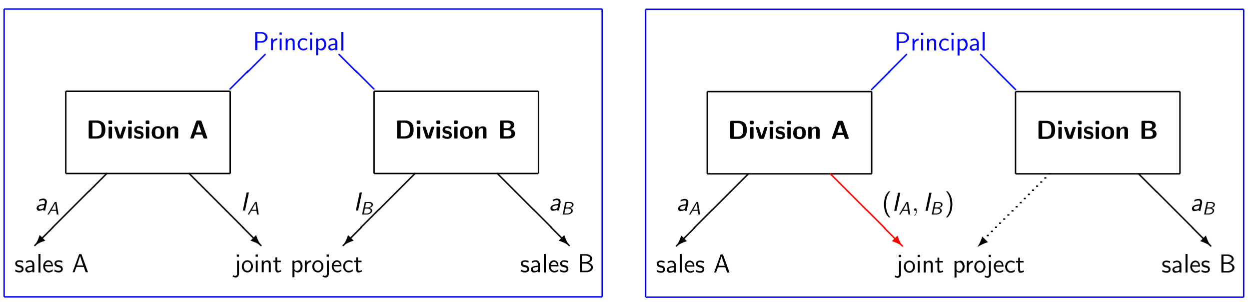


Fig.2a: Symmetric regime **Fig.2b**: Bundling with $\ell = A$

Figure 2: Organizational Modes

We build on earlier studies of settings in which decisions themselves may not be contractible, but decision rights are, e.g., Aghion and Tirole (1997), Hart and Holmstrom (2002), Bester and Krahmer (2008). Note that whenever investments are non-contractible, decision rights over them are inextricably tied to the associated fixed cost charges: if Manager i has the authority to choose k_j, then the

[12] Given our assumption that the investments are equally productive, it is without loss of generality under the symmetric regime that Manager i chooses k_i, $i = A, B$, rather than k_j, $j \neq i$.

fixed cost $F(k_j)$ will reduce his own divisional income measure, π_i, i.e.:[13]

$$FC_i(\mathbf{k}) = \begin{cases} F(k_i), & \text{under the symmetric regime,} \\ (F(k_A) + F(k_B)) \times \mathbb{1}_{i=\ell}, & \text{under bundling with } \ell = i, \end{cases}$$

where $\mathbb{1}_{i=\ell} \in \{0,1\}$ is the indicator function. Because the managers split the project surplus equally irrespective of the regime choice, we can restate Manager i's gross expected payoff from the project (ignoring the fixed cost and omitting project-irrelevant terms from (3)) as[14]

$$\Gamma(\mathbf{k} \mid \beta_i) \equiv \beta_i \left(\frac{E[M(\theta, \mathbf{k})]}{2} - \frac{\rho}{8} \beta_i Var(\mathbf{k}) \right). \tag{8}$$

The corresponding net payoff to Manager i under the symmetric regime is derived by subtracting the manager's internalized portion of investment fixed costs, $\beta_i FC_i(\cdot)$:

$$\Lambda_i(\mathbf{k} \mid \beta_i) \equiv \Gamma(\mathbf{k} \mid \beta_i) - \beta_i F(I_i)$$

under the symmetric regime, and

$$\Lambda_i^{\ell}(\mathbf{k} \mid \beta_i) \equiv \Gamma(\mathbf{k} \mid \beta_i) - \beta_i \sum_{m=A,B} F(I_m) \times \mathbb{1}_{i=\ell}$$

under bundling.

Under the symmetric regime, at Date 2, the managers choose their respective investments simultaneously in form of a pure-strategy Nash equilibrium: for given $\boldsymbol{\beta}$,

$$\max_{k_i \in \mathcal{K}_i} \Lambda_i(k_i, k_j \mid \beta_i), \quad i, j = A, B, \quad i \neq j. \tag{9}$$

Denote the equilibrium investments for given $\boldsymbol{\beta}$ by $\mathbf{k}^S(\boldsymbol{\beta})$ (superscript "S" denotes the *symmetric* regime). At Date 1, the principal anticipates the Date-2

[13]We ignore here other aspects affecting the delegation of decision rights over investments that, e.g., related to optionality as in Arya et al. (2002).

[14]There is no need to index the $\Gamma(\cdot)$-function because the managers are identical (except for their operating uncertainty). Also, in our model investments are of equal productivity and therefore $\Gamma(x, y \mid \beta_i) \equiv \Gamma(y, x \mid \beta_i)$, for any x, y, β_i.

investment subgame outcome and solves

$$\textbf{Program } \mathcal{P}^S: \quad \max_{\boldsymbol{\beta}\in[0,1]^2} W(\boldsymbol{\beta}) \equiv W(\mathbf{k}, \boldsymbol{\beta}^S(\boldsymbol{\beta})).$$

We assume an interior solution and denote it by $\boldsymbol{\beta}^S = (\beta_A^S, \beta_B^S)$. The equilibrium investments under the symmetric regime then are $\mathbf{k}^S \equiv (\mathbf{k}_A^S, \mathbf{k}_B^S) \equiv \mathbf{k}^S(\boldsymbol{\beta}^S)$.

Under bundling, Manager ℓ, the designated investment center manager, chooses both investments ("$\widehat{\ }^{\ell}$" denotes the bundling regime):

$$\widehat{\mathbf{k}}^{\ell}(\beta_\ell) \in \arg\max_{\mathbf{k}\in\mathcal{K}} \Lambda_\ell^\ell(\mathbf{k} \mid \beta_\ell). \tag{10}$$

A key difference to the symmetric regime, as in (9), is that the profit center manager's PPS, $\beta_{j\neq\ell}$, no longer affects any investments. The principal's Date-1 contracting problem for given allocation of decision rights, ℓ, reads

$$\textbf{Program } \widehat{\mathcal{P}}^\ell: \quad \max_{\boldsymbol{\beta}\in[0,1]^2} W^\ell(\boldsymbol{\beta}) \equiv W(\boldsymbol{\beta}, \widehat{\mathbf{k}}^\ell(\beta_\ell)).$$

Denote the solution to this program by $\widehat{\boldsymbol{\beta}}^\ell$ and the resulting investments by $\widehat{\mathbf{k}}^\ell \equiv (\widehat{k}_A^\ell, \widehat{k}_B^\ell) \equiv \widehat{\mathbf{k}}^\ell(\widehat{\beta}_\ell^\ell)$.

At Date 0, the principal asks which Manager $\ell \in \{A, B\}$ delivers a higher value of the respective programs $\widehat{\mathcal{P}}^\ell$ under bundling, and then compares the maximum achievable surplus with that under the symmetric regime. As we will show now, the nature of the investments—continuous or lumpy—critically affects the regime comparison.

3 Continuous investments

We begin by assuming investments are perfectly scalable, i.e., $\mathcal{K} = \mathbb{R}_+^2$. The investment fixed costs are given by $F(k_i) = \frac{fk_i^2}{2}$. We start with the benchmark case and decompose the principal's problem into two steps: First, it is easy to see that the conditionally optimal PPS for given investments $\mathbf{k}$ is

$$\beta_i^o(\mathbf{k}) = \left(1 + \rho v \left(\sigma_i^2 + \frac{Var(\mathbf{k})}{4}\right)\right)^{-1}. \tag{11}$$

Let $\boldsymbol{\beta}^o(\mathbf{k}) = (\beta_A^o(\mathbf{k}), \beta_B^o(\mathbf{k}))$. Second, the optimal investment, $\mathbf{k}^* \in \mathbb{R}_+^2$, maximizes the value function $W^*(\mathbf{k}) \equiv W(\boldsymbol{\beta}^o(\mathbf{k}), \mathbf{k})$; hence, $\beta_i^* = \beta_i^o(\mathbf{k}^*)$. Because both investments come at identical fixed cost and risk premium effects, by (6) and (1), the benchmark investments are identical: $k_A^* = k_B^* \equiv k^*$. The investment-risk link implies that $\beta_i^o(\mathbf{k})$ is decreasing in k_j, for any i, j. Accounting for the project risk reduces the PPS below $\beta_i^{MH} = (1+\rho v \sigma_i^2)^{-1}$, the PPS in a pure moral hazard model without joint projects. Thus, investments and PPS are substitutes for the principal in the benchmark case. By $\sigma_A^2 > \sigma_B^2$, we have $\beta_A^* \leq \beta_B^*$. We assume f is sufficiently high to ensure all investment problems studied below are well-behaved.[15] Moreover, we assume the project uncertainty is bounded from above:

Assumption 1 $\eta \leq \min\{\eta_{risk}, \eta_{pos}\}$, *where* $\eta_{risk} = 4\sigma_B^2 \left(\frac{f-2}{\mu f}\right)^2$ *and* $\eta_{pos} \equiv \frac{1}{\rho}$.

Assuming $\eta \leq \eta_{risk}$ ensures the project risk for each manager is less than his operating risk; for high project risk, one would expect the divisions to be merged.[16] Note that $\lim_{\eta \to \eta_{risk}} \beta_i^* = \beta_i^{min} \equiv (1 + 2\rho v \sigma_i^2)^{-1}$ and, therefore, $\beta_i^* \in [\beta_i^{min}, \beta_i^{MH}]$. The restriction $\eta \leq \eta_{pos}$ ensures the principal chooses a positive benchmark investment level, $k_i^* > 0$.[17]

[15] Specifically, assuming $f > 6$ ensures global concavity of the expected payoffs of the principal (in the contractible benchmark case) and of the division managers (under non-contractibility), respectively.

[16] To provide intuition for the bound η_{risk} in Assumption 1, note that the hypothetical investments in a risk-free world (where $\eta \to 0$), $\mathbf{k}^{rf} \in \arg\max_{k_A, k_B} M(\mu, \mathbf{k}) - \frac{f \sum_i k_i^2}{2}$ (omitting irrelevant terms), are given by $k_i^{rf} = k^{rf} = \frac{\mu}{f-2}$. By (1), $k^* \leq k^{rf}$. Assuming $\eta < \eta_{risk}$ then ensures the project-related risk would be less than the operating risk for each manager, even if these risk-free investments were chosen. From a technical standpoint, $\eta \leq \eta_{risk}$ is sufficient also for the project-related risk premium for Manager i, $\frac{\rho}{8}(\beta_i^o(\mathbf{k}))^2 \cdot Var(\mathbf{k})$, to be increasing in k_i for any $k_i \leq k^{rf}$. Under this condition, the indirect effect in form of a reduced PPS is dominated by the direct effect on the variance of the surplus.

[17] Taking the derivative of the principal's expected utility, $W_{k_i}^* = E[M_{k_i}(\theta, \mathbf{k})] - \frac{\rho}{8} \sum_i (\beta_i^o(\mathbf{k}))^2 Var_{k_i}(\mathbf{k}) - F'(k_i) = q^*(\mu, \mathbf{k}) \left(1 - \frac{\rho\eta}{4} \sum_j (\beta_j^o(\mathbf{k}))^2\right) - f k_i$. Hence, for $\eta \leq \eta_{pos}$, the marginal investment return for small k_i is positive because $\beta_j \leq 1,\ j = A, B$. As we show, this condition also ensures that the pressing investment distortion is underinvestment. For cases in which overinvestment arises, see Baldenius and Michaeli (2017).

In earlier incomplete contracting models that have ignored project risk, bilateral investments tend to be mutually reenforcing: efficiency-enhancing investments by Manager i increase the expected project scale, which in turn raises the marginal investment return to Manager j, and vice versa. That is, the firmwide expected contribution margin, $E[M(\cdot)]$, displays increasing differences in the investments. However, by the investment-risk link as in (1), the same is true for the managers' project-related risk premium. Given Assumption 1, it is easy to show though that the first-moment effect dominates, making contractible investments complements at the margin:

Lemma 1 *Given Assumption 1 and for given $\boldsymbol{\beta}$, both $W(\mathbf{k}, \boldsymbol{\beta})$ and $\Gamma(\mathbf{k} \mid \beta_i)$ have strictly increasing differences in $\mathbf{k}$.*

Both the expected surplus in the benchmark case as well as the *gross* investment return functions under the two decentralized regimes (and thereby also the managers' *net* investment returns, $\Lambda_i(\mathbf{k} \mid \beta_i)$ and $\Lambda_i^{\ell}(\mathbf{k} \mid \beta_i)$) display investment complementarity for given PPS. We now turn to the manager's investment incentives under the symmetric regime.

Lemma 2 *Given Assumption 1 and $\mathbf{k} \in \mathbb{R}_+^2$, under the symmetric regime:*

(a) For given $\boldsymbol{\beta}$, there exists a unique equilibrium investment profile with $k_i^S(\boldsymbol{\beta}) = \frac{\mu(2-\rho\eta\beta_i)}{\rho\eta(\beta_i+\beta_j)+4(f-1)}$, $i = A, B,\ j \neq i$. Each investment level is decreasing in the PPS of either manager: $\frac{dk_i^S(\boldsymbol{\beta})}{d\beta_j} < 0,\ i, j = A, B$.

(b) In equilibrium, $k_i^* > k_A^S > k_B^S,\ i = A, B$.

With cash flows equally scaled by his PPS, the sole first-order effect of an increase in PPS, is that the investing manager becomes more sensitive to the investment-risk link, and thus reluctant to invest. Strategic complementarity reinforces the investment-suppressing effect of PPS and implies that Manager i

will invest less, too, if his counterpart's PPS increases.[18] Comparing across divisions, in equilibrium, Manager A will invest more than Manager B because of the differential operating volatility, $\sigma_A^2 > \sigma_B^2$: greater uncertainty is associated with relatively lower PPS for Manager A, which makes the latter more tolerant to the incremental investment-related project risk.[19] Yet, even Manager A underinvests relative to the benchmark level because of the holdup problem.

We now turn to the bundling regime, in which Manager ℓ, the designated investment center manager, chooses both k_A and k_B. As argued in connection with (10), above, Manager ℓ's choice of $\mathbf{k}$ is affected only by his own PPS. In contrast to the symmetric regime (Lemma 2), therefore, the resulting investment profile under bundling is always symmetric: $\widehat{k}_A^\ell = \widehat{k}_B^\ell$, for any ℓ. The arguments in Lemma 2 for underinvestment and the investment-suppressing effect of the PPS apply with only minor modifications to bundling (proof omitted):

Lemma 3 *Given Assumption 1, under the bundling regime with Manager ℓ choosing both $\mathbf{k} \in \mathbb{R}_+^2$:*

(a) For given $\boldsymbol{\beta}$, Manager ℓ chooses $\widehat{k}_i^\ell(\boldsymbol{\beta}) = \frac{\mu(2-\rho\eta\beta_\ell)}{2\rho\eta\beta_\ell+4(f-1)}$, for any i, where $k_i^\ell(\boldsymbol{\beta})$ is decreasing in β_ℓ.

(b) In equilibrium, for any i and ℓ, $k_i^* > \widehat{k}_i^\ell$.

We now ask which regime maximizes the principal's expected payoff. A remaining design issue for the principal under bundling, is in whose hands to concentrate the decision rights, i.e., which of the managers to designate the investment center manager (whether to set $\ell = A$ or $\ell = B$):

[18] As one would expect, the closed-form term for $k_i^S(\boldsymbol{\beta})$ indicates that Manager i responds more sensitively to changes in β_i (the direct interaction between k_i and β_i) than to changes in β_j (the indirect effect through investment complementarity).

[19] Lemma 2 is silent on how the PPS under the symmetric regime, β_i^S, compares with the benchmark one, β_i^*. As Baldenius and Michaeli (2017) have shown in a simpler unilateral investment setting, this comparison can go either way because of two countervailing effects: the investment suppressing effect of effort incentives calls for lowering the PPS if investment is noncontractible; on the other hand, equilibrium underinvestment implies that the marginal risk premium is reduced, which calls for increasing the PPS. Baldenius and Michaeli (2017) derive conditions that predict the direction of the net effect.

Proposition 1 *Given Assumption 1 and* $\mathbf{k} \in \mathbb{R}_+^2$*, bundling with* $\ell = A$ *(high volatility) dominates both bundling with* $\ell = B$ *as well as the symmetric regime.*

Because any decentralized regime results in underinvestment relative to the benchmark solution, the question is which regime is most effective in alleviating this distortion. With scalable investments, the answer hinges solely on the induced risk tolerance arguments, above: Manager A faces the more volatile environment and therefore has muted PPS to begin with; hence, he is less sensitive to the investment-induced project risk and should be assigned all investment decision rights. Put differently, the key to stimulating delegated investment is muted PPS, and the opportunity cost of muted PPS is minimized by designating Manager A the investment center manager under bundling.

We now turn to non-scalable investments. Lumpiness in project investments will amplify the role of strategic complementarity, with drastic consequences for the ranking of the organizational modes.

4 Lumpy investments

We now consider investments of fixed size, normalized so that $\mathcal{K} = \{0, 1\}^2$, i.e., each investment can either be undertaken or not. Examples are the replacement of existing equipment, M&A, or the decision to develop a new product or to enter a new market. The assumption that both investments are of similar size is solely for notational convenience. We continue to assume that each investment is equally productive by normalizing the marginal gross return to one, and the fixed cost per unit of investment to $\phi > 0$. (None of our results hinges qualitatively on this symmetry restriction.) The principal's objective remains to maximize $W(\mathbf{k}, \boldsymbol{\beta})$, as in (7), now with $\mathcal{K} = \{0, 1\}^2$ and fixed costs $F_i(k_i) = \phi k_i$.

4.1 Investment incentives for given PPS at Date 2

It is useful to begin the analysis of lumpy investments by studying the outcome of the Date-2 investment subgame for given PPS and only then endogenizing the PPS. The *conditionally optimal* investments chosen by the principal in the contractible benchmark setting, holding fixed the PPS, are given by $\mathbf{k}^*(\boldsymbol{\beta}) \in \arg\max_{\mathbf{k}\in\{0,1\}^2} W(\mathbf{k}, \boldsymbol{\beta})$. Using the investment complementarity (Lemma 1), the benchmark investment profile will be "all or nothing:"[20]

$$\mathbf{k}^*(\boldsymbol{\beta}) = \begin{cases} (1,1), \text{ if } \phi \le \phi^*(\boldsymbol{\beta}) \equiv \frac{1}{2}\{E_\theta[M((1,1),\theta) - M((0,0),\theta)] \\ \qquad\qquad\qquad\qquad\qquad -\frac{\rho}{8}(\beta_A^2 + \beta_B^2)[Var(1,1) - Var(0,0)]\}, \\ (0,0), \text{ if } \phi > \phi^*(\boldsymbol{\beta}). \end{cases} \tag{12}$$

We refer to $\phi^*(\boldsymbol{\beta})$ as the benchmark fixed cost threshold for given $\boldsymbol{\beta}$.

Strategic complementarity affects also the set of investment profiles to arise in equilibrium under delegation with non-contractible investments. Under bundling, Manager ℓ's optimization problem remains as in (10), with investments now chosen from the discrete set $\mathcal{K} = \{0,1\}^2$ at fixed cost $\phi(k_A + k_B)$. Because Manager ℓ's investment return also displays complementarity in investments (again, Lemma 1), the equilibrium investment profile will again be "all or nothing:" the investment center manager under bundling chooses $(1,1)$ for given PPS if

$$\Gamma(1,1 \mid \beta_\ell) - 2\beta_\ell\phi \ge \Gamma(0,0 \mid \beta_\ell), \tag{13}$$

and $(0,0)$ otherwise. Therefore,

$$\widehat{\mathbf{k}}^\ell(\beta_\ell) = \begin{cases} (1,1), & \text{if } \phi \le \phi^\ell_{(1,1)}(\beta_\ell) \equiv \frac{1}{2\beta_\ell}[\Gamma(1,1 \mid \beta_\ell) - \Gamma(0,0 \mid \beta_\ell)], \\ (0,0), & \text{if } \phi > \phi^\ell_{(1,1)}(\beta_\ell). \end{cases} \tag{14}$$

[20]It is easy to see that the principal would be indifferent between the "mixed" investment profiles $(1,0)$ and $(0,1)$. However, by Lemma 1, such a mixed investment profile can never be optimal.

Below the fixed cost threshold $\phi^{\ell}_{(1,1)}(\beta_\ell)$, Manager ℓ makes both investments; beyond this threshold, he foregoes any investment.

Under the symmetric regime, a Nash equilibrium in investments, $k_i^S(\boldsymbol{\beta})$, is determined by (9), now with $k_i \in \{0,1\}$ at fixed cost $F(k_i) = \phi k_i$. For the sake of illustration, and with slight abuse of notation, for now denote the PPS profile by the non-ordered pair $\boldsymbol{\beta} = (\underline{\beta}, \overline{\beta})$ where $\underline{\beta} < \overline{\beta}$. Recall that, by Lemma 2 (which applies qualitatively also to lumpy investments), greater PPS lowers a manager's investment incentive by increasing his exposure to the marginal investment-induced risk. All else equal, therefore, the bottleneck in terms of eliciting investments is the manager with the greater PPS. Hence, the investment profile $\mathbf{k}^S(\boldsymbol{\beta}) = (1,1)$ constitutes an equilibrium under the symmetric regime for ϕ low enough such that even the high-PPS manager has no incentive to deviate:

$$\Gamma(1,1 \mid \overline{\beta}) - \overline{\beta}\phi \geq \Gamma(1,0 \mid \overline{\beta}). \tag{15}$$

Denote by $\phi_{(1,1)}(\boldsymbol{\beta})$ the fixed cost value at which (15) becomes binding. At the same time, $\mathbf{k}^S(\boldsymbol{\beta}) = (0,0)$ is an equilibrium for ϕ high enough such that even the low-PPS manager has no incentive to deviate by investing unilaterally:

$$\Gamma(1,0 \mid \underline{\beta}) - \underline{\beta}\phi \leq \Gamma(0,0 \mid \underline{\beta}). \tag{16}$$

Denote by $\phi_{(0,0)}(\boldsymbol{\beta})$ the fixed cost threshold at which (16) becomes binding.

Clearly, for sufficiently low fixed costs $(1,1)$ is the unique investment equilibrium under the symmetric regime; for fixed costs high enough $(0,0)$ is the unique equilibrium. For intermediate ϕ-values one of two cases may arise: either both (15) and (16) hold simultaneously, resulting in multiple symmetric equilibria; or (15) and (16) are both violated, permitting only asymmetric equilibria in which exactly one manager undertakes the investment. To characterize the set of equilibria, we show in the proof of Lemma 4 that the fixed cost threshold differential for given PPS, $\phi_{(1,1)}(\boldsymbol{\beta}) - \phi_{(0,0)}(\boldsymbol{\beta})$, is proportional to some function

$$X(\boldsymbol{\beta}) \equiv \left[(2\mu+3)\left(1 - \frac{\overline{\beta}\eta\rho}{2}\right) - (2\mu+1)\left(1 - \frac{\underline{\beta}\eta\rho}{2}\right)\right].$$

Note that $X(\boldsymbol{\beta})$ is increasing in $\underline{\beta}$ and decreasing in $\overline{\beta}$—and therefore decreasing in the PPS differential—and it is decreasing in η. We then have:

Lemma 4 *Suppose Assumption 1 holds. For* $\mathbf{k} \in \{0,1\}^2$ *and given* $\boldsymbol{\beta} = (\underline{\beta}, \overline{\beta})$, *the (Pareto-dominant) investment equilibrium under the symmetric regime is:*

(a) For $X(\boldsymbol{\beta}) \geq 0,\ \phi_{(0,0)}(\boldsymbol{\beta}) \leq \phi_{(1,1)}(\boldsymbol{\beta})$, *and* $\mathbf{k}^S(\boldsymbol{\beta}) = \begin{cases} (1,1), & \text{if } \phi \leq \phi_{(1,1)}(\boldsymbol{\beta}), \\ (0,0), & \text{if } \phi > \phi_{(1,1)}(\boldsymbol{\beta}). \end{cases}$

A sufficient condition for $X(\boldsymbol{\beta}) \geq 0$ *is that the PPS differential is small (or the project risk* η *is small); specifically,* $\overline{\beta} - \underline{\beta} \leq \frac{1}{\mu}\left(\frac{3}{2} + \frac{2}{\rho\eta}\right)$.

(b) For $X(\boldsymbol{\beta}) < 0,\ \phi_{(0,0)}(\boldsymbol{\beta}) > \phi_{(1,1)}(\boldsymbol{\beta})$, *and* $\mathbf{k}^S(\boldsymbol{\beta}) = \begin{cases} (1,1), & \text{if } \phi \leq \phi_{(1,1)}(\boldsymbol{\beta}), \\ (1,0), & \text{if } \phi \in \left(\phi_{(1,1)}(\boldsymbol{\beta}), \phi_{(0,0)}(\boldsymbol{\beta})\right], \\ (0,0), & \text{if } \phi > \phi_{(0,0)}(\boldsymbol{\beta}). \end{cases}$

Games of strategic complementarity are routinely afflicted by multiple equilibria. This is true also for the symmetric regime if managers face fairly similar PPS (Lemma 4a): for intermediate values of fixed costs, both (15) and (16) hold simultaneously, making $(1,1)$ and $(0,0)$ each a Nash equilibrium. To predict which of these the managers will play, we note that the investment subgame satisfies the conditions for a supermodular game as in Milgrom and Roberts (1990); hence, we can invoke their Theorem 7 stating that the highest equilibrium—here, $(1,1)$—is the Pareto-dominant one. For $\phi \in (\phi_{(0,0)}(\boldsymbol{\beta}), \phi_{(1,1)}(\boldsymbol{\beta}))$ in Lemma 4a, we can therefore ignore the no-investment equilibrium.

Lemma 4b states a condition for an asymmetric equilibrium to obtain for intermediate fixed cost values. In that equilibrium, only the manager with the lower PPS will invest. While his investment raises the investment incentive also for the other manager, if the PPS differential is large enough, then this strategic complementarity effect is insufficient to compensate the high-PPS manager for the incremental risk premium associated with investing.[21]

Turning now to the regime comparison for lumpy investments, we first approach this issue heuristically by asking which regime implements the investment profile $(1,1)$ for a wider range of fixed cost parameters—i.e., we compare the ϕ-thresholds in (14) and Lemma 4 for given $\boldsymbol{\beta} = (\underline{\beta}, \overline{\beta})$. Both regimes fall short of providing efficient investment incentives by this criterion (holding fixed the PPS):

Lemma 5 *Suppose Assumption 1 holds. For* $\mathbf{k} \in \{0,1\}^2$ *and given* $\boldsymbol{\beta} = (\underline{\beta}, \overline{\beta})$*:*

(a) Under the symmetric regime, $\phi_{(1,1)}(\boldsymbol{\beta}) < \phi^*(\boldsymbol{\beta})$*.*

(b) Under bundling, $\phi^{\ell}_{(1,1)}(\beta_\ell) < \phi^*(\boldsymbol{\beta})$*, for any* ℓ*.*

Either regime leads the managers to underinvest for given contracts, by this heuristic criterion. Which regime is best suited to alleviate the prevailing underinvestment problem? Conditional on bundling decision rights, to best alleviate the underinvestment problem, the principal would designate the manager with the lower PPS the investment center manager. For the remainder of this subsection, we index by ℓ^* the manager facing the lower of the two PPS, $\underline{\beta}$. Given Lemma 5, we say the symmetric regime *investment-dominates (for given PPS)* the bundling regime whenever $\phi_{(1,1)}(\boldsymbol{\beta}) \geq \phi^{\ell^*}_{(1,1)}(\underline{\beta})$, and vice versa.[22]

There are two conceptual differences between the regimes, which jointly determine the performance comparison: (a) the game forms differ—a simultaneous-move game under the symmetric regime versus a single-agent optimization problem under the bundling regime; (b) the *risk tolerance benefit* under the bundling regime resulting from assigning the investment authority to the agent who is more willing to invest because of his lower PPS. Our next result identifies conditions for the difference in game forms to be the dominant force:

[21] In Lemma 4b, it is without loss of generality to write the investment equilibrium for intermediate fixed costs as $(1,0)$. This equilibrium is payoff-equivalent for all parties to $(0,1)$.

[22] On a technical level, we use here the fact that $W(\boldsymbol{\beta}, \mathbf{k})$ is concave in $\mathbf{k}$ for given $\boldsymbol{\beta}$.

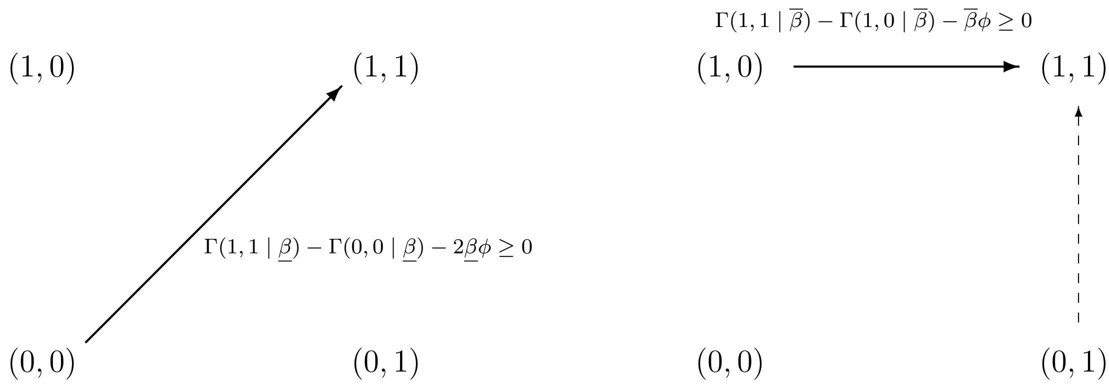
Lemma 6 *Suppose Assumption 1 holds,* $\mathbf{k} \in \{0,1\}^2$*, and* $X(\boldsymbol{\beta}) > 1$*. Then, for any given PPS of* $\boldsymbol{\beta} = (\underline{\beta}, \overline{\beta})$*,* $\phi_{(1,1)}(\boldsymbol{\beta}) > \phi^{\ell*}_{(1,1)}(\underline{\beta})$*, and hence the symmetric regime investment-dominates bundling. A sufficient condition for* $X(\boldsymbol{\beta}) > 1$ *is that the PPS differential is small; specifically,* $\overline{\beta} - \underline{\beta} \leq \frac{1}{\mu}\left(\frac{3}{2} + \frac{1}{\rho\eta}\right)$.

Lemma 6 is in stark contrast to Proposition 1, which dealt with continuous investments. Why, with lumpy investments, does the symmetric regime generate stronger investment incentives than the bundling regime if the PPS differential is limited? As $\overline{\beta} - \underline{\beta}$ becomes small, the risk tolerance benefit of bundling vanishes; this leaves the different game forms. Consider the limit case where both PPS levels converge to the same value, say x. Comparing (13) with (15), holding the PPS for each manager fixed at x, we find that inducing $(1,1)$ as a Nash equilibrium under the symmetric regime is a less demanding condition than inducing some Manager ℓ under bundling to invest two units rather than none. The reason is that, for any given x,

$$\underbrace{\Gamma(1,1 \mid \beta_i = x) - \Gamma(1,0 \mid \beta_i = x)}_{\text{as in (15)}} - \underbrace{\frac{\Gamma(1,1 \mid \beta_\ell = x) - \Gamma(0,0 \mid \beta_\ell = x)}{2}}_{\text{as in (13)}} > 0, \tag{17}$$

by strategic complementarity (Lemma 2). The symmetric regime generates strong investment incentives in aggregate by requiring that investing only be each manager's best response to the other manager also investing. At a fixed cost of $\beta_i \phi$, Manager i reaps his share of the return from changing the investment profile from $(0,1)$ to $(1,1)$. Investment incentives under bundling, in contrast, are muted by the fact that the investment center manager has to pay for the *total* fixed cost, $2\beta_\ell \phi$, to change investments from $(0,0)$ to $(1,1)$.

Loosely put, if the project proceeds are split by the managers, eliciting high levels of inputs from two players in form of a Nash equilibrium is relatively "cheap" if these inputs are strategic complements. We henceforth label the game form effect the *strategic complementarity effect*: the symmetric regime takes advantage of the strategic investment complementarity; bundling does not.



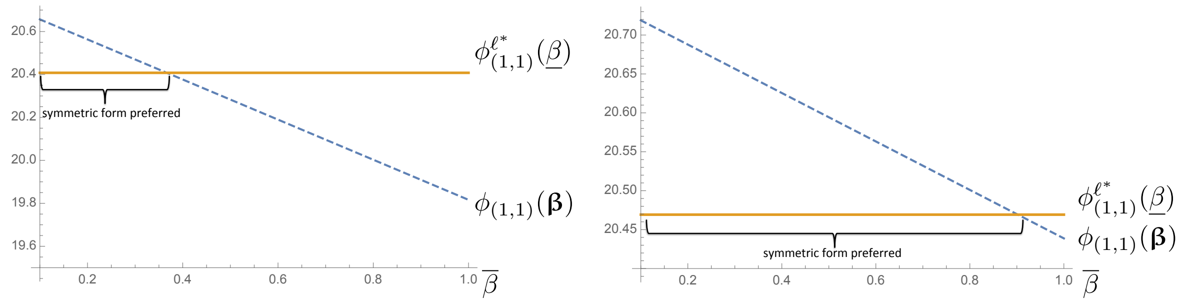
(a) Bundling: low-PPS manager chooses k_A and k_B

(b) Symmetric form: Nash equilibrium (binding constraint is high-PPS manager)

Figure 3: Comparison of investment incentives with exogenous PPS
Solid arrows indicate the binding constraints in order to elicit the investment profile $(1,1)$.

Figure 3 illustrates the risk tolerance and strategic complementarity effects, with bold arrows indicating the binding investment incentive constraints.

Now consider the case of a large PPS differential, $\overline{\beta}-\underline{\beta}$; specifically, fix $\underline{\beta}$ while increasing $\overline{\beta}$. Investment incentives under the bundling regime are unaffected by this change, because only the low-PPS manager matters for investments. The bottleneck under the symmetric regime is to get the high-PPS manager to invest; this constraint becomes tighter as $\overline{\beta}$ increases. Letting η vary, as a measure of the intrinsic project uncertainty, Figure 4 plots the difference in the fixed cost thresholds as $\overline{\beta}$ increases, holding $\underline{\beta}$ fixed. High η values (Fig. 4a) boost the induced risk tolerance effect and dampen the strategic complementarity by strengthening the increasing differences of the risk premium in $\mathbf{k}$, as per (8). Both effects work in tandem to make the symmetric regime the investment-dominant one for a wider range of PPS differentials $\overline{\beta} - \underline{\beta}$, for small η values (Fig. 4b).



(a) High project risk: $\eta = 0.09$ **(b)** Low project risk: $\eta = 0.03$

Figure 4: Fixed cost thresholds comparison for $\mu = 40$, $\underline{\beta} = 0.1$, $\rho = 1$.
Dashed line represents $\phi_{(1,1)}(\boldsymbol{\beta})$. Solid line represents $\phi^{\ell^*}_{(1,1)}(\underline{\beta})$.

4.2 Regime comparison with equilibrium contracts

As before in the case of scalable investments, the optimal PPS for given $\mathbf{k}$ under the contractible benchmark is $\boldsymbol{\beta}^o(\mathbf{k})$ as derived in (11) but with $\mathcal{K}_i = \{0, 1\}$; and the optimal $\mathbf{k}$ maximizes $W^*(\mathbf{k}) \equiv W(\mathbf{k}, \boldsymbol{\beta}^o(\mathbf{k}))$. As we show in the proof of Lemma 7, the expected surplus continues to display investment complementarity even after endogenizing $\boldsymbol{\beta}$, i.e., $W^*(\mathbf{k})$ has increasing differences in $\mathbf{k}$. As a result, the principal's choice of lumpy investments under the benchmark case with endogenous contracts continues to be "all or nothing."

Lemma 7 *If Assumption 1 holds and $\mathbf{k} \in \{0, 1\}^2$, then the contractible benchmark solution is:*

$$(\mathbf{k}^*, \boldsymbol{\beta}^*) = \begin{cases} ((1,1), \boldsymbol{\beta}^o(1,1)), & \text{if } \phi \leq \phi^* \equiv \phi^*(\boldsymbol{\beta}^o(1,1)), \\ ((0,0), \boldsymbol{\beta}^o(0,0)), & \text{otherwise.} \end{cases}$$

In the main (decentralized) setting, the managers' contracts again are chosen by the principal at Date 1, anticipating the induced Date-2 investment and general effort choices. By Lemma 4, with lumpy investments, this causes a technical challenge under the symmetric regime, as the PPS profile $\boldsymbol{\beta}$ affects qualitatively the set of possible equilibria in the investment subgame in which exactly one manager invests. To address this issue, for the remainder of this section we tighten the upper bound on the intrinsic project risk:

Assumption 2 $\eta \leq \min\{\eta_{risk}, \eta_{pos}, \eta_{symm}\}$, *where* $\eta_{symm} \equiv \frac{4}{\rho(2\mu+3)}$.

Assuming $\eta \leq \eta_{symm}$ ensures the sufficient condition for $X(\boldsymbol{\beta}) > 0$ in Lemma 4 holds for any PPS.[23] Hence, when optimizing over $\boldsymbol{\beta}$, the principal only needs to consider symmetric equilibrium investment profiles, $(0,0)$ or $(1,1)$, in the subgame played by the managers.

Under the symmetric regime, if the principal were to set the PPS equal to $\boldsymbol{\beta}^o(1,1)$, then the managers would play the $(1,1)$ investment equilibrium up to a fixed cost level of $\phi_{(1,1)}(\boldsymbol{\beta}^o(1,1))$. To reduce clutter, let $\overline{\phi}^S \equiv \phi_{(1,1)}(\boldsymbol{\beta}^o(1,1))$. At the same time, as described above, the solution to the benchmark problem entails a fixed cost threshold ϕ^* such that $\mathbf{k}^* = (1,1)$ if and only if $\phi \leq \phi^*$. Beyond this fixed cost level, investments are lost—and the PPS adjusted to $\boldsymbol{\beta}^o(0,0)$—even under the benchmark solution; a fortiori, the same holds under the symmetric regime. Hence, for any $\phi \notin (\overline{\phi}^S, \phi^*]$, there is no cost to the principal as a result of incomplete contracting under the symmetric regime. However, for intermediate fixed cost values, $\phi \in (\overline{\phi}^S, \phi^*]$, Lemma 5 suggests weak underinvestment. The only way for the principal to induce the investment profile (1,1) in this fixed cost range is by lowering the PPS, so that the investment incentive condition,

$$\Gamma(1,1 \mid \beta_i) - \beta_i \phi \geq \Gamma(1,0 \mid \beta_i) \tag{18}$$

holds for any Manager i. Denote by $\breve{\beta}^S(\phi)$ the PPS that satisfies this condition as an equality.

Under the symmetric regime, the principal will choose the optimization program with the greater value from among the following:

[23] The condition $\eta < \frac{4}{\rho(2\mu+3)}$ is derived by setting the left-hand side of the sufficient condition for $X(\boldsymbol{\beta}) > 0$ in Lemma 4 equal to its maximal value of one.

$\mathcal{P}^S_{(1,1)}$ *(Induce investment under symmetric regime):* For any $\phi \in (\overline{\phi}^S, \phi^*]$,

$$\max_{\boldsymbol{\beta}} \; W(\boldsymbol{\beta} \mid \mathbf{k} = (1,1)),$$
$$\text{subject to (5) and } \beta_i \le \breve{\beta}^S(\phi), \text{ for any } i.$$

$\mathcal{P}^S_{(0,0)}$ *(Forestall investment under symmetric regime):* For any $\phi \in (\overline{\phi}^S, \phi^*]$,

$$\max_{\boldsymbol{\beta}} \; W(\boldsymbol{\beta} \mid \mathbf{k} = (0,0)),$$
$$\text{subject to (5) and } \beta_i > \breve{\beta}^S(\phi), \text{ for any } i.$$

Evaluating these two programs yields our next result:

Proposition 2 (Symmetric regime) *Suppose* $\mathbf{k} \in \{0,1\}^2$ *and Assumption 2 holds. Under the symmetric regime, there exists a unique* $\phi^S \in (\overline{\phi}^S, \phi^*]$, *such that:*

(i) If $\phi \in (\overline{\phi}^S, \phi^S]$, *then* $\beta_B = \breve{\beta}^S(\phi) < \beta^o_B(1,1)$ *and* $\beta_A = \min\{\breve{\beta}^S(\phi), \beta^o_A(1,1)\}$. *Both managers invest, as they would under the benchmark solution.*

(ii) If $\phi \in (\phi^S, \phi^*]$, *then* $\beta_i = \beta^o_i(0,0) > \beta^o_i(1,1), \; i = A, B$. *Neither manager invests, whereas* $\mathbf{k}^* = (1,1)$ *under the benchmark solution.*

Figure 5 illustrates this result graphically, with increasing fixed costs along the x-axis. The manager with less volatile operations (Manager B) has a higher benchmark PPS. As fixed cost increase, his PPS has to be muted first to $\beta_B = \breve{\beta}^S(\phi)$ to elicit his investment. As fixed costs grow further, at some point the investment constraint (18) may become binding even for Manager A. Now both managers' incentives must be muted in lockstep; hence, $\beta_A = \min\{\breve{\beta}^S(\phi), \beta^o_A(1,1)\}$. At fixed cost of ϕ^S, the net investment benefit falls short of the opportunity cost of foregone general effort, so the principal gives up on investments. This in turn allows her to expose both managers to higher-powered incentives because of the reduced project risk.

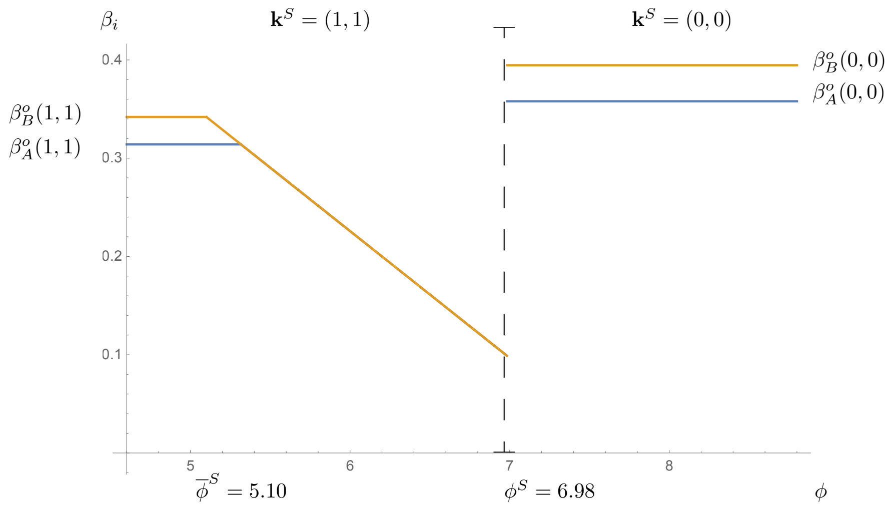


Figure 5: Optimal PPS and induced investments under the symmetric regime
Numerical example with $\mu = 14$, $\rho = 2$, $v = 0.013$, $\sigma_A^2 = 20$, $\sigma_B^2 = 10$, $\eta = 1$. In this example, under the contractible benchmark, $\mathbf{k}^* = (1,1)$ and $\boldsymbol{\beta}^* = \boldsymbol{\beta}^o(1,1) = (0.31, 0.34)$ if $\phi \leq \phi^* = 13.14$. Otherwise, $\mathbf{k}^* = (0,0)$ and $\boldsymbol{\beta}^* = \boldsymbol{\beta}^o(0,0) = (0.36, 0.39)$.

Under bundling, incomplete contracting imposes costs on the principal also only for intermediate fixed cost levels. Recall, to capitalize on the induced risk tolerance effect, the principal chooses $\ell = A$. For $\phi \in (\overline{\phi}^A, \phi^*]$, however, Lemma 5 suggests weak underinvestment, where $\overline{\phi}^A \equiv \phi^A_{(1,1)}(\boldsymbol{\beta}^o(1,1))$. Manager A will choose $(1,1)$ whenever

$$\Gamma(1,1 \mid \beta_A) - 2\beta_A\phi \geq \Gamma(0,0 \mid \beta_A). \tag{19}$$

Denote by $\breve{\beta}^A(\phi)$ the PPS level that satisfies this condition as an equality. By strategic complementarity, it is immediate that $\breve{\beta}^A(\phi) < \breve{\beta}^S(\phi)$ for any ϕ.

Under bundling the principal compares the values of the following two optimization programs:

$\widehat{\mathcal{P}}^A_{(1,1)}$ *(Induce investment under bundling, $\ell = A$):* For any $\phi \in (\overline{\phi}^A, \phi^*]$,

$$\max_{\boldsymbol{\beta}} \; W(\boldsymbol{\beta} \mid \mathbf{k} = (1,1)),$$
$$\text{subject to (5) and } \beta_A \leq \breve{\beta}^A(\phi).$$

$\widehat{\mathcal{P}}^{A}_{(0,0)}$ *(Forestall investment under bundling, $\ell = A$):* For any $\phi \in (\overline{\phi}^{A}, \phi^*]$,

$$\max_{\boldsymbol{\beta}} \; W(\boldsymbol{\beta} \mid \mathbf{k} = (0,0)),$$
$$\text{subject to (5) and } \beta_A > \breve{\beta}^{A}(\phi).$$

Evaluating these two programs yields:

Proposition 3 (Bundling) *Suppose $\mathbf{k} \in \{0,1\}^2$. Under bundling, Manager A (high operating volatility) is designated investment center manager ($\ell = A$), and there exists a unique $\widehat{\phi}^{A} \in (\overline{\phi}^{A}, \phi^*]$ such that:*

(i) If $\phi \in (\overline{\phi}^{A}, \widehat{\phi}^{A}]$, then $\beta_A = \breve{\beta}^{A}(\phi) < \beta^{o}_{A}(1,1)$ and $\beta_B = \beta^{o}_{B}(1,1)$. Manager A chooses $(1,1)$, as would be the case under the benchmark solution.

(ii) If $\phi \in (\widehat{\phi}^{A}, \phi^*]$, then $\beta_i = \beta^{o}_{i}(0,0) > \beta^{o}_{i}(1,1)$, $i = A, B$. Manager A chooses $(0,0)$, whereas $\mathbf{k}^* = (1,1)$ under the benchmark solution.

Propositions 2 and 3 have in common that for intermediate fixed cost levels, the principal trades off investment and effort distortions. Yet, the nature of the contract adjustments necessary to elicit investments differs qualitatively across the regimes (see Figures 5 and 6). Under the symmetric regime, incomplete contracting leads to PPS *convergence* across divisions because the high-PPS manager is the bottleneck whose PPS needs to be muted (first). Under bundling, in contrast, the low-PPS manager is designated investment center manager: only his PPS needs to be muted as the incentive constraint becomes binding, resulting in further PPS *divergence* across divisions. Throughout the paper we assume that decision rights can be moved across divisions at no direct cost. However, this may not always be the case; instead some firms may be "stuck" with the symmetric regime for technological reasons. In this case, our model predicts harmonized incentives and thus sheds light on the puzzle of "corporate socialism."[24]

[24] Levine (1991), Zenger and Hesterly (1997), Shaw et al. (2002), Siegel and Hambrick (2005).

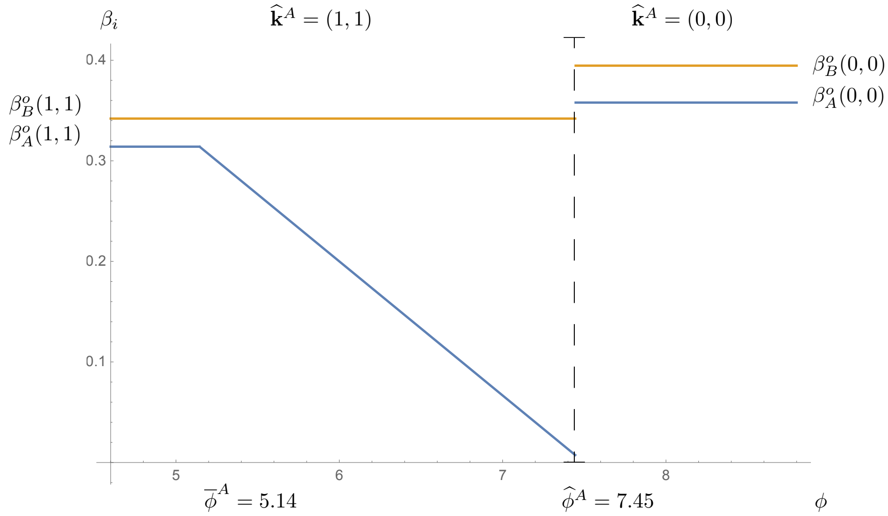


Figure 6: Optimal PPS and induced investments under bundling
Numerical example with $\mu = 14$, $\rho = 2$, $\upsilon = 0.013$, $\sigma_A^2 = 20$, $\sigma_B^2 = 10$, $\eta = 1$. In this example, under the contractible benchmark, $\mathbf{k}^* = (1,1)$ and $\boldsymbol{\beta}^* = \boldsymbol{\beta}^o(1,1) = (0.31, 0.34)$ if $\phi \leq \phi^* = 13.14$. Otherwise, $\mathbf{k}^* = (0,0)$ and $\boldsymbol{\beta}^* = \boldsymbol{\beta}^o(0,0) = (0.36, 0.39)$.

Having characterized the optimal contractual adjustments and attendant equilibrium investments under the two regimes, we are now in a position to generalize the result of Lemma 6 regarding the regime comparison.

Proposition 4 *Suppose* $\mathbf{k} \in \{0,1\}^2$ *and Assumption 2 holds. For given* σ_B^2*, the principal prefers the symmetric regime to the bundling regime if* $\sigma_A^2 \in (\sigma_B^2, \sigma_B^2 + \delta)$*, for some* $\delta > 0$*.*

The message from our heuristic performance comparison for given PPS generalizes to optimal contracts (within the class of linear schemes relying on divisional performance measures). For lumpy capital investments, the principal trades off taking advantage of strategic complementarity under the symmetric regime, against utilizing the greater induced risk tolerance associated with low-powered PPS under bundling. As the volatility levels converge across divisions, the risk tolerance benefit becomes negligible and the symmetric regime is preferred. The principal could assign all investment authority to one manager (bundling) and mute that manager's PPS to boost both investments. The associated opportunity cost in terms of foregone general effort exerted by Manager ℓ, however, makes this less advantageous than splitting decision rights between the managers (the symmetric regime).

5 Personally costly project-specific efforts

We now consider the case where the relationship-specific inputs are not paid for with divisional funds but instead are personally costly to the respective manager who chooses them. Examples are foregone perquisites or private benefits from pet projects, or simply the disutility of engaging in time-consuming market research. To accommodate such personally-costly relationship-specific efforts (henceforth "project efforts"), we modify the notation. At Date 2, project efforts $\mathbf{e} = (e_A, e_B)$ are chosen at personal disutility of $G(e_i)$, respectively (replacing $\mathbf{k}$ chosen at monetary divisional fixed cost of $F(k_i)$). For simplicity, we assume the project's contribution margin has the same functional form as before in that $q^*(\theta, \mathbf{e}) = \theta + \sum_i e_i$ and $M(\theta, \mathbf{e}) = \frac{1}{2}\left(\theta + \sum_i e_i\right)^2$.

We again compare the symmetric regime (Manager i exerts effort e_i) and the bundling regime (Manager ℓ exerts both (e_A, e_B)) in settings where the project efforts are either scalable or lumpy. The performance measures now read $\pi_i = a_i + \varepsilon_i + \frac{M(\theta,\mathbf{e})}{2}$, regardless of the regime, as the project effort cost is borne privately by the managers. The manager's expected utility is, accordingly,

$$EU_i = \alpha_i + \beta_i \left(a_i + \frac{E[M(\theta, \mathbf{e})]}{2}\right) - \widetilde{FC}_i(\mathbf{e}) - \frac{v}{2}a_i^2 - \frac{\rho}{2}\beta_i^2 \left(\sigma_i^2 + \frac{Var(\mathbf{e})}{4}\right), \quad (20)$$

where $Var(\mathbf{e}) \equiv Var(M(\theta, \mathbf{e})) = (q^*(\mu, \mathbf{e}))^2\, \eta$ and

$$\widetilde{FC}_i(\mathbf{e}) = \begin{cases} G(e_i), & \text{under the symmetric regime,} \\ (G(e_A) + G(e_B)) \times \mathbb{1}_{i=\ell}, & \text{under bundling with } \ell = i. \end{cases}$$

By moving from a setting of monetary to one of personally-costly project inputs, the managers' general effort incentive constraints in (5) are unaffected, but their choice of the project-specific inputs is affected in that the input cost, $\widetilde{FC}_i(\cdot)$, is no longer scaled by the PPS (contrast (20) with (3)).

In the benchmark case of contractible project efforts, the principal simply maximizes $(\mathbf{e}^*, \boldsymbol{\beta}^*) \in \arg\max_{\mathbf{e},\boldsymbol{\beta}} W(\mathbf{e}, \boldsymbol{\beta})$. As before, $\beta^* \equiv \beta^o(\mathbf{e}^*)$ where $\beta_i^o(\mathbf{e}) = \left(1 + \rho v \left[\sigma_i^2 + \frac{Var(\mathbf{e})}{4}\right]\right)^{-1}$ is the conditionally optimal PPS. For noncontractible project efforts, Manager i's expected gross payoff from the project is $\Gamma(\mathbf{e} \mid \beta_i)$, with $\Gamma(\cdot)$ as defined in (8) with $\mathbf{e}$ replacing $\mathbf{k}$. The corresponding net payoff to Manager i is denoted by $\widetilde{\Lambda}_i(\mathbf{e} \mid \beta_i) = \Gamma(\mathbf{e} \mid \beta_i) - G(e_i)$ under the symmetric regime and by $\widetilde{\Lambda}_i^\ell(\mathbf{e} \mid \beta_i) = \Gamma(\mathbf{e} \mid \beta_i) - \sum_{m=A,B} G(e_m) \times \mathbb{1}_{i=\ell}$ under bundling. Under the symmetric regime, at Date 2, the managers choose project efforts simultaneously such that, for given $\boldsymbol{\beta}$,

$$\max_{e_i} \ \widetilde{\Lambda}_i(e_i, e_j \mid \beta_i), \quad i, j = A, B, \quad i \neq j. \tag{21}$$

As before, we focus on pure-strategy Nash equilibria. The subgame equilibrium project efforts for given $\boldsymbol{\beta}$ are $\mathbf{e}^S(\boldsymbol{\beta})$. At Date 1, the principal maximizes $W(\boldsymbol{\beta}) \equiv W(\boldsymbol{\beta}, \mathbf{e}^S(\boldsymbol{\beta}))$ over $\boldsymbol{\beta}$. We assume an interior solution and denote it by $\boldsymbol{\beta}^S = (\beta_A^S, \beta_B^S)$, resulting in equilibrium project efforts of $\mathbf{e}^S \equiv (e_A^S, e_B^S) \equiv \mathbf{e}^S(\boldsymbol{\beta}^S)$. Under bundling, at Date 2, Manager ℓ solves, for given $\boldsymbol{\beta}$,

$$\max_{\mathbf{e}} \ \widetilde{\Lambda}_\ell^\ell(\mathbf{e} \mid \beta_\ell), \tag{22}$$

which yields the solution $\widehat{\mathbf{e}}^\ell(\beta_\ell)$. At Date 1, the principal maximizes $W(\boldsymbol{\beta}, \widehat{\mathbf{e}}^\ell(\beta_\ell))$ over $\boldsymbol{\beta}$. Assuming an interior solution, we denote it by $\widehat{\boldsymbol{\beta}}^\ell = (\widehat{\beta}_A^\ell, \widehat{\beta}_B^\ell)$ and the resulting equilibrium project efforts by $\widehat{\mathbf{e}}^\ell \equiv (\widehat{e}_A^\ell, \widehat{e}_B^\ell) \equiv \widehat{\mathbf{e}}^\ell(\beta_\ell)$. Lastly we adapt Assumption 1 to project efforts:

Assumption 1′ $\eta \leq \min\{\widetilde{\eta}_{risk}, \eta_{pos}\}$, *where* $\widetilde{\eta}_{risk} = 4\sigma_B^2 \left(\frac{g-2}{\mu g}\right)^2$ *and* $\eta_{pos} \equiv \frac{1}{\rho}$.

5.1 Continuous project efforts

Suppose project efforts are scalable, $\mathbf{e} \in \mathbb{R}^2_+$, at personal disutility $G(e_i) = \frac{ge_i^2}{2}$ with g sufficiently high.[25] Recall that monetary investments were decreasing in the managers' PPS, because all cash flows were scaled by the PPS, so at the margin greater PPS merely made the managers more sensitive to the investment-related project risk. With project-specific personally-costly efforts, this result flips:

Lemma 8 *Given Assumption 1′ and* $\mathbf{e} \in \mathbb{R}^2_+$*:*

(a) For given $\boldsymbol{\beta}$*, both* $W(\mathbf{e}, \boldsymbol{\beta})$ *and* $\Gamma(\mathbf{e} \mid \beta_i)$ *have strictly increasing differences in* $\mathbf{e}$*.*

(b) $\widetilde{\Lambda}_i(\mathbf{e} \mid \beta_i)$ *and* $\widetilde{\Lambda}^{\ell}_{i=\ell}(\mathbf{e} \mid \beta_i)$ *have strictly increasing differences in* $(\mathbf{e}, \beta_i)$*, for any* i, ℓ*.*

(c) $\frac{d\widetilde{\Lambda}_i(\mathbf{e}|\beta_i)}{de_i} \leq \frac{dW(\mathbf{e},\boldsymbol{\beta})}{de_i}$ *and* $\frac{d\widetilde{\Lambda}^{\ell}_i(\mathbf{e}|\beta_\ell)}{de_i} \leq \frac{dW(\mathbf{e},\boldsymbol{\beta})}{de_i}$ *, for any* $\mathbf{e}, \boldsymbol{\beta}, i, \ell$*.*

(d) For any $\boldsymbol{\beta}$*: under the symmetric regime there exists a unique Nash equilibrium in efforts solving* (21) *such that* $\frac{de_i^S(\boldsymbol{\beta})}{d\beta_j} > 0$ *for any* i, j*; under bundling, for any* ℓ*, there exists a unique maximizer to* (22) *such that* $\frac{d\widehat{e}^{\ell}_i(\beta_\ell)}{d\beta_\ell} > 0$ *and* $\frac{d\widehat{e}^{\ell}_i(\beta_\ell)}{d\beta_{j\neq\ell}} \equiv 0$ *for any* i*.*

With project efforts being personally costly, the classic moral hazard argument that stronger PPS elicits greater (project) effort is merely weakened but not overturned by the input-risk link (the risk premium is again increasing in e_i, as $\frac{\partial}{\partial e_i} Var(\mathbf{e}) = 2q^*(\mu, \mathbf{e})\eta$, for any i). Put differently, the first moment-effect of an increase in the PPS now dominates the second moment-effect. When designing incentive contracts, the principal no longer has to trade off general effort and project-specific inputs: a_i is increasing in β_i, while $\mathbf{e}$ is increasing in β_ℓ under

[25] Similar to the case of monetary investments, $g > 6$ ensures global concavity of the expected payoffs of the principal (in the contractible benchmark case) and of the division managers (under non-contractibility), respectively.

bundling and increasing in both $\boldsymbol{\beta}$ under the symmetric regime (again, strategic complementarity).[26]

Accordingly, we find for the regime comparison for scalable project efforts:

Proposition 5 *Given Assumption 1′ and* $\mathbf{e} \in \mathbb{R}_{+}^{2}$*, bundling with* $\ell = B$ *(low volatility) outperforms both bundling with* $\ell = A$ *and the symmetric regime.*

As with monetary investments, surplus splitting implies that either regime will lead to underprovision of project efforts in equilibrium. Bundling decision rights mitigates this problem most effectively, *but now the manager facing the more stable operating environment should be designated investment center manager.* A more stable environment calls for higher-powered PPS for Manager B which, by Lemma 8, delivers greater general-purpose *and* project-specific effort levels in tandem.

5.2 Lumpy project efforts

Now consider lumpy personally costly project efforts, $e_i \in \{0, 1\}$. The effort disutility per unit of project effort is given by $\gamma > 0$. The principal's objective remains to maximize $W(\boldsymbol{\beta}, \mathbf{e})$, with modified input costs $G_i(k_i) = \gamma e_i$. Solving this program, by strategic complementarity (Lemma 8a), optimal project efforts are $\mathbf{e}^* = (1, 1)$ for any effort cost below a threshold γ^*, and $\mathbf{e} = (0, 0)$ otherwise.

To study noncontractible project efforts, we begin again by taking the PPS $\boldsymbol{\beta} = (\underline{\beta}, \overline{\beta})$ as given, where $\underline{\beta} < \overline{\beta}$. Following similar arguments as above, both decentralization regimes result in underinvestment. But contrary to monetary investments, recall the PPS now stimulates, rather than depresses, project specific inputs. Therefore, under bundling the principal designates the manager

[26] In analogy with Lemmas 2a and 3a, the proof of Lemma 8d derives closed-form expressions for the resulting project effort profile for given $\boldsymbol{\beta}$ under the two regimes as

$$e_i^S(\boldsymbol{\beta}) = \frac{\beta_i \mu \left(\frac{1}{2} - \frac{\eta\rho}{4}\beta_i\right)}{g + \frac{\eta\rho}{4}\left(\beta_i^2 + \beta_j^2\right) - \frac{1}{2}(\beta_i + \beta_j)}, \; j \neq i, \quad \text{and} \quad \widehat{e}_i^{\ell}(\beta_\ell) = \frac{\mu\left(\frac{1}{2} - \frac{\eta\rho}{4}\beta_\ell\right)}{\frac{g}{\beta_\ell} + \frac{\eta\rho}{2}\beta_\ell - 1} \quad \text{for} i, j = A, B.$$

facing higher-powered PPS the investment center manager ($\beta_\ell = \underline{\beta}$), while under the symmetric regime the manager with the *lower* PPS now is the bottleneck in terms of eliciting project efforts (Fig. 7).

Under bundling the investment center manager chooses $(1,1)$ for given PPS if and only if

$$\Gamma(1,1 \mid \beta = \underline{\beta}) - 2\gamma \geq \Gamma(0,0 \mid \beta = \underline{\beta}), \tag{23}$$

or equivalently, if $\gamma \leq \gamma^{\ell}_{(1,1)}(\underline{\beta})$; and $(0,0)$ otherwise.[27] Under the symmetric regime, $\mathbf{e}^S(\boldsymbol{\beta}) = (1,1)$ constitutes an equilibrium under the symmetric regime for γ low enough such that

$$\Gamma(1,1 \mid \underline{\beta}) - \gamma \geq \Gamma(1,0 \mid \underline{\beta}). \tag{24}$$

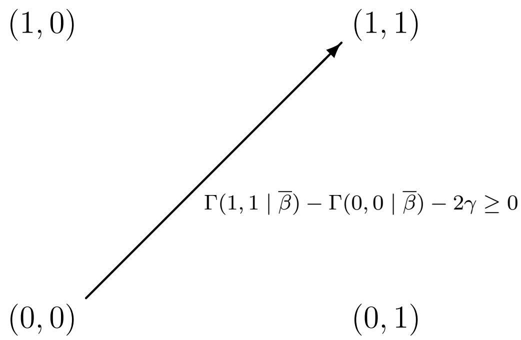
Denote by $\gamma_{(1,1)}(\boldsymbol{\beta})$ the effort cost at which (24) becomes binding. At the same time, $\mathbf{e}^S(\boldsymbol{\beta}) = (0,0)$ is an equilibrium for $\gamma \geq \gamma_{(0,0)}(\boldsymbol{\beta})$, where the latter makes the constraint $\Gamma(1,0 \mid \overline{\beta}) - \gamma \leq \Gamma(0,0 \mid \overline{\beta})$ binding. As in Section 4.1, one can show (see proof of Proposition 6) that only symmetric equilibria exist provided the managers face sufficiently similar PPS, with a sufficient condition being that

$$\frac{\beta_B^{MH} - \beta_A^{MH}}{\beta_B^{MH}} \leq \frac{2 - \eta\rho}{2\mu + 3}. \tag{25}$$

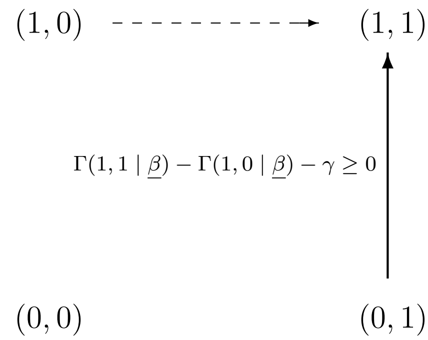
This condition is met if the divisions face sufficiently similar levels of volatility, i.e., small $(\sigma_A^2 - \sigma_B^2)$, and η is small. It rules out asymmetric project effort equilibria under the symmetric regime and ensures $\mathbf{e}^S(\boldsymbol{\beta}) = (1,1)$ if $\gamma \leq \gamma_{(1,1)}(\boldsymbol{\beta})$, and $\mathbf{e}^S(\boldsymbol{\beta}) = (0,0)$ otherwise, even for optimally chosen PPS, i.e., at $\boldsymbol{\beta} = \boldsymbol{\beta}^S$.[28]

[27] Because we have assumed similar functional forms for all key constructs across the (monetary and personally costly) input scenarios, the effort cost threshold equals $\gamma^{\ell}_{(1,1)}(\beta_\ell) = \beta_\ell \phi^{\ell}_{(1,1)}(\beta_\ell)$, with $\phi^{\ell}_{(1,1)}(\beta_\ell)$ as defined in (14). (As argued above, however, the optimal assignment of decision rights will differ from that for monetary inputs.) This reflects the fact that project effort costs are incurred privately by the manager. In the benchmark solution, on the other hand, $\gamma^* = \phi^*$, as in Section 4.2, because *ex ante* the principal ultimately pays for all project input costs, whether monetary in nature or effort disutility.

[28] In the proof of Proposition 6 we show that if $(\overline{\beta} - \underline{\beta})/\overline{\beta} \leq \frac{2-\eta\rho}{2\mu+3}$, only symmetric equilibria exist under the symmetric regime. In equilibrium, $\beta_A^S \leq \beta_B^S$ and the relative PPS differential,



(a) Bundling: high-PPS manager chooses e_A and e_B



(b) Symmetric form: Nash equilibrium (binding constraint is low-PPS manager)

Figure 7: Comparison of project effort incentives with exogenous PPS
Solid arrows indicate the binding constraints in order to elicit the project effort profile $(1, 1)$.

We turn now to the principal's Date-1 contracting problem with noncontractible, lumpy project efforts. Because at the benchmark PPS levels either delegation regime would elicit suboptimal levels of project effort, any contract adjustments will be geared toward stimulating investment. For the symmetric regime, denote by $\breve{\beta}^S(\gamma)$ the PPS level that makes (24) binding for given effort cost, and by $\overline{\gamma}^S \equiv \gamma_{(1,1)}(\boldsymbol{\beta}^o(1,1))$ the effort cost level up to which high project efforts are elicited without adjusting the PPS. The optimal contract under the symmetric regime then is a straightforward adaptation of that in Proposition 2:

$(\beta_B^S - \beta_A^S)/\beta_B^S$ is bounded from above by $(\beta_B^{MH} - \beta_A^{MH})/\beta_B^{MH}$. To see why, note that for sufficiently low γ, both managers invest even at $\boldsymbol{\beta}^S = \boldsymbol{\beta}^o(1,1)$; for sufficiently high γ, even the principal prefers $(0,0)$, and so $\boldsymbol{\beta}^S = \boldsymbol{\beta}^o(0,0)$. Inducing $(1,1)$ for intermediate values of γ requires strengthening incentives—first to the low-PPS "bottleneck" Manager A. This results in convergence of the PPS for intermediate γ. Therefore, $(\beta_B^S - \beta_A^S)/\beta_B^S \leq (\beta_B^o(0,0) - \beta_A^o(0,0))/\beta_B^o(0,0) \leq (\beta_B^{MH} - \beta_A^{MH})/\beta_B^{MH}$. Note that the sufficient condition to rule out asymmetric equilibria for monetary investments in Section 4.1 (e.g., in Lemma 4) was an upper bound on the absolute PPS differential, whereas (25) bounds the relative differential. This is again a consequence of the fact that input costs are scaled by the PPS for monetary investments but not for project efforts.

Proposition 6 (Symmetric regime, project efforts) *Given Assumption 1′ and* $\mathbf{e} \in \{0,1\}^2$*, suppose (25) holds. Under the symmetric regime, there exists a unique* $\gamma^S \in (\overline{\gamma}^S, \gamma^*)$*, such that:*

(i) If $\gamma \in (\overline{\gamma}^S, \gamma^S]$*, then* $\beta_A = \breve{\beta}^S(\gamma) > \beta_A^o(1,1)$ *and* $\beta_B = \max\{\breve{\beta}^S(\gamma), \beta_B^o(1,1)\}$*. Both managers exert project effort, as would be the case under the benchmark solution.*

(ii) If $\gamma \in (\gamma^S, \gamma^*]$*, then* $\beta_i = \beta_i^o(0,0) > \beta_i^o(1,1),\ i = A, B$*. Neither manager exerts project effort, whereas* $\mathbf{e}^* = (1,1)$ *under the benchmark solution.*

Under bundling, designating Manager B the investment center manager most effectively stimulates project efforts, as his stable operating environment calls for high-powered PPS to begin with. Given $\ell = B$, denote by $\breve{\beta}^B(\gamma)$ the PPS that makes (23) binding for given effort cost, and by $\overline{\gamma}^B \equiv \gamma^B_{(1,1)}(\boldsymbol{\beta}^o(1,1))$ the fixed cost level up to which bundling would implement high efforts absent any PPS adjustment, i.e., if $\beta_B = \beta_B^o(1,1)$. The optimal contract under bundling then is:

Proposition 7 (Bundling, project efforts) *Given Assumption 1′ and* $\mathbf{e} \in \{0,1\}^2$*, under bundling, Manager B (low operating volatility) is designated investment center manager, and there exists a unique* $\widehat{\gamma}^B \in (\overline{\gamma}^B, \gamma^*]$ *such that:*

(i) If $\gamma \in (\overline{\gamma}^B, \widehat{\gamma}^B]$*, then* $\beta_B = \breve{\beta}^B(\gamma) > \beta_B^o(1,1)$ *and* $\beta_A = \beta_A^o(1,1)$*. Manager B chooses* $(1,1)$*, as would be the case under the benchmark solution.*

(ii) If $\gamma \in (\widehat{\gamma}^B, \gamma^*]$*, then* $\beta_i = \beta_i^o(0,0) > \beta_i^o(1,1),\ i = A, B$*. Manager B chooses* $(0,0)$*, whereas* $\mathbf{e}^* = (1,1)$ *under the benchmark solution.*

Under either regime, any contract adjustments to stimulate project efforts take the shape of strengthening the PPS for one or both of the managers. While this is in contrast to monetary investments which called for muted PPS, the feature that contract adjustments lead to PPS convergence under the symmetric regime and to PPS divergence under bundling carries over.

The tradeoff faced by the principal at Date 0 is as follows: Bundling decision rights in the hands of the manager with a more stable environment takes advantage of the standard moral hazard logic that higher PPS elicits greater effort, both general as well as project-specific. On the other hand, the symmetric regime better utilizes the strategic complementarity. We therefore arrive at a regime comparison that is qualitatively similar to that for monetary inputs (Proposition 4):

Proposition 8 *Given Assumption 1′ and* $\mathbf{e} \in \{0, 1\}^2$*, suppose (25) holds. For given* σ_B^2*, the principal prefers the symmetric regime to bundling if* $\sigma_A^2 \in (\sigma_B^2, \sigma_B^2 + \delta)$*, for some* $\delta > 0$*.*

In summary, while changing the nature of specific inputs flips the relation between PPS and equilibrium input levels, the optimal allocation of decision rights remains qualitatively unchanged except for the prediction which manager will be designated investment center manager under bundling; see Figure 8.

6 Concluding Remarks

This paper derives new predictions for the optimal assignment of decision rights across business unit managers in multidivisional firms that exhibit synergies. With scalable project-specific inputs, it is always optimal for the principal to bundle decision rights in the hands of one division manager—but which manager it is, hinges on the nature of the input. If the project input is a monetary investment, e.g., PP&E, the manager facing more volatile operations is designated investment center manager; if the input is personally costly (effort), it is the manager facing more stable general operations. In either case, the PPS differential attributable to the different volatility levels assures the underprovision of specific inputs due to holdup is alleviated. With lumpy project inputs, on the other hand, it is always optimal to split the decision rights symmetrically be-

Project inputs	**Monetary investments**	**Personally-costly efforts**
Scalable	**Bundling** the decision rights in the hands of the manager facing ***high general volatility***	**Bundling** the decision rights in the hands of the manager facing ***low general volatility***
Lumpy	**Symmetric allocation** of decision rights across managers if they face similar general volatilities	

Figure 8: Optimal allocation of decision rights

tween the managers, provided they face comparable levels of operating volatility. This holds for monetary investments as well as personally-costly project efforts.

Moreover, the model sheds light on the effect of contractual incompleteness on the managers' *relative* incentive strength: bundling of decision rights results in PPS *divergence* across divisions; the symmetric regime results in PPS *convergence*. To test our predictions, it would be useful to adapt earlier empirical studies on incentives and organizational processes, e.g., Nagar (2002), by distinguishing between the delegation of tasks that are personally costly to managers and those that call for managers to invest the firm's funds in joint projects.

It is useful to clarify the role the investment-risk link (Baldenius and Michaeli 2017)—or, more generally, input-risk link—plays for the findings. Our results for scalable monetary investments are indeed driven by this second-moment effect: investment incentives are depressed by high-powered PPS because of the incremental risk exposure; decision rights therefore should be bundled in the hands of the manager whose PPS is muted to insure him against a high-volatility environment. The result that scalable project efforts should be bundled in the hands of the manager facing the more stable environment, on the other hand, is due to a more conventional (first-moment) moral hazard argument: greater PPS elicits both general-purpose and project-specific efforts in lockstep. Finally, the result that a symmetric split of decision rights elicits greater lumpy inputs is also unrelated to the input-risk link and instead is driven mainly by the model feature that the party that chooses any action in our model also has to bear its cost. In an incomplete contracting setting where actions are not verifiable but decision rights are, this seems the most plausible assumption.

Appendix

Proof of Lemma 1: Given Assumption 1, it is straightforward to show that $\frac{\partial^2 W(\mathbf{k},\boldsymbol{\beta})}{\partial k_A \partial k_B} = 1 - \frac{\rho\eta}{4}(\beta_A^2 + \beta_B^2) \geq 1 - \frac{\rho\eta}{2} > 0$ and $\frac{\partial^2 \Gamma(\mathbf{k}|\beta_i)}{\partial k_A k_B} = \beta_i \left(\frac{1}{2} - \frac{\rho\eta}{4}\beta_i\right) \propto 1 - \frac{\rho\eta}{2}\beta_i \geq 1 - \frac{\rho\eta}{2} > 0, \quad i = A, B.$

Proof of Lemma 2:

Part (a): For given PPS, the best response for Manager i to an anticipated investment k_j by his counterpart, $k_i(k_j \mid \beta_i)$, is found by setting the first-order conditions corresponding to (9) equal to zero; upon rearranging:

$$k_i(k_j \mid \beta_i) = \frac{(\mu + k_j)\left(\frac{1}{2} - \frac{\rho\eta}{4}\beta_i\right)}{f + \frac{\rho\eta}{4}\beta_i - \frac{1}{2}}, \quad i = A, B, \quad j \neq i.$$

Because $k_i(k_j = 0 \mid \beta_i) > 0$, and $\frac{d}{dk_j} k_i(k_j \mid \beta_i) < 1$ for any $k_j, \boldsymbol{\beta}, i$ and $j \neq i$, we have a unique Nash equilibrium in investments for any PPS. (To bound the slope of the reaction curves by one, we use the maintained assumption that $f > 6$.) Solving for the equilibrium in closed form:

$$k_i^S(\boldsymbol{\beta}) = \frac{\mu\,(2 - \rho\eta\beta_i)}{\rho\eta(\beta_i + \beta_j) + 4(f-1)}, \ i = A, B, \ j \neq i.$$

The equilibrium investments are decreasing in either manager's PPS:

$$\frac{dk_i^S(\boldsymbol{\beta})}{d\beta_i} = \frac{-\mu\rho\eta(\rho\eta\beta_j + 4f - 3)}{(\rho\eta(\beta_i + \beta_j) + 4(f-1))^2} < \frac{dk_i^S(\boldsymbol{\beta})}{d\beta_j} = \frac{-\mu\rho\eta(1 - \rho\eta\beta_i)}{(\rho\eta(\beta_i + \beta_j) + 4(f-1))^2} < 0. \quad (26)$$

Part (b): We first show that $k_A^S > k_B^S$. Recall that $\boldsymbol{\beta}^S \in \arg\max_{\boldsymbol{\beta}} W(\boldsymbol{\beta})$, where:

$$\begin{aligned} W(\boldsymbol{\beta}) &\equiv W(\boldsymbol{\beta}, \mathbf{k}^S(\boldsymbol{\beta})) \\ &= \sum_i \Phi_i(\beta_i) + E\left[M(\theta, \mathbf{k}^S(\boldsymbol{\beta}))\right] - \frac{\rho}{8}\sum_i \beta_i^2 Var(\mathbf{k}) - \sum_i F(k_i^S(\boldsymbol{\beta})). \end{aligned}$$

Taking derivative,

$$\begin{aligned}
\frac{\partial W(\boldsymbol{\beta})}{\partial \beta_i} &= \Phi_i'(\beta_i) - \frac{\rho}{4}\beta_i Var(\mathbf{k}^S(\boldsymbol{\beta}) \\
&\quad + \sum_{j=A,B}\left[\frac{dk_j^S(\boldsymbol{\beta})}{d\beta_i}\left(E\left[M_{k_j}(\theta,\mathbf{k}^S(\boldsymbol{\beta})\right] - \frac{\rho}{8}\sum_l \beta_l^2 \frac{\partial Var(\mathbf{k}^S(\boldsymbol{\beta}))}{\partial k_j} - F'(k_j^S(\boldsymbol{\beta}))\right)\right] \\
&= \Phi_i'(\beta_i) - \frac{\rho\eta}{4}\beta_i(q^*(\mu,\mathbf{k}^S(\boldsymbol{\beta})))^2 \\
&\quad + \sum_{j=A,B}\left[\frac{dk_j^S(\boldsymbol{\beta})}{d\beta_i}\left(q^*(\mu,\mathbf{k}^S(\boldsymbol{\beta})) - \frac{\rho\eta}{4}\sum_l \beta_l^2 q^*(\mu,\mathbf{k}^S(\boldsymbol{\beta})) - fk_j^S(\boldsymbol{\beta})\right)\right] \\
&\equiv w_i^S(\boldsymbol{\beta}) \qquad (27)
\end{aligned}$$

The optimal PPS vector $\boldsymbol{\beta}^S$ satisfy the system of equations:

$$w_i^S(\boldsymbol{\beta}^S) = 0, \quad i = A, B.$$

Note that $w_A^S(\boldsymbol{\beta})$ differs from $w_B^S(\boldsymbol{\beta})$ only in σ_i^2, $i = A, B$. Because by assumption, $\sigma_A > \sigma_B$, for $\beta_A^S < \beta_B^S$ to hold, we only need to show that β_i^S is decreasing in σ_i^2. Applying the Implicit Function Theorem to (27):

$$\frac{\partial \beta_i^S}{\partial \sigma_i^2} = -\frac{\frac{\partial w_i^S(\boldsymbol{\beta})}{\partial \sigma_i^2}}{\frac{\partial w_i^S(\boldsymbol{\beta})}{\partial \beta_i^2}} \propto \frac{\partial w_i^S(\boldsymbol{\beta})}{\partial \sigma_i^2} = -\rho\beta_i < 0.$$

Now note that $k_A^S > k_B^S$ by (26) and the fact that $k_i^S = k_i^S(\boldsymbol{\beta}^S)$.

Next, we show that $k_i^* > k_i^S$. Recall that under the benchmark case, the principal contracts on $k_A^* = k_B^* = k^*$. We fix k_B (the investment undertaken by the manager facing less volatile general operations) at $k_B = k^*$ and consider the best response of Manager A, described by

$$y_A^S(k_A, k_B^*) \equiv q^*(\mu, k_A, k_B^*)\left(\frac{1}{2} - \frac{\rho\eta}{4}\beta_A\right) - fk_A = 0.$$

Contrast that with the conditionally optimal choice of contractible investment on the part of the principal (benchmark case):

$$y_A^*(k_A, k_B^*) \equiv q^*(\mu, k_A, k_B^*)\left(1 - \frac{\rho\eta}{4}\sum_i(\beta_i^o(k_A, k_B^*))^2\right) - fk_A = 0.$$

Now note that:

$$\begin{aligned} y_A^*(k_A, k_B^*) - y_A^S(k_A, k_B^*) &= q^*(\mu, k_A, k_B^*)\left(1 - \frac{\rho\eta}{4}\sum_i(\beta_i^o(k_A, k_B^*))^2 - \frac{1}{2} + \frac{\rho\eta}{4}\beta_A\right) \\ &\propto 1 - \frac{\rho\eta}{2}\left(\sum_i(\beta_i^o(k_A, k_B^*))^2 - \beta_A\right) \\ &> 1 - \rho\eta \\ &\geq 0. \end{aligned}$$

It follows that, even if Manager B invests the benchmark, Manager A will underinvest. Similar arguments apply to Manager B's best response if Manager A were to choose his benchmark investment k_A^*. Combined with strategic complementarity (Lemma 1), both managers thus underinvest under the symmetric regime.

Proof of Proposition 1: In Step 1, we show the bundling regime with $\ell = A$ outperforms the symmetric regime; in Step 2, that it outperforms the bundling regime with $\ell = B$.

Step 1. Fix the PPS at $\boldsymbol{\beta} = \boldsymbol{\beta}^S$ and consider the equilibrium under the bundling regime with $\ell = A$ at this PPS level. Then, $\widehat{k}_i^A(\boldsymbol{\beta}^S) > k_i^S(\boldsymbol{\beta}^S)$, for any i, by $\beta_A^S < \beta_B^S$ and strategic complementarity. For $W^S(\boldsymbol{\beta}) \equiv W(\mathbf{k}^S(\boldsymbol{\beta}), \boldsymbol{\beta})$ and $W^\ell(\boldsymbol{\beta}) \equiv W(\widehat{\mathbf{k}}^\ell(\boldsymbol{\beta}), \boldsymbol{\beta})$, it follows that $W^S(\boldsymbol{\beta}^S) \leq W^A(\boldsymbol{\beta}^S)$, because: (i) $W(\cdot)$ is concave in (k_A, k_B) for given $\boldsymbol{\beta}$, and (ii) $\widehat{k}_i^A \leq k_i^*$, for any i, as per Lemma 3. By revealed preference, $W^A(\boldsymbol{\beta}^S) \leq W^A(\widehat{\boldsymbol{\beta}}^A) \equiv W(\widehat{\mathbf{k}}^A(\widehat{\boldsymbol{\beta}}^A), \widehat{\boldsymbol{\beta}}^A)$.

Step 2. Note that $q^*(\theta, \mathbf{k})$ depends solely on the sum of the investments, henceforth denoted $n \equiv \sum_{i=A,B} k_i$, and so does the conditionally optimal PPS of Manager i in the benchmark solution. Therefore, slightly abusing notation, we write $q^*(\theta, n)$ (instead of $q^*(\theta, \mathbf{k})$) and $\beta_i^o(n)$ (instead $\beta_i^o(\mathbf{k})$).

Denote the optimal PPS for Manager B under the bundling regime with $\ell = B$ by $\widehat{\beta}_B^B = \overline{\beta}$, resulting in total units of equilibrium investment $\overline{n} \equiv \sum_{i=A,B} \widehat{k}_i^B(\overline{\beta})$. The optimal PPS vector under the bundling regime with $\ell = B$ then is $\widehat{\boldsymbol{\beta}}^B = (\beta_A^o(\overline{n}), \overline{\beta})$ where $\overline{n} \equiv \sum_i \widehat{k}_i^B(\overline{\beta})$. Note that $\overline{\beta} < \beta_B^o(\overline{n})$. Now consider the bundling regime with $\ell = A$ and suppose the principal chooses a PPS vector of $\boldsymbol{\beta} = (\beta_A = \overline{\beta}, \beta_B = \beta^o_B(\overline{n}))$, resulting again in total investment of $\sum_i \widehat{k}^A_i(\overline{\beta}) = \overline{n}$. Because $\sigma^2_A > \sigma^2_B$, $\beta^o_B(\overline{n}) > \beta^o_A(\overline{n})$, but there are two possible cases regarding the ranking of $\beta^o_A(\overline{n})$ and $\overline{\beta}$.

Case 1: We claim that for $\beta^o_B(\overline{n}) > \beta^o_A(\overline{n}) > \overline{\beta}$ bundling with $\ell = A$ dominates bundling with $\ell = B$. Denoting $\Phi_i(\beta_i) \equiv a_i(\beta_i) - \frac{v}{2}(a_i(\beta_i))^2 - \frac{\rho}{2}\beta_i^2\sigma_i^2$; $\chi(\beta_i \mid \overline{n}, \sigma^2_i) \equiv \Phi_i(\beta_i) - \frac{\rho m}{8}\beta_i^2(q^*(\mu, \overline{n}))^2$ and $W^\ell(\boldsymbol{\beta}) \equiv W(\widehat{\mathbf{k}}^\ell(\beta_\ell), \boldsymbol{\beta})$, we have:

$$
\begin{aligned}
W^A(\overline{\beta}, \beta^o_B(\overline{n})) - W^B(\beta^o_A(\overline{n}), \overline{\beta}) &= \chi(\overline{\beta} \mid \overline{n}, \sigma^2_A) + \chi(\beta^o_B(\overline{n}) \mid \overline{n}, \sigma^2_B) \\
&\quad - \chi(\beta^o_A(\overline{n}) \mid \overline{n}, \sigma^2_A) - \chi(\overline{\beta} \mid \overline{n}, \sigma^2_B) \\
&> \chi(\overline{\beta} \mid \overline{n}, \sigma^2_A) + \chi(\beta^o_A(\overline{n}) \mid \overline{n}, \sigma^2_B) \\
&\quad - \chi(\beta^o_A(\overline{n}) \mid \overline{n}, \sigma^2_A) - \chi(\overline{\beta} \mid \overline{n}, \sigma^2_B) \\
&= -\frac{\rho}{2}\overline{\beta}^2\left(\sigma^2_A - \sigma^2_B\right) + \frac{\rho}{2}(\beta^o_A(\overline{n}))^2\left(\sigma^2_A - \sigma^2_B\right) \\
&\propto (\beta^o_A(\overline{n}))^2 - \overline{\beta}^2 \\
&> 0,
\end{aligned}
$$

because $\beta^o_j(n) \in \arg\max_{\beta_j} \chi(\beta_j \mid n, \sigma^2_j)$, $j = A, B$ and $\beta^o_B(\overline{n}) > \beta^o_A(\overline{n}) > \overline{\beta}$. By revealed preference, $W^A(\widehat{\boldsymbol{\beta}}^A) > W^A(\overline{\beta}, \beta^o_B(\overline{n}))$, i.e., the bundling regime with $\ell = A$ outperforms the bundling regime with $\ell = B$.

Case 2: We claim that for $\beta^o_B(\overline{n}) > \overline{\beta} > \beta^o_A(\overline{n})$ the symmetric regime dominates bundling with $\ell = B$. Again, fix $\boldsymbol{\beta} = \widehat{\boldsymbol{\beta}}^B = (\beta^o_A(\overline{n}), \overline{\beta})$ at the level optimal under the bundling regime with $\ell = B$. If the principal were to choose this PPS vector under the symmetric regime, the managers' investment reaction functions would be:

$$
\begin{aligned}
k^S_B(k_A \mid \beta_B = \overline{\beta}) &\equiv \widehat{k}^B_B(k_A \mid \beta_B = \overline{\beta}), \\
k^S_A(k_B \mid \beta_A = \beta^o_A(\overline{n})) &> \widehat{k}^B_A(k_B \mid \beta_B = \overline{\beta}).
\end{aligned}
$$

By strategic complementarity, $\mathbf{k}^S(\beta^o_A(\overline{n}), \overline{\beta}) > \widehat{\mathbf{k}}^B$. Recall that $W(\cdot)$ is concave in (k_A, k_B) for given $\boldsymbol{\beta}$. Hence, $W^S(\widehat{\boldsymbol{\beta}}^B) > W^B(\widehat{\boldsymbol{\beta}}^B)$ because $\widehat{k}^B_A < k^S_i \leq k^*_i$, $i =$

A, B (by the observation above, Lemma 2 and Lemma 3). By revealed preference, $W^S(\boldsymbol{\beta}^S) > W^S(\widehat{\boldsymbol{\beta}}^B)$, which together with Step 1 above establishes the result.

Proof of Lemma 4: Using (15) and (16), if $\phi_{(1,1)}(\boldsymbol{\beta}) - \phi_{(0,0)}(\boldsymbol{\beta}) < 0$, the investment profile (1,0) can be a Nash equilibrium for any $\phi \in (\phi_{(1,1)}(\boldsymbol{\beta}), \phi_{(0,0)}(\boldsymbol{\beta}))$. Otherwise, if $\phi_{(1,1)}(\boldsymbol{\beta}) - \phi_{(0,0)}(\boldsymbol{\beta}) \geq 0$, then for any $\phi \in (\phi_{(0,0)}(\boldsymbol{\beta}), \phi_{(1,1)}(\boldsymbol{\beta}))$, both $(1,1)$ and $(0,0)$ can be equilibria simultaneously. We note that:

$$\begin{aligned}
\phi_{(1,1)}(\boldsymbol{\beta}) - \phi_{(0,0)}(\boldsymbol{\beta}) &= \frac{\Gamma(1,1 \mid \overline{\beta}) - \Gamma(1,0 \mid \overline{\beta})}{\overline{\beta}} - \frac{\Gamma(1,0 \mid \underline{\beta}) - \Gamma(0,0 \mid \underline{\beta})}{\underline{\beta}} \\
&= \frac{E[(q^*(\theta,1,1))^2 - (q^*(\theta,1,0))^2]}{4} - \frac{\rho\eta}{2}\overline{\beta}\frac{(q^*(\mu,1,1))^2 - (q^*(\mu,1,0))^2}{4} \\
&\quad - \frac{E[(q^*(\theta,1,0))^2 - (q^*(\theta,0,0))^2]}{4} + \frac{\rho\eta}{2}\underline{\beta}\frac{(q^*(\mu,1,0))^2 - (q^*(\mu,0,0))^2}{4} \\
&= \frac{1}{4}\left(E[(\theta+2)^2 - (\theta+1)^2] - \frac{\rho\eta}{2}\overline{\beta}((\mu+2)^2 - (\mu+1)^2)\right) \\
&\quad - \frac{1}{4}\left(E[(\theta+1)^2 - \theta^2] + \frac{\rho\eta}{2}\underline{\beta}((\mu+1)^2 - \mu^2)\right) \\
&= \underbrace{\frac{1}{4}\left[(2\mu+3)\left(1 - \frac{\overline{\beta}\eta\rho}{2}\right) - (2\mu+1)\left(1 - \frac{\underline{\beta}\eta\rho}{2}\right)\right]}_{\equiv X(\boldsymbol{\beta})}
\end{aligned}$$

Because $\beta \in [0,1]$, a sufficient condition for $X(\boldsymbol{\beta}) > 0$ is that $\overline{\beta} - \underline{\beta} < \frac{1}{\mu}\left(\frac{3}{2} + \frac{2}{\rho\eta}\right)$.

Note that if $X(\boldsymbol{\beta}) \geq 0$, then for any $\phi \in (\phi_{(0,0)}(\boldsymbol{\beta}), \phi_{(1,1)}(\boldsymbol{\beta}))$, both $(1,1)$ and $(0,0)$ can be Nash equilibria simultaneously. To predict which of these equilibria the managers will play, we invoke Theorem 7 in Milgrom and Roberts (1990): if multiple equilibria exist in a supermodular game (such as the managers' Date-2 investment game) with positive spillovers (i.e., player i's payoff is increasing in player j's action), then the highest equilibrium—here, $(1,1)$—Pareto-dominates the other equilibria. That is, for $\phi \in (\phi_{(0,0)}(\boldsymbol{\beta}), \phi_{(1,1)}(\boldsymbol{\beta}))$ we can ignore the shirking equilibrium.

Proof of Lemma 5: We need to show that $\phi^*(\boldsymbol{\beta}) - \phi_{(1,1)}(\boldsymbol{\beta}) > 0$ and $\phi^*(\boldsymbol{\beta}) -$

$\phi^{\ell}_{(1,1)}(\beta_\ell) > 0$ for any given PPS profile, $\boldsymbol{\beta} = (\underline{\beta}, \overline{\beta})$. Simplifying,

$$\begin{aligned}
\phi^*(\boldsymbol{\beta}) &= \frac{1}{2}\sum_{i=A,B}\left[\frac{E[M(\theta,1,1)]}{2} - \frac{\rho}{8}\beta_i^2 \cdot Var(1,1) - \left(\frac{E[M(\theta,0,0)]}{2} - \frac{\rho}{8}\beta_i^2 \cdot Var(0,0)\right)\right] \\
&= \frac{E[(q^*(\theta,1,1))^2 - (q^*(\theta,0,0))^2]}{4} - \frac{\rho\eta}{2}(\underline{\beta}^2 + \overline{\beta}^2)\frac{(q^*(\mu,1,1))^2 - (q^*(\mu,0,0))^2}{8} \\
&= \frac{E[((\theta+2)^2 - \theta^2]}{4} - \frac{\rho\eta}{2}(\underline{\beta}^2 + \overline{\beta}^2)\frac{(\mu+2)^2 - \mu^2}{8} \\
&= (\mu+1)\left(1 - \frac{\rho\eta}{4}(\underline{\beta}^2 + \overline{\beta}^2)\right).
\end{aligned}$$

Similarly,

$$\begin{aligned}
\phi^{\ell}_{(1,1)}(\underline{\beta}) &= \frac{\Gamma(1,1|\underline{\beta}) - \Gamma(0,0|\underline{\beta})}{2\underline{\beta}} = \frac{1}{2}(\mu+1)\left(1 - \frac{\rho\eta}{2}\underline{\beta}\right); \\
\phi^{\ell}_{(1,1)}(\overline{\beta}) &= \frac{\Gamma(1,1|\overline{\beta}) - \Gamma(0,0|\overline{\beta})}{2\overline{\beta}} = \frac{1}{2}(\mu+1)\left(1 - \frac{\rho\eta}{2}\overline{\beta}\right);
\end{aligned}$$

$$\phi_{(1,1)}(\boldsymbol{\beta}) = \frac{\Gamma(1,1|\overline{\beta}) - \Gamma(0,1|\overline{\beta})}{\overline{\beta}} = \frac{1}{2}\left(\mu + \frac{3}{2}\right)\left(1 - \frac{\rho\eta}{2}\overline{\beta}\right).$$

Now note that $\phi^{\ell}_{(1,1)}(\underline{\beta}) > \phi^{\ell}_{(1,1)}(\overline{\beta})$ and so

$$\begin{aligned}
\phi^*(\boldsymbol{\beta}) - \phi^{\ell}_{(1,1)}(\overline{\beta}) &> \phi^*(\boldsymbol{\beta}) - \phi^{\ell}_{(1,1)}(\underline{\beta}) \\
&= (\mu+1)\left(1 - \frac{\rho\eta}{4}(\underline{\beta}^2 + \overline{\beta}^2)\right) - (\mu+1)\left(\frac{1}{2} - \frac{\rho\eta}{4}\underline{\beta}\right) \\
&= \frac{1}{2}(\mu+1)\left(1 - \frac{\rho\eta}{2}(\underline{\beta}^2 + \overline{\beta}^2 - \underline{\beta})\right) \\
&\propto 1 - \frac{\rho\eta}{2}(\overline{\beta}^2 - \underline{\beta}(1 - \underline{\beta})) \\
&> 1 - \frac{\rho\eta}{2}\overline{\beta}^2 \\
&> 0.
\end{aligned}$$

Lastly,

$$\begin{aligned}
\phi^*(\boldsymbol{\beta}) - \phi_{(1,1)}(\boldsymbol{\beta}) &= (\mu+1)\left(1 - \frac{\rho\eta}{4}(\underline{\beta}^2 + \overline{\beta}^2)\right) - \left(\mu + 1 + \frac{1}{2}\right)\left(\frac{1}{2} - \frac{\rho\eta}{4}\overline{\beta}\right) \\
&= \frac{1}{2}(\mu+1)\left(1 - \frac{\rho\eta}{2}(\underline{\beta}^2 + \overline{\beta}^2 - \underline{\beta})\right) - \frac{1}{2}\left(\frac{1}{2} - \frac{\rho\eta}{4}\overline{\beta}\right) \\
&> \frac{1}{2}\left(1 - \frac{\rho\eta}{2}\overline{\beta}^2\right) - \frac{1}{2}\left(\frac{1}{2} - \frac{\rho\eta}{4}\overline{\beta}\right) \\
&> \left(\frac{1}{2} - \frac{\rho\eta}{4}\overline{\beta}\right) - \frac{1}{2}\left(\frac{1}{2} - \frac{\rho\eta}{4}\overline{\beta}\right) \\
&> 0.
\end{aligned}$$

Proof of Lemma 6: We need to show that $\phi^A_{(1,1)}(\boldsymbol{\beta}) \leq \phi_{(1,1)}(\boldsymbol{\beta})$. Using the proof of Lemma 5,

$$\phi^{\ell^*}_{(1,1)}(\boldsymbol{\beta}) = \frac{1}{2}(\mu+1)\left(1 - \frac{\rho\eta}{2}\underline{\beta}\right) \qquad \text{and} \qquad \phi_{(1,1)}(\boldsymbol{\beta}) = \frac{1}{2}\left(\mu + \frac{3}{2}\right)\left(1 - \frac{\rho\eta}{2}\overline{\beta}\right)$$

so that

$$\begin{aligned}
\phi_{(1,1)}(\boldsymbol{\beta}) - \phi^{\ell^*}_{(1,1)}(\boldsymbol{\beta}) &= \frac{1}{2}\left(\mu + \frac{3}{2}\right) - \frac{1}{2}(\mu+1) - \frac{\rho\eta}{2}\left(\overline{\beta}\frac{1}{2}\left(\mu + \frac{3}{2}\right) - \underline{\beta}\frac{1}{2}(\mu+1)\right) \\
&\propto \mu + \frac{3}{2} - \mu - \frac{2}{2} - \frac{\rho\eta}{2}\left(\overline{\beta}\left(\mu + \frac{3}{2}\right) - \underline{\beta}(\mu+1)\right) \\
&= \underbrace{1 - \frac{\rho\eta}{2}\left(2\mu(\overline{\beta} - \underline{\beta}) + 3\overline{\beta} - 2\underline{\beta}\right)}_{\equiv Y}
\end{aligned}$$

Note that

$$\begin{aligned}
X(\boldsymbol{\beta}) - Y &= 2 - \frac{\rho\eta}{2}\left(2\mu(\overline{\beta} - \underline{\beta}) + 3\overline{\beta} - \underline{\beta}\right) - 1 + \frac{\rho\eta}{2}\left(2\mu(\overline{\beta} - \underline{\beta}) + 3\overline{\beta} - 2\underline{\beta}\right) \\
&= 1 - \frac{\rho\eta}{2}\underline{\beta},
\end{aligned}$$

where Y is defined above and $X(\boldsymbol{\beta})$ is defined in Lemma 5. It is straightforward that $X(\boldsymbol{\beta}) - Y \in (0,1)$. It follows that a sufficient condition for $Y > 0$ is $X(\boldsymbol{\beta}) > 1$. Lastly, note that a sufficient condition for $X(\boldsymbol{\beta}) > 1$ is that $\overline{\beta} - \underline{\beta} \leq \frac{1}{\mu}\left(\frac{3}{2} + \frac{1}{\rho\eta}\right)$.

Proof of Lemma 7: Recall that the principal's payoff under the benchmark case can be conveniently presented as $W^*(\mathbf{k}) \equiv W(\boldsymbol{\beta}^o(\mathbf{k}), \mathbf{k})$, where $\boldsymbol{\beta}^o(\mathbf{k})$ is as defined in (11) (because for given $\mathbf{k}$, the optimal PPS is $\boldsymbol{\beta}^o(\mathbf{k})$). Note that $\frac{\partial}{\partial\phi}W^*(1,1) = -2$; $\frac{\partial}{\partial\phi}W^*(1,0) = -1$; and $\frac{\partial}{\partial\phi}W^*(0,0) = 0$. Further, by Assumption 1, $\lim_{\phi\to 0} W^*(1,1) > \lim_{\phi\to 0} W^*(1,0) > \lim_{\phi\to 0} W^*(0,0)$. Hence, for sufficiently low ϕ, the optimal solution is $\mathbf{k}^* = (1,1)$ and $\boldsymbol{\beta}^* = \boldsymbol{\beta}^o(1,1)$. Moreover, $\lim_{\phi\to\infty} W^*(1,1) < \lim_{\phi\to\infty} W^*(1,0) < \lim_{\phi\to\infty 0} W^*(0,0)$. Hence, for sufficiently high ϕ, the optimal solution is $\mathbf{k}^* = (0,0)$ and $\boldsymbol{\beta}^* = \boldsymbol{\beta}^o(0,0)$.

It remains to consider the solution for intermediate values of ϕ. To show that it is never optimal for the principal to contract on $\mathbf{k} = (1,0)$ and $\boldsymbol{\beta} = \boldsymbol{\beta}^o(1,0)$ we need to show that $\phi_{2\to 1} \geq \phi_{2\to 0}$, where

$$\phi_{2\to 1} \equiv W^*(1,1) - W^*(1,0) \qquad \text{and} \qquad \phi_{2\to 0} \equiv \frac{W^*(1,1) - W^*(0,0)}{2}.$$

Note that

$$\begin{aligned}
\phi_{2\to 1} - \phi_{2\to 0} &\propto 2(W^*(1,1) - W^*(1,0)) - (W^*(1,1) - W^*(0,0)) \\
&= W^*(1,1) - W^*(1,0) - (W^*(1,0) - W^*(0,0)) \\
&\geq 0,
\end{aligned}$$

because $W^*(1,1) - W^*(1,0) \geq W^*(1,0) - W^*(0,0)$. To see why, treat for a moment $\mathbf{k}$ as continuous variable and note that $W^*(\mathbf{k})$ has strictly increasing differences in $\mathbf{k}$. Specifically, using the Envelope Theorem,

$$\begin{aligned}
\frac{dW^*}{dk_A} = \frac{\partial W^*}{k_A} &= E\left[M_{k_A}(\theta, \mathbf{k})\right] - \frac{\rho}{8}\sum_i (\beta_i^o(\mathbf{k}))^2 \frac{\partial Var(\mathbf{k})}{\partial k_A} - F'(k_A) \\
&= q(\mu, \mathbf{k})\left(1 - \frac{\rho\eta}{4}\sum_i (\beta_i^o(\mathbf{k}))^2\right) - f k_A.
\end{aligned}$$

Taking cross-partial derivative,

$$\begin{aligned}
\frac{d^2W^*}{dk_A dk_B} &= q_{k_B}(\mu,\mathbf{k})\left(1-\frac{\rho\eta}{4}\sum_i(\beta_i^o(\mathbf{k}))^2\right)-q(\mu,\mathbf{k})\frac{\rho\eta}{2}\sum_i\left(\beta_i^o(\mathbf{k})\frac{\partial\beta_i^o(\mathbf{k})}{\partial k_B}\right). \\
&= 1-\frac{\rho\eta}{4}\sum_i(\beta_i^o(\mathbf{k}))^2+\frac{\rho\eta}{4}\sum_i(\beta_i^o(\mathbf{k}))^3\rho v Var(\mathbf{k}) \\
&\geq 1-\frac{\rho\eta}{2} \\
&> 0.
\end{aligned}$$

In summary, the contractible benchmark solution is:

$$(\mathbf{k}^*,\boldsymbol{\beta}^*)=\begin{cases}((1,1),\boldsymbol{\beta}^o(1,1)), & \text{if } \phi\leq\phi^*\equiv\phi^*(\boldsymbol{\beta}^o(1,1)),\\ ((0,0),\boldsymbol{\beta}^o(0,0)), & \text{otherwise.}\end{cases}$$

Proof of Proposition 2: In order to elicit the investments (Program $\mathcal{P}^S_{(1,1)}$), the principal optimally sets $\beta_B=\breve{\beta}^S(\phi)$ and $\beta_A=\min\{\breve{\beta}^S(\phi),\beta_A^o(1,1)\}$, where $\breve{\beta}^S(\phi)$ is such that the investment constraint (18) is just binding. Under the no-investment program $\mathcal{P}^S_{(0,0)}$, $\beta_i=\beta_i^o(0,0)$ for each manager $i=A,B$.

Denote by K^* the value of program $\mathcal{P}^*$ and by $K^S_{\mathbf{k}}$ the value of program $\mathcal{P}^S_{\mathbf{k}}$, $\mathbf{k}=(0,0),(1,1)$. For any $\phi\in(\phi^S,\phi^*]$:

$$\begin{aligned}
K^*(\phi) &= W(\boldsymbol{\beta}^o(1,1),\mathbf{k}=(1,1)\mid\phi),\\
K^S_{(1,1)}(\phi) &= W(\min\{\breve{\beta}^S(\phi),\beta_A^o(1,1)\},\breve{\beta}^S(\phi),\mathbf{k}=(1,1)\mid\phi),\\
K^S_{(0,0)} &= W(\boldsymbol{\beta}^o(0,0),\mathbf{k}=(0,0)).
\end{aligned}$$

Begin by considering fixed cost values $\phi=\overline{\phi}^S+\nu$ for $\nu\to 0$. Then, $K^S_{(1,1)}(\phi)=K^*(\phi)-\delta,\ \delta\to 0$, because $\breve{\beta}^S(\phi)$ is a continuous and decreasing function of ϕ and $W(\min\{\breve{\beta}^S(\phi),\beta_A^o(1,1)\},\breve{\beta}(\phi),\mathbf{k}=(1,1)\mid\phi)$ is continuous and decreasing in ϕ and increasing in β_A. That is, the value of program $\mathcal{P}^S_{(1,1)}$ converges to that of the benchmark program $\mathcal{P}^*$ as δ becomes small. At the same time, $K^S_{(0,0)}$ is bounded away from $K^*(\phi)$ for ϕ close to ϕ^S. This holds because, by Lemma 5, $\phi^S<\phi^*$, together with the observations that at ϕ^* we have $K^S_{(0,0)}=K^*(\phi)$

and that $K^*(\phi)$ is monotonically decreasing in ϕ. Thus, we have shown that for $\phi \downarrow \phi^S$, $K^S_{(1,1)}(\phi) > K^S_{(0,0)}$, whereas for $\phi \uparrow \phi^*$, $K^S_{(1,1)}(\phi) < K^S_{(0,0)}$.

Lastly, since $K^S_{(1,1)}(\phi)$ is monotonically decreasing in ϕ whereas $K^S_{(0,0)}$ is independent of ϕ, it follows that there exists a unique indifference value ϕ^S at which $K^S_{(1,1)}(\phi^S) = K^S_{(0,0)}$.

Proof of Proposition 3: The proof follows similar steps as that of Proposition 2 and is available upon request.

Proof of Proposition 4: We need to show that, given σ_A^2 sufficiently close to σ_B^2, the principal's payoff under the symmetric regime is (weakly) larger than under bundling with $\ell = A$, i.e., $W(\mathbf{k}^S, \boldsymbol{\beta}^S \mid \phi) \geq W(\widehat{\mathbf{k}}^A, \widehat{\boldsymbol{\beta}}^A \mid \phi)$ for any ϕ.

We first show that $\overline{\phi}^A \leq \overline{\phi}^S$ if σ_A^2 is sufficiently close to σ_B^2. Using the proof of Lemma 6,

$$\overline{\phi}^S - \overline{\phi}^A \;=\; \frac{1}{2}\left((\mu+1)\frac{\rho\eta}{2}\left(\beta^o_A(1,1) - \beta^o_B(1,1)\right) + \frac{1}{2}\left(1 - \frac{\rho\eta}{2}\beta^o_B(1,1)\right)\right).$$

Fix σ_B^2 and note that $\frac{\partial(\overline{\phi}^S - \overline{\phi}^A)}{\partial \sigma_A^2} = \frac{1}{2}(\mu+1)\frac{\rho\eta}{2}\frac{\beta^o_A(1,1)}{\partial \sigma_A^2} < 0$. Further, $\lim_{\sigma_A^2 \to \sigma_B^2}(\overline{\phi}^S - \overline{\phi}^A) = \frac{1}{4}\left(1 - \frac{\rho\eta}{2}\beta^o_B(1,1)\right) > 0$. Hence, for given σ_B^2, there exists a threshold $\delta_A > 0$ such that, if $\sigma_A^2 - \sigma_B^2 < \delta_A$, then $\overline{\phi}^S > \overline{\phi}^A$. Depending on the parameters, the threshold δ_A may be finite or infinite; in the latter case $\overline{\phi}^S > \overline{\phi}^A$ for any $\sigma_A^2 > 0$.

For any $\phi \in [\overline{\phi}^A, \overline{\phi}^S]$ the principal's payoff under bundling with $\ell = A$ is lower than under the symmetric regime, because under bundling she needs to reduce the investing manager's PPS to induce $(1,1)$, whereas under symmetric regime the same investments are induced with the benchmark PPS.

Next, we again fix σ_B^2 and show that $\widehat{\phi}^A \leq \phi^S$ for σ_A^2 sufficiently close to σ_B^2. Suppose not. Then, at $\widehat{\phi}^A$ it must be that the payoff of the principal when she induces investment under the symmetric regime is smaller than her payoff when she foregoes investment, or,

$$W((1,1), (\beta^m, \breve{\beta}^S(\widehat{\phi}^A)) \mid \widehat{\phi}^A) < W((0,0), \boldsymbol{\beta}^o(0,0)), \tag{28}$$

where $\beta^m \equiv \min\{\beta_A^o(1,1), \breve{\beta}^S(\widehat{\phi}^A)\}$. For the principal to induce $(1,1)$ at $\widehat{\phi}^A$ under bundling she needs to contract on $(\breve{\beta}^A(\widehat{\phi}^A), \beta_B^o(0,0))$, whereas under the symmetric regime she needs to contract on $(\beta^m, \breve{\beta}^S(\widehat{\phi}^A))$. As we will show below, the principal's expected payoff when she induces investment under the symmetric regime is larger than her payoff when she induces investment under bundling with $\ell = A$, i.e.,

$$W((1,1),(\beta^m, \breve{\beta}^S(\widehat{\phi}^A)) \mid \widehat{\phi}^A) \geq W((1,1),(\breve{\beta}^A(\widehat{\phi}^A), \beta_B^o(1,1)) \mid \widehat{\phi}^A), \quad (29)$$

if σ_A^2 sufficiently close to σ_B^2. This contradicts (28), because by definition of $\widehat{\phi}^A$, it has to be that $W((1,1),(\breve{\beta}^A(\widehat{\phi}^A), \beta_B^o(1,1)) \mid \widehat{\phi}^A) = W((0,0), \boldsymbol{\beta}^o(0,0))$. It follows that $\widehat{\phi}^A \leq \phi^S$.

We now verify that (29) holds. Substituting the optimal general effort choice, $a_i(\beta_i) = \frac{\beta_i}{v}$, and the effort-related payoff, $\Phi(\beta_i) = a(\beta_i) - \frac{v}{2}(a(\beta_i))^2 - \frac{\rho}{2}\beta_i^2\sigma_i^2$, into (29) and rearranging, gives:

$$\begin{aligned} &[\beta^m - \breve{\beta}^A(\widehat{\phi}^A)]\left[\frac{1}{v} - \left(\frac{\rho\sigma_A^2}{2} + \frac{1}{2v} + \frac{\rho}{8}Var(1,1)\right)(\beta^m + \breve{\beta}^A(\widehat{\phi}^A))\right] \\ +&[\breve{\beta}^S(\widehat{\phi}^A) - \beta_B^o(1,1)]\left[\frac{1}{v} - \left(\frac{\rho\sigma_B^2}{2} + \frac{1}{2v} + \frac{\rho}{8}Var(1,1)\right)(\breve{\beta}^S(\widehat{\phi}^A) + \beta_B^o(1,1))\right] \geq 0. \end{aligned} \quad (30)$$

Next, we note that:

$$\beta^m \geq \breve{\beta}^A(\widehat{\phi}^A) \quad \text{and} \quad \breve{\beta}^S(\widehat{\phi}^A) \leq \beta_B^o(1,1) \quad (31)$$

because: (i) $\beta^m = \min\{\beta_A^o(1,1), \breve{\beta}^S(\widehat{\phi}^A)\}$ by definition; (ii) $\breve{\beta}^S(\widehat{\phi}^A) \geq \breve{\beta}^A(\widehat{\phi}^A)$ by strategic complementarity; and (iii) $\beta_A^o(1,1) \geq \breve{\beta}^A(\widehat{\phi}^A)$ because $\widehat{\phi}^A > \overline{\phi}^A$. Hence, (30) holds if the following inequalities hold simultaneously:

$$-\left(\frac{1}{v} - \left[\frac{\rho\sigma_A^2}{2} + \frac{1}{2v} + \frac{\rho}{8}Var(1,1)\right](\beta^m + \breve{\beta}^A(\widehat{\phi}^A))\right) \leq 0, \quad (32)$$

$$\frac{1}{v} - \left[\frac{\rho\sigma_B^2}{2} + \frac{1}{2v} + \frac{\rho}{8}Var(1,1)\right](\breve{\beta}^S(\widehat{\phi}^A) + \beta_B^o(1,1)) \leq 0. \quad (33)$$

We note that (33) and (32) are proportional to

$$-\frac{1}{v(\beta^m+\breve{\beta}^A(\widehat{\phi}^A))}+\left(\frac{\rho\sigma_A^2}{2}+\frac{1}{2v}+\frac{\rho}{8}Var(1,1)\right) \leq 0, \tag{34}$$

$$\frac{1}{v(\breve{\beta}^S(\widehat{\phi}^A)+\beta_B^o(1,1))}-\left(\frac{\rho\sigma_B^2}{2}+\frac{1}{2v}+\frac{\rho}{8}Var(1,1)\right) \leq 0. \tag{35}$$

Summing (35) and (34) yields

$$Q(\sigma_A^2,\sigma_B^2)\equiv\frac{\rho}{2}\left(\sigma_A^2-\sigma_B^2\right)-\frac{1}{v}\left(\frac{1}{\beta^m+\breve{\beta}^A(\widehat{\phi}^A)}-\frac{1}{\breve{\beta}^S(\widehat{\phi}^A)+\beta_B^o(1,1)}\right)\leq 0. \tag{36}$$

A sufficient condition for (29) to hold is that (36) holds. Using (31) and the fact that $\lim_{\sigma_A^2\to\sigma_B^2}\beta_A^o(1,1)=\beta_B^o(1,1)$, we have

$$\begin{aligned}
\lim_{\sigma_A^2\to\sigma_B^2} Q(\sigma_A^2,\sigma_B^2) &\propto -\left(\breve{\beta}^S(\widehat{\phi}^A)+\beta_B^o(1,1)\right)+\left(\beta^m+\breve{\beta}^A(\widehat{\phi}^A)\right)\\
&\leq 2\left(\beta^m-\beta_B^o(1,1)\right)\\
&= 2\left(\beta^m-\beta_A^o(1,1)\right)\\
&\leq 0.
\end{aligned}$$

By continuity, it follows that (29) holds if $\sigma_A^2\in(\sigma_B^2,\sigma_B^2+\delta_B)$, $\delta_B>0$.

For any $\phi\in[\widehat{\phi}^A,\phi^S]$ it follows by revealed preference that the principal's payoff under the symmetric regime is higher than her respective payoff under bundling. For any $\phi>\phi^S$, under both regimes, the principal finds it too costly to induce $(1,1)$ and instead acquiesces to $(0,0)$ by contracting on the benchmark PPS, $\boldsymbol{\beta}^o(0,0)$. Hence, for $\phi>\phi^S$ the principal's payoff is equal under both regimes. We cannot rank unambiguously ϕ^S and $\widehat{\phi}^A$. If $\widehat{\phi}^A\leq\phi^S$, no further proof is required. However, if $\widehat{\phi}^A>\phi^S$, we need to show that $W((1,1),\widehat{\boldsymbol{\beta}}|\phi)\geq W((1,1),\widehat{\boldsymbol{\beta}}^A|\phi)$ for any $\phi\in[\phi^S,\widehat{\phi}^A]$. The proof of this inequality follows similar steps as the derivation of (29) and is hence omitted. Lastly, let $\delta\equiv\min\{\delta_A,\delta_B\}$.

Proof of Lemma 8:

Part (a): Given Assumption 1′, it is straightforward to show that $\frac{\partial^2 W(\mathbf{e},\boldsymbol{\beta})}{\partial e_A\partial e_B}=1-\frac{\rho\eta}{4}(\beta_A^2+\beta_B^2)\geq 1-\frac{\rho\eta}{2}>0$ and $\frac{\partial^2\Gamma(\mathbf{e}|\beta_i)}{\partial e_A e_B}=\beta_i\left(\frac{1}{2}-\frac{\rho\eta}{4}\beta_i\right)\propto 1-\frac{\rho\eta}{2}\beta_i\geq 1-\frac{\rho\eta}{2}>0,\quad i=A,B.$

Part (b): Using $\eta \leq \frac{1}{\rho}$, by Assumption 1′, and taking cross-partial derivatives:

$$
\begin{aligned}
\frac{\partial^2 \widetilde{\Lambda}_i(\mathbf{e} \mid \beta_i)}{\partial e_A \partial e_B} &= \beta_i \left(\frac{1}{2} - \frac{\rho\eta}{4} \beta_i \right) \propto 1 - \frac{\rho\eta}{2} \beta_i \geq 0, \\
\frac{\partial^2 \widetilde{\Lambda}_i(\mathbf{e} \mid \beta_i)}{\partial e_m \partial \beta_i} &= \frac{q^*(\mu, \mathbf{e})}{2} - \frac{\rho\eta}{2} \beta_i q^*(\mu, \mathbf{e}) \propto 1 - \rho\eta\beta_i \geq 0, \\
\frac{\partial^2 \widetilde{\Lambda}_i^\ell(\mathbf{e} \mid \beta_i)}{\partial e_A \partial e_B} &= \beta_i \left(\frac{1}{2} - \frac{\rho\eta}{4} \beta_i \right) \propto 1 - \frac{\rho\eta}{2} \beta_i \geq 0, \\
\frac{\partial^2 \widetilde{\Lambda}_i^\ell(\mathbf{e} \mid \beta_i)}{\partial e_m \partial \beta_i} &= \frac{q^*(\mu, \mathbf{e})}{2} - \frac{\rho\eta}{2} \beta_i q^*(\mu, \mathbf{e}) \propto 1 - \rho\eta\beta_i \geq 0, \quad m = A, B, i = A, B.
\end{aligned}
$$

Part (c): Using $\eta \leq \frac{1}{\rho}$, by Assumption 1′:

$$
\begin{aligned}
\frac{\partial W(\mathbf{e}, \boldsymbol{\beta})}{\partial e_i} - \frac{\partial \widetilde{\Lambda}_i(\mathbf{e} \mid \beta_i)}{\partial e_i} &= q^*(\mu, \mathbf{e}) \left(1 - \frac{\beta_i}{2} - \frac{\rho\eta}{4} (\beta_i^2 + \beta_{-i}^2 - \beta_i) \right) \\
&\propto 1 - \frac{\beta_i}{2} - \frac{\rho\eta}{4} (\beta_i^2 + \beta_{-i}^2 - \beta_i) \geq \frac{1}{2} - \frac{\rho\eta}{2} \geq 0, \\
\frac{\partial W(\mathbf{e}, \boldsymbol{\beta})}{\partial e_i} - \frac{\partial \widetilde{\Lambda}_i^\ell(\mathbf{e} \mid \beta_\ell)}{\partial e_i} &= q^*(\mu, \mathbf{e}) \left(1 - \frac{\beta_\ell}{2} - \frac{\rho\eta}{4} (\beta_\ell^2 + \beta_{-\ell}^2 - \beta_\ell) \right) \geq 0, \quad i = A, B.
\end{aligned}
$$

Part (d): For given PPS, the best response for Manager i to an anticipated effort e_j by his counterpart, $e_i(e_j \mid \beta_i)$, is found by setting the first-order conditions equal to zero:

$$
\frac{\partial \Gamma(\mathbf{e} \mid \beta_i)}{\partial e_i} - g e_i = \beta_i \left(\frac{q^*(\mu, \mathbf{e})}{2} - \frac{\rho\eta}{4} \beta_i q^*(\mu, \mathbf{e}) \right) - g e_i = 0, \quad i = A, B.
$$

Upon rearranging:

$$
e_i(e_j \mid \beta_i) = \frac{(e_j + \mu) \left(\frac{1}{2} - \frac{\rho\eta}{4} \beta_i \right)}{\frac{g}{\beta_i} + \frac{\rho\eta}{4} \beta_i - \frac{1}{2}}, \quad i = A, B, \quad j \neq i.
$$

Because $e_i(e_j = 0 \mid \beta_i) > 0$, and $\frac{d}{de_j} e_i(e_j \mid \beta_i) < 1$ for any $e_j, \boldsymbol{\beta}, i$ and $j \neq i$, we have a unique Nash equilibrium in investments for any PPS. (To bound the slope of the reaction curves by one, we use the fact that $g > 6$.) Solving for the equilibrium in closed form:

$$
e_i^S(\boldsymbol{\beta}) = \frac{\beta_i \mu \left(\frac{1}{2} - \frac{\eta\rho}{4} \beta_i \right)}{g + \frac{\eta\rho}{4} \left(\beta_i^2 + \beta_j^2 \right) - \frac{1}{2} (\beta_i + \beta_j)}, \quad i = A, B, \ j \neq i.
$$

The equilibrium efforts are increasing in either manager's PPS:

$$\begin{aligned}\frac{\partial e_i^S(\boldsymbol{\beta})}{\partial \beta_i} &= \frac{2\mu(1-\eta\rho\beta_i)(4g+\eta\rho\beta_j^2-2\beta_j)}{\left(\eta\rho\left(\beta_i^2+\beta_j^2\right)-2(\beta_i+\beta_j-2g)\right)^2} \propto g+\frac{\eta\rho}{4}\beta_j^2-\frac{\beta_j}{2}\geq g-\frac{1}{2}>0.\\ \frac{\partial e_i^S(\boldsymbol{\beta})}{\partial \beta_j} &= \frac{2\beta_i\mu(2-\beta_i\eta\rho)(1-\beta_j\eta\rho)}{\left(\eta\rho\left(\beta_i^2+\beta_j^2\right)-2(\beta_i+\beta_j 2g)\right)^2} \propto (2-\beta_i\eta\rho)(1-\beta_j\eta\rho)\geq(2-1)(1-1)=0.\end{aligned}$$

Under bundling, for given PPS, the first-order conditions yield:

$$\frac{\partial\Gamma(\mathbf{e}\mid\beta_\ell)}{\partial e_i}-ge_i=\beta_\ell\left(\frac{q^*(\mu,\mathbf{e})}{2}-\frac{\rho\eta}{4}\beta_\ell q^*(\mu,\mathbf{e})\right)-ge_i=0,\quad i=A,B.$$

Solving and rearranging:

$$\widehat{e}_i^\ell(\beta_\ell) = \frac{\mu\left(\frac{1}{2}-\frac{\eta\rho}{4}\beta_\ell\right)}{\frac{g}{\beta_\ell}+\frac{\eta\rho}{2}\beta_\ell-1},\quad i=A,B,\ j\neq i.$$

It is immediate that $\frac{d\widehat{e}_i^\ell(\beta_\ell)}{d\beta_{j\neq\ell}}\equiv 0$ and, using $\eta\leq\frac{1}{\rho}$, by Assumption $1'$,

$$\frac{d\widehat{e}_i^\ell(\beta_\ell)}{d\beta_\ell} = \frac{2g\mu(1-\beta_\ell\eta\rho)}{(\beta_\ell(\beta_\ell\eta\rho-2)+2g)^2}\propto 1-\beta_\ell\eta\rho\geq 0.$$

Proof of Proposition 5: The proof follows similar steps as that of Proposition 1 and is available upon request.

Proof of Proposition 6: We begin by deriving a sufficient condition to rule out asymmetric project effort equilibria under the symmetric regime. Recall that $\mathbf{e}^S(\boldsymbol{\beta})=(1,1)$ is an equilibrium for $\gamma\leq\gamma_{(1,1)}(\boldsymbol{\beta})$, whereas $\mathbf{e}^S(\boldsymbol{\beta})=(0,0)$ is an equilibrium for $\gamma>\gamma_{(0,0)}(\boldsymbol{\beta})$. If $\gamma_{(1,1)}(\boldsymbol{\beta})-\gamma_{(0,0)}(\boldsymbol{\beta})<0$, then $\mathbf{e}^S(\boldsymbol{\beta})=(1,0)$ can be a Nash equilibrium for any $\gamma\in(\gamma_{(1,1)}(\boldsymbol{\beta}),\gamma_{(0,0)}(\boldsymbol{\beta}))$. On the other hand, if $\gamma_{(1,1)}(\boldsymbol{\beta})-\gamma_{(0,0)}(\boldsymbol{\beta})\geq 0$, then for any $\gamma\in(\gamma_{(0,0)}(\boldsymbol{\beta}),\gamma_{(1,1)}(\boldsymbol{\beta}))$, both $(1,1)$ and

$(0,0)$ can be equilibria simultaneously.

$$
\begin{aligned}
\gamma_{(1,1)}(\boldsymbol{\beta}) - \gamma_{(0,0)}(\boldsymbol{\beta}) &= \Gamma(1,1 \mid \underline{\beta}) - \Gamma(1,0 \mid \underline{\beta}) - \big(\Gamma(1,0 \mid \overline{\beta}) - \Gamma(0,0 \mid \overline{\beta})\big) \\
&= \underline{\beta}\left(\frac{E[(q^*(\theta,1,1))^2 - (q^*(\theta,1,0))^2]}{4} - \frac{\rho\eta}{2}\underline{\beta}\frac{(q^*(\mu,1,1))^2 - (q^*(\mu,1,0))^2}{4}\right) \\
&\quad -\overline{\beta}\left(\frac{E[(q^*(\theta,1,0))^2 - (q^*(\theta,0,0))^2]}{4} - \frac{\rho\eta}{2}\overline{\beta}\frac{(q^*(\mu,1,0))^2 - (q^*(\mu,0,0))^2}{4}\right) \\
&= \frac{\underline{\beta}}{4}\left(E[(\theta+2)^2 - (\theta+1)^2] - \frac{\rho\eta}{2}\underline{\beta}((\mu+2)^2 - (\mu+1)^2)\right) \\
&\quad -\frac{\overline{\beta}}{4}\left(E[(\theta+1)^2 - \theta^2] + \frac{\rho\eta}{2}\overline{\beta}((\mu+1)^2 - \mu^2)\right) \\
&= \frac{1}{4}\left[\underline{\beta}(2\mu+3)\left(1 - \frac{\underline{\beta}\eta\rho}{2}\right) - \overline{\beta}(2\mu+1)\left(1 - \frac{\overline{\beta}\eta\rho}{2}\right)\right] \\
&\propto \underline{\beta}(2\mu+3)\left(1 - \frac{\underline{\beta}\eta\rho}{2}\right) - \overline{\beta}(2\mu+1)\left(1 - \frac{\overline{\beta}\eta\rho}{2}\right) \\
&> \underline{\beta}(2\mu+3) - \overline{\beta}(2\mu+3) + 2\overline{\beta}\left(1 - \frac{\eta\rho}{2}\right) \\
&> 0,
\end{aligned}
$$

if

$$
\frac{\overline{\beta} - \underline{\beta}}{\overline{\beta}} \leq \frac{2 - \eta\rho}{2\mu + 3}. \tag{37}
$$

Suppose (37) holds. For any $\gamma \in (\gamma_{(0,0)}(\boldsymbol{\beta}), \gamma_{(1,1)}(\boldsymbol{\beta}))$, to predict which of the symmetric equilibria, $(1,1)$ or $(0,0)$, the managers will play, we invoke Theorem 7 in Milgrom and Roberts (1990): if multiple equilibria exist in a supermodular game (such as the managers' Date-2 effort game) with positive spillovers (i.e., player i's payoff is increasing in player j's action), then the highest equilibrium—here, $(1,1)$—Pareto-dominates the other equilibria. That is, for $\gamma \in (\gamma_{(0,0)}(\boldsymbol{\beta}), \gamma_{(1,1)}(\boldsymbol{\beta}))$ we can ignore the shirking equilibrium. Now note that the left-hand side of (37) in the optimal solution under the symmetric regime is bounded from above by $(\beta_B^{MH} - \beta_A^{MH})/\beta_B^{MH}$; therefore a sufficient condition for asymmetric equilibria not to arise in equilibrium is Condition (25), which will hold for sufficiently small volatility differential, $\sigma_A^2 - \sigma_B^2$, and small η.

Assuming (25) holds, under the symmetric regime the principal will choose the optimization program with the greater value from among the following:

$\mathcal{P}^S_{(1,1)}$ *(Induce effort under symmetric regime):* For any $\gamma \in (\overline{\gamma}^S, \gamma^*]$,

$$\max_{\boldsymbol{\beta}} \; W(\boldsymbol{\beta} \mid \mathbf{e} = (1,1)),$$
$$\text{subject to (5) and } \beta_i \geq \breve{\beta}^S(\gamma), \text{ for any } i.$$

$\mathcal{P}^S_{(0,0)}$ *(Forestall effort under symmetric regime):* For any $\gamma \in (\overline{\gamma}^S, \gamma^*]$,

$$\max_{\boldsymbol{\beta}} \; W(\boldsymbol{\beta} \mid \mathbf{e} = (0,0)),$$
$$\text{subject to (5) and } \beta_i < \breve{\beta}^S(\gamma), \text{ for any } i.$$

The remainder of the proof follows similar steps as the proof of Proposition 2 and is available upon request.

Proof of Proposition 7: The proof follows similar steps to those in the proof of Proposition 2 and is available upon request.

Proof of Proposition 8: The proof follows similar steps as that of Proposition 4 and is available upon request.

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