Platform Competition under Network Effects: Piggybacking and Optimal Subsidization

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A repeated challenge in launching a two-sided market platform is how to solve the "chicken-and-egg" problem. The solution often suggested in the literature is subsidizing one side of the market to jumpstart adoption of the platform. In this paper, using a game-theoretic framework, we study piggybacking – importing users from external networks – as a new approach to launching platforms. First, in the presence of piggybacking, we solve for the platforms' optimal pricing/subsidization strategies. Benchmarked with the case of no piggybacking, we find that, although piggybacking changes the degree of platform subsidization, it does not change the conditions for doing so. Second, we show that piggybacking can either intensify or mitigate price competition among platforms and we identify under which conditions each scenario happens. We also show when subsidization can complement piggybacking. Third, we show that these findings are robust to an extension when piggybacking is endogenized (i.e., external users need to be purchased). Finally, we depart from authentic piggybacking by examining fabricated piggybacking, that is, when imported external users (e.g., zombies or fake users) generate network effects but no revenue. We show that fabricated piggybacking, in contrast to authentic piggybacking, affects the platform's subsidization conditions and undermines profits for the competing platform. Managerial implications for platform practitioners are also discussed.

Key words: Analytical Modeling, Economics of IS, Network Effects, Piggybacking, Platform Competition, Pricing, Subsidization

1. Introduction

As more and more businesses (both physical and digital) search for their multi-sided platform business models (i.e., intermediaries that connect two or more distinct groups of users and enable their direct interactions), a primary challenge is how to expand the user bases in view of the interdependence issue among different user groups – known as the "chicken-and-egg" problem (Caillaud and Jullien 2003). The key solution proposed in the extant literature is subsidizing one or some user groups to jumpstart adoption of the platform (e.g., Rochet and Tirole 2003, Parker and Van Alstyne 2005, Eisenmann et al. 2006, Bolt and Tieman 2008). These strategies are widely used by practical platforms: for example, Microsoft took a total loss of over US $4 billion in the first four years after launching its Xbox gaming platform, primarily by allowing consumers to pay a market price below the cost of manufacturing[1].

To incentivize adoption by users, in addition to price controls such as subsidies, platforms are increasingly embracing non-pricing controls such as optimizing platform features. For example, Hagiu and Spulber (2013) suggest that, to attract early consumers, platforms themselves could offer contents. Anderson Jr. et al. (2013) investigate whether video-game console platforms should maintain low development costs for game developers. We contribute to this literature by examining a new non-pricing control – user traffic management in general, and piggybacking in particular: importing external user traffic to the platform. This enables us to study a platform's decision on optimal subsidization when piggybacking is viable. By comparing our insights to Anderson Jr. et al. (2013) and Hagiu and Spulber (2013), we show that the conventional wisdom of a substitution relationship between non-pricing features and subsidies may not always hold. We reveal a host of new insights into platforms' competing strategies when they are able to engage in piggybacking, with more complex characterizations on the interplay among traffic volume, cross-side network effects, and pricing/subsidization.

Many examples exist of platforms tapping into external networks to attract early traffic, rather than using subsidies alone to build an installed base from scratch. Google, for example, has had

[1] https://venturebeat.com/2011/11/15/the-making-of-the-xbox-part-2/

great success with this strategy. In its early days, Google was the third-party search engine service for Yahoo!'s portal website, which helped Google gain popularity and exposure among Yahoo! users. In 2007, to counter the success of Apple's mobile operating system iOS, Google partnered with handset manufacturers and carriers to create the Open Handset Alliance (OHA) in order to promote Google's Android operating system among all its partners' customers. In this way, Google was able to achieve a market share of more than 80% of the worldwide mobile market. Other examples abound: before going independent, Zynga was a gaming subsidiary of Facebook; YouTube provided video tools for MySpace; PayPal started as a payment solution affiliated with eBay. In addition, firms undergoing a business model transformation often migrate consumer traffic from old products to new platforms (Zhu and Furr 2016). For example, OpenTable, as it transformed from a restaurant customer relationship management (CRM) software vendor to an online reservation platform, leveraged its existing restaurant clients to attract booking consumers. Similar strategies were employed when Valve expanded from a game developer to the leading gaming platform (i.e., Steam) as well as when Qihoo 360 launched the largest software marketplace in China (i.e., 360 Software Manager) based on its success with anti-virus software.

More interestingly, firms take advantage of opportunities to acquire consumer traffic without corporate collaborations or alliances, which is often known as "growth hacking". Airbnb, for example, used this tactic to boost its growth early on with the button "publish on Craigslist." By clicking on this button, Airbnb hosts could immediately publish their Airbnb listings on Craigslist, and anyone responding to the listing could still reach the host through Airbnb.[2] Similarly, a growing number of firms promote their platforms by posting or answering on social media websites, such as LinkedIn, Reddit, and Quora, to generate consumer traffic. In fact, many social media websites open their traffic to startups through social logins (e.g., Facebook Connect[3] and Google Sign-In[4]), which calls for research on traffic-based strategies for launching a platform.

[2] https://hbswk.hbs.edu/item/how-uber-airbnb-and-etsy-attracted-their-first-1-000-customers

[3] https://www.similartech.com/technologies/facebook-connect

[4] https://developers.google.com/identity/

Following the literature, we define piggybacking as the ability of a platform to "connect with an existing user base from a different platform and stage the creation of value unit in order to recruit those users to participate" (Parker et al. 2016, page 91). We summarize examples of piggybacking in Table 1. In addition, consumer traffic can be purchased through professional digital marketing services (e.g., fiverr.com, fuwu.taobao.com). Other variations of piggybacking strategies also exist. For example, unlike the examples in Table 1 in which platforms are adopted by real human beings, the user traffic on some digital platforms is reportedly mixed with fake accounts and bots. These fabricated users stimulate interactions on the digital platform, but contribute no monetary value to the platform directly. Parker et al. (2016) express this as "fake it until you make it," noting the following examples: Paypal created bots that made purchases on eBay, thereby attracting sellers to the PayPal platform; dating services often simulate initial attraction by creating fake profiles and conversations; Reddit fakes profiles by posting links to the kind of content the founders wanted to see on the site over time; and editors on Quora ask questions and then answer the questions themselves, to simulate activity on the platform.

Although piggybacking is popular in practice, particularly in solving the "chicken-and-egg" problem, it has received little, if any, formal analysis in the academic literature. This paper aims to take the first step toward filling this gap by developing an analytical modeling framework for studying platform competition in the presence of piggybacking. To the best of our knowledge, this paper is among the first to formally study optimal piggybacking strategies in the context of platform competition under network effects.

Our key research question is: *Under platform competition and network effects, how does piggybacking affect the platforms' optimal pricing/subsidization strategy and profits?* Further, in our extensions, we address this research question in alternative forms of piggybacking, including single-sided market piggybacking, endogenous piggybacking when piggybacking traffic can be purchased (e.g., fivrr.com, fuwu.taobao.com), and fabricated piggybacking, in which external fabricated users (e.g., bots or fake users) generate network effects but no revenue.

Approach	Definition	Examples
Direct importing	Offer direct gateways to users from external networks	Social log-in with Facebook or Google
Business affiliation	Acquire new users by serving as alliances or subsidiaries	Embedded Google search on Yahoo! YouTube video tools on MySpace Zygna games on Facebook PayPal payment services on eBay
Growth hacking	Convert users from other networks without upfront agreements	Airbnb home-rental listings on Craigslist
Business model transformation	Extend or switch to new business model with existing users	OpenTable's switch from CRM to a resturant reservation platform

Table 1 Summary of Different Approaches of Piggybacking in Platform Practice

In the presence of piggybacking, we solve for the platforms' optimal pricing/subsidization strategies. Our study yields many interesting findings. First, benchmarked with the case of no-piggybacking, we find that piggybacking, while changing the degree of platform subsidization, does not change the conditions of platform subsidization. Second, we show that piggybacking can either intensify or mitigate platforms' pricing competition, and we identify the conditions under which each scenario happens. We also find when subsidization can complement piggybacking. These results demonstrate the strategic importance of piggybacking as a non-pricing platform control, which appears new in the literature. Third, we show that these findings are robust to an extension when piggybacking is endogenized (i.e., external users need to be purchased). Finally, we depart from the setting of authentic piggybacking by examining fabricated piggybacking, that is, when imported external users (e.g., zombies or fake users) generate network effects but no revenue. Unlike authentic piggybacking, fabricated piggybacking affects the platform's subsidization conditions and undermines the competing platform's profit.

The rest of this paper is organized as follows: Section 2 reviews related literature, and Section 3 introduces our model. In Section 4, we show the results of our baseline model and extend it in several ways in Section 5. Section 6 outlines the managerial implications and concludes.

2. Related Literature

Our research is related to three research streams that we briefly review below. The first stream is the literature on launching a two-sided platform (e.g., Rochet and Tirole 2003, Parker and Van Alstyne 2005, Parker et al. 2016). A repeated challenge in launching a platform is the "chicken-and-egg" problem (Caillaud and Jullien 2003), discussed in Section 1. Subsidizing one side of the market, or the "seesaw principle," has been suggested as the solution (e.g., Wright 2004, Bhargava and Choudhary, 2004, Parker and Van Alstyne 2005, Rochet and Tirole 2006, Hagiu 2007, Bolt and Tieman 2008, Hagiu 2009). For example, Parker and Van Alstyne (2005) recommend giving away free access/products to either providers or consumers, depending on the cross-sided elasticities. Rochet and Tirole (2006) introduce the seesaw principle, in which a profit-maximizing platform charges a high price on one side and a low price on the other side. Although both papers (Parker and Van Alstyne 2005, Rochet and Tirole 2006) assumed that users single-home, the seesaw principle is also shown to be optimal when users multi-home (Armstrong 2006). Armstrong (2006) shows that in the "competitive bottleneck" setting, when only one side multi-homes and the other side single-homes, the platform can charge a higher price on the multi-homing side because of its monopoly power in providing access to the single-homing side. The "competitive bottleneck" has become the standard setting in this literature (e.g., Rochet and Tirole 2003, Armstrong 2006, Economides and Tåg 2012, Hagiu and Hałaburda 2014). For example, Hagiu and Hałaburda (2014) consider different types of user expectations when they join the platform. They find that platforms might be better off when users are less informed and form their expectations passively. Using the "competitive bottleneck" setting, we formally study the strategic implications of piggybacking and optimal subsidization in launching a platform.

Prior studies related to piggybacking are mostly empirical observations (e.g., Boudreau 2010). This second line of research has provided rich evidence that motivates formal modeling, as we attempt to do. Scenarios related to piggybacking include building market momentum (e.g., Gawer and Cusumano 2008), adding initial developers to the software platform (e.g., Boudreau 2012), attracting early users via single-sided features (e.g., Hagiu and Eisenmann 2007, Hagiu and Spulber 2013, Anderson Jr. et al. 2013) or advertising (e.g., Tucker and Zhang 2010), and integrating the user base with a complementary platform (e.g., Li and Agarwal 2016). In these studies, non-price controls other than piggybacking are often the driving force for additional user participation (e.g., more functionality or platform contents from complementary platforms or a new group of developers). However, traditional platform controls often incur costs for the platform (e.g., advertising, content creation), in exchange for additional user participation on one side of the market. In contrast, the role of piggybacking is much richer. Piggybacking might be either costly or beneficial for the platform, depending on whether platforms charge or subsidize piggybacking consumers. In addition, piggybacking strategies involve managerial challenges for user participation not only on both sides of the market but also for the external networks. As we show later, it is non-trivial to address such challenges in the presence of cross-side network effects under platform competition.

We also extend our model to consider fabricated piggybacking. Although the computer science literature – the third related research stream – focuses on the technical side of identifying fabricated piggybacking (Chu et al. 2010, Hu et al. 2011, Hu et al. 2012, and Haustein et al. 2016) such as zombie users, fake profiles, and review frauds, economic analysis of fabricated piggybacking is rare other than documenting some suggestive empirical evidence. For example, Aral (2014) suggests that online review fraud is one of the prominent reasons for the J-shape distribution of online consumer ratings discovered by Hu et al. (2009). Luca and Zervas (2016) provide empirical evidence from Yelp that a restaurant is more likely to commit review fraud when its reputation is weak and is more likely to receive unfavorable fake reviews under strong competition. This line of research has motivated us to investigate formally the impact of fabricated piggybacking, such as bots, compared with non-fabricated piggybacking, in which case users are real. To the best of our knowledge, this is among the first to examine the issue, and we provide new insights on platforms to understand the economics of fabricated piggybacking, such as different strategic implications of fabricated versus authentic piggybacking for competing platforms.

3. Model

Consider a duopoly between Platforms A and B in a two-sided market consisting of consumers (denoted by superscript c) and providers (denoted by superscript d). Consumers consist of those who are from the focal market (called "focal consumers") and those who are redirected from external networks as a result of piggybacking (called "piggybacking consumers"). We normalize the population of focal consumers to be 1. We assume the population of piggybacking consumers to be $N_0 \in [0,1)$. Furthermore, focal consumers are uniformly distributed in a linear city between 0 and 1. Focal consumers located at $x \in [0,1]$ receive the following utilities from adopting Platforms A and B, respectively:

$$\begin{aligned} U_A^c(x) &= V - p_A^c - tx + \beta N_A^d; \\ U_B^c(x) &= V - p_B^c - t(1-x) + \beta N_B^d, \end{aligned} \tag{1}$$

where V represents the standalone value of adopting either Platform A or Platform B. p_k^c is the consumer-side fee to access Platform $k \in \{A,B\}$. tx ($t(1-x)$) is the transportation cost for consumers at x to adopt Platform A (Platform B). Following the classic Hotelling setup (Hotelling 1929), coefficient t measures the degree of horizontal differentiation between the two platforms: A higher t indicates that it is more difficult for platforms to attract focal consumers via low prices or subsidies. In other words, t can be viewed as a proxy for consumer-side price elasticity. $\beta \geq 0$ represents the degree of consumer-side network effects. N_k^d is the number of providers in equilibrium who participate in Platform $k \in \{A,B\}$.

Given the utility functions above, the number of focal consumers adopting Platform k, N_k^c, can be obtained by solving $\hat{x}$ from $U_A^c(\hat{x}) = U_B^c(\hat{x})$.

$$\begin{aligned} N_A^c &= \hat{x} = \frac{1}{2} - \frac{p_A^c - p_B^c}{2t} + \frac{\beta(N_A^d - N_B^d)}{2t}; \\ N_B^c &= 1 - \hat{x} = \frac{1}{2} - \frac{p_B^c - p_A^c}{2t} + \frac{\beta(N_B^d - N_A^d)}{2t}. \end{aligned} \tag{2}$$

Meanwhile, platforms are able to acquire consumers from external networks through piggybacking. Compared with focal consumers, piggybacking consumers are often more closely connected to the platform, possibly because of significant switching costs (e.g., shared user accounts or credits with the external network), such that piggybacking consumer adoptions are not deterred by price competition. We assume that among N_0 ($N_0 \geq 0$) piggybacking consumers, rN_0 (respectively, $(1-r)N_0$) consumers adopt Platform A (Platform B). Both r and N_0 are exogenous and reflect the platforms' ability to generate additional consumer traffic from external networks. Without loss of generality, we assume $r \in [\frac{1}{2}, 1]$, meaning Platform A has an advantage over Platform B in piggybacking.

On the provider side, following the literature, we assume the standard "competitive bottleneck" setting (e.g., Rochet and Tirole 2003, Armstrong 2006, Economides and Tåg, 2012, Hagiu and Hałaburda 2014) in which providers multi-home (i.e., they can join a platform as long as they pay for platform access and there is no price competition between platforms for providers). Following Hagiu and Hałaburda (2014), in equilibrium, we have the following demand functions on the provider side,

$$N_k^d = \alpha N_k^c - p_k^d, \tag{3}$$

where $\alpha \geq 0$ represents the degree of provider-side network effects. Equation (3) suggests that adoptions on the provider-side are driven purely by consumer adoptions because a platform has no standalone value for providers if it has no consumers.

Following the literature (e.g., Hagiu and Hałaburda 2014), we assume $\frac{(\alpha+\beta)^2}{4} \leq t$ to maintain a reasonable degree of network effects for the profit function to be well-behaved.

The timeline of events has three stages: in stage 0, each platform obtains a group of piggybacking consumers from external networks; in stage 1, two platforms simultaneously announce their prices on both sides; and in stage 2, focal consumers choose to join either Platform A or Platform B, while each provider decides whether to participate in Platform A, Platform B, or both. We are particularly interested in how the presence of piggybacking consumers in stage 0 affects optimal platform pricing strategies in equilibrium in stage 2. The timeline of events is illustrated in Figure 1.

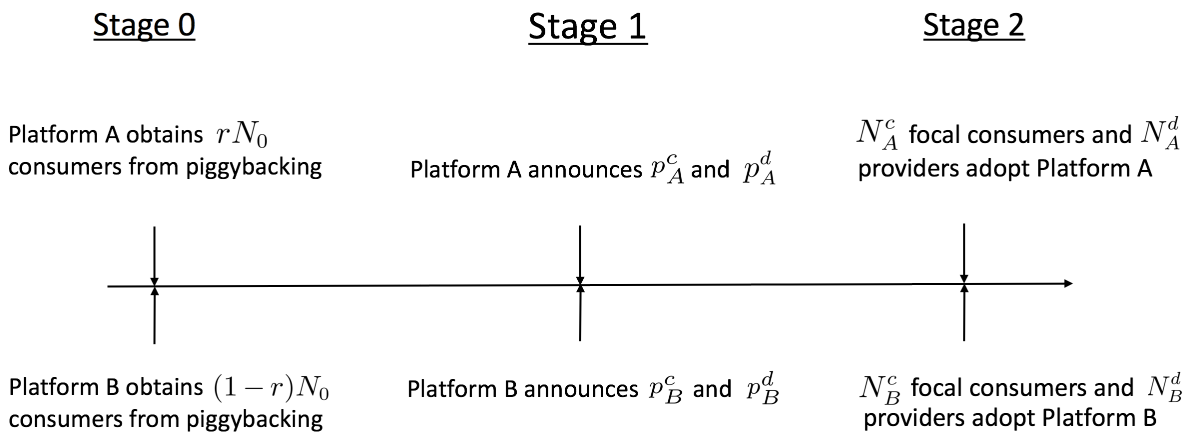

Figure 1 The timeline of events

Platform k determines optimal prices on both sides simultaneously to maximize its overall profit Π_k from both sides. A platform will stay in the market as long as a non-negative number of focal consumers or providers adopts it in equilibrium, that is, equilibrium exists when $N_k^c \geq 0$ and $N_k^d \geq 0$ for $k \in \{A, B\}$. Specifically, each platform's profit maximization problem is Equation (4) and (5).

$$\max_{p_A^c, p_A^d} \quad \Pi_A = p_A^c\,(rN_0 + N_A^c) + p_A^d N_A^d, \tag{4}$$

$$s.t. \quad N_A^c = \frac{1}{2} - \frac{p_A^c - p_B^c}{2t} + \frac{\beta(N_A^d - N_B^d)}{2t};$$

$$N_B^c = \frac{1}{2} - \frac{p_B^c - p_A^c}{2t} + \frac{\beta(N_B^d - N_A^d)}{2t};$$

$$N_A^d = \alpha(N_A^c + rN_0) - p_A^d, \;\; N_B^d = \alpha[N_B^c + (1-r)N_0] - p_B^d;$$

$$N_k^c \geq 0, \;\; N_k^d \geq 0, \;\; \frac{(\alpha+\beta)^2}{4} \leq t, \;\; r \in [\frac{1}{2}, 1].$$

$$\max_{p_B^c, p_B^d} \quad \Pi_B = p_B^c\,[(1-r)N_0 + N_B^c] + p_B^d N_B^d, \tag{5}$$

$$s.t. \quad N_A^c = \frac{1}{2} - \frac{p_A^c - p_B^c}{2t} + \frac{\beta(N_A^d - N_B^d)}{2t};$$

$$N_B^c = \frac{1}{2} - \frac{p_B^c - p_A^c}{2t} + \frac{\beta(N_B^d - N_A^d)}{2t};$$

$$N_A^d = \alpha(N_A^c + rN_0) - p_A^d, \;\; N_B^d = \alpha[N_B^c + (1-r)N_0] - p_B^d;$$

$$N_k^c \geq 0, \;\; N_k^d \geq 0, \;\; \frac{(\alpha+\beta)^2}{4} \leq t, \;\; r \in [\frac{1}{2}, 1].$$

We summarize our key notation in Table 2.

k	Platform index, $k \in \{A, B\}$;
t	Transportation cost, $t > 0$;
$\beta(\alpha)$	Degree of the consumer (provider) side network effects, $\frac{(\alpha+\beta)^2}{4} \leq t$;
p_k^c (p_k^d)	Equilibrium price of Platform k on the consumer (provider) side;
N_k^c (N_k^d)	Number of adopting consumers (providers) of Platform k, N_k^c, $N_k^d \geq 0$;
Π_k	Equilibrium profit of Platform k;
N_0	Overall population of piggybacking consumers, $N_0 \in [0, 1)$;
r	Platform A's share in N_0, $r \in [\frac{1}{2}, 1]$.

Table 2 Summary of Key Notation

4. Analysis and Findings

In this section, we first characterize the duopoly equilibrium in Section 4.1. Then we present our analysis and findings in Section 4.2 and Section 4.3 on the properties of equilibrium pricing and profits.

4.1. Equilibrium

We first note that, because of piggybacking, it is possible for Platform B to exit the focal market completely, such that $N_B^c < 0$. The following proposition identifies conditions that ensure the existence of the duopoly equilibrium.

Proposition 1. *Denote $T_1 = \frac{\alpha^2+4\alpha\beta+\beta^2}{4}$. Platform B stays in the market in equilibrium if and only if (iff):*

(a) $t \geq \frac{4T_1}{5}$ or

(b) $t < \frac{4T_1}{5}$ and $r \leq \bar{r} = \frac{1}{2} + \frac{2T_1-3t}{4(T_1-t)N_0}$.

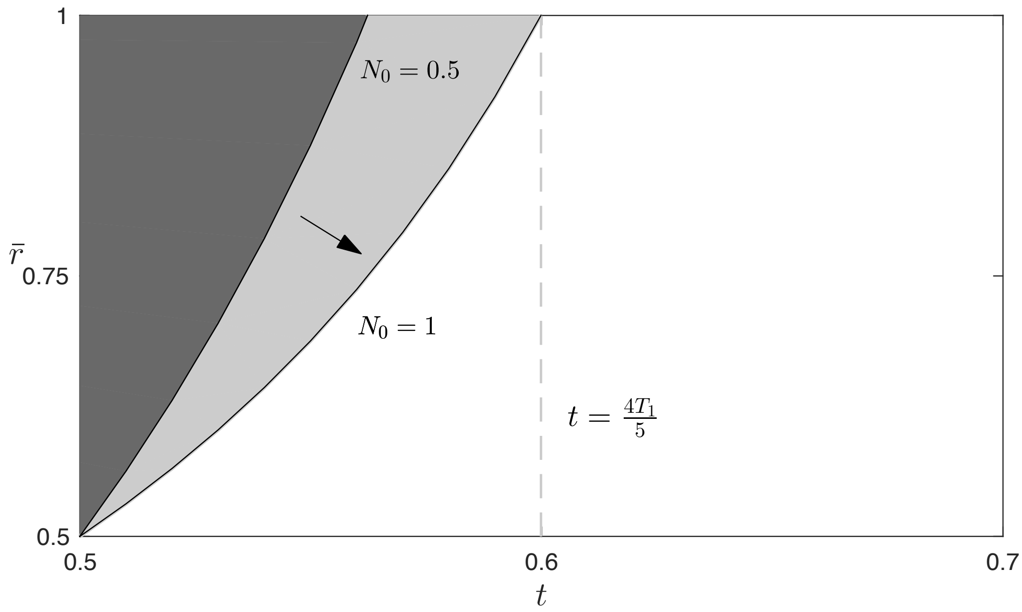

Figure 2 The Existence of Equilibrium (Platform B exits the market in the shaded regions)

Figure 2 illustrates Proposition 1. The white region in the Figure 2 shows the parameter space in which Platform B stays in the focal market. Increases in N_0, indicating growth in the external user pool, pushes the threshold curve of $\bar{r}(N_0)$ to the bottom right, that is, the white region (in which Platform B remains) shrinks. However, when t exceeds a threshold such that $t > \frac{4T_1}{5}$, Platform B will not exit. In this case, Platform B will always stay in the market and compete with Platform A, even Platform A has a piggybacking advantage. Intuitively, this is because Platform B has offered significantly more differentiated services than Platform A. When both platforms stay in the market, Proposition 2 gives the equilibrium results.

Proposition 2. *Denote* $T_2 = \frac{\alpha(\alpha+3\beta)}{4}$, $T_3 = \frac{\alpha^2+6\alpha\beta+\beta^2}{4}$. *Table 3 summarizes the equilibrium results: optimal pricing strategies, market size, and the profit of each platform.*

4.2. Piggybacking and optimal subsidization

We first examine how piggybacking affects optimal pricing/subsidization strategies. Proposition 3 and Table 4 provide comparative statics on r, Platform A's piggybacking advantage over Platform B.

	Market Side	Platform A	Platform B
Pricing	consumer	$(t-T_2)\left(1+\frac{2[(1+r)t-T_1]}{3t-2T_1}\times N_0\right)$	$(t-T_2)\left(1+\frac{2[(2-r)t-T_1]}{3t-2T_1}\times N_0\right)$
	provider	$\frac{\alpha-\beta}{4}\left(1+\frac{2[(1+r)t-T_1]}{3t-2T_1}\times N_0\right)$	$\frac{\alpha-\beta}{4}\left(1+\frac{2[(2-r)t-T_1]}{3t-2T_1}\times N_0\right)$
Market Size	consumer	$\frac{1}{2}-\frac{(2r-1)(t-T_1)}{3t-2T_1}\times N_0$	$\frac{1}{2}+\frac{(2r-1)(t-T_1)}{3t-2T_1}\times N_0$
	provider	$\frac{\alpha+\beta}{4}\left(1+\frac{2[(1+r)t-2T_1]}{3t-2T_1}\times N_0\right)$	$\frac{\alpha+\beta}{4}\left(1+\frac{2[(2-r)t-2T_1]}{3t-2T_1}\times N_0\right)$
Profit		$\frac{(2t-T_3)(3t-2T_1+2[(1+r)t-T_1]\times N_0)^2}{4(3t-T_1)^2}$	$\frac{(2t-T_3)(3t-2T_1+2[(2-r)t-T_1]\times N_0)^2}{4(3t-T_1)^2}$

Table 3 Equilibrium Results of Platform Duopoly

Proposition 3. *Assuming a fixed external piggybacking pool N_0, Table 4 summarizes equilibrium sensitivity analysis on r.*

	Market Side	Platform A ($r\uparrow$)	Platform B ($r\uparrow$)
$\alpha \geq \beta$	consumer	$(p_A^c)^* \downarrow$ iff $t < T_2$	$(p_B^c)^* \uparrow$ iff $t < T_2$
	provider	$(p_A^d)^* \uparrow$	$(p_B^d)^* \downarrow$
$\alpha < \beta$	consumer	$(p_A^c)^* \uparrow$	$(p_B^c)^* \downarrow$
	provider	$(p_A^d)^* \downarrow$	$(p_B^d)^* \uparrow$

Table 4 Comparative Statics on r

Corollary 1 highlights three key insights from Proposition 3. Specifically, we are interested in whether piggybacking affects platforms' optimal subsidization strategies.

Corollary 1. *The following hold true in equilibrium:*

(a) ***(Unaffected Subsidization Conditions)*** *Benchmarked with the case of no-piggybacking, piggybacking does not change the conditions of platform subsidization. Specifically,* $(p_k^c)^* < 0$ *iff* $t < T_2$ *and* $(p_k^d)^* < 0$ *iff* $\alpha < \beta$ *for* $k \in \{A, B\}$*;*

(b) ***(Strategy Complementarity)*** *For Platform A, the consumer-side subsidization complements its piggybacking advantage. Specifically,* $\frac{\partial (p_A^c)^*}{\partial r} < 0$ *always holds if* $(p_A^c)^* < 0$*;*

(c) ***(Impact on Pricing Strategies)*** *On each side, platforms' prices move in the opposite direction as* r *increases.*

First, Corollary 1a suggests that the conditions of subsidization (i.e., $p_k^c < 0$ or $p_k^d < 0$) are not affected by piggybacking. Recall that $T_2 = \frac{\alpha(\alpha+3\beta)}{4}$, thus subsidization conditions are determined only by the strength of cross-side network effects. Figures 3 and 4 illustrate subsidization conditions and how the platforms adjust the magnitude of subsidization when Platform A is able to piggyback more than Platform B (i.e., r increases).

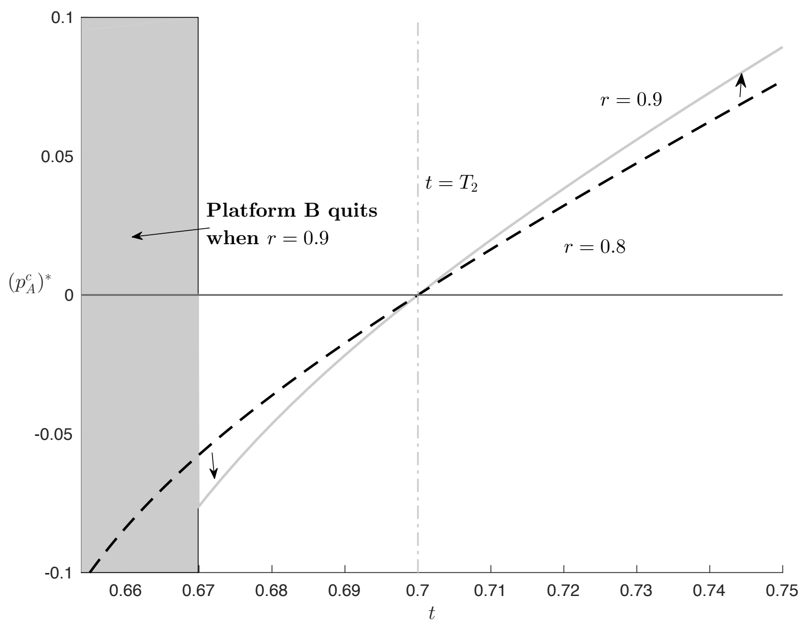

Figure 3 **Platform A's Consumer-Side Equilibrium Price. The shaded area represents regions where Platform B exits the market when r increases from 0.8 to 0.9 ($N_0 = 0.3$, $\alpha = 1$, $\beta = 0.6$)**

As shown in Figure 3, for Platform A, the price curves rotate counter-clockwise when r increases.

However, the rotation does not change the sign of equilibrium prices, illustrating that piggybacking does not change the conditions for Platform A's subsidization strategy. Note that when Platform A is able to piggyback more consumers from the external network, it subsidizes more on the consumer side, indicating complementarity between piggybacking and subsidization. We highlight this strategy complementarity in Corollary 1b.

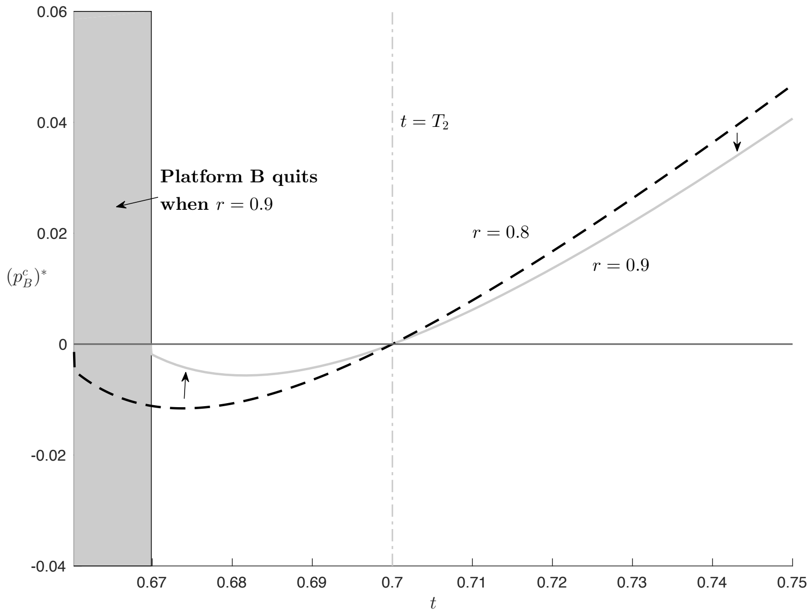

Figure 4 **Platform B's Consumer-Side Equilibrium Price. The shaded area represents regions where Platform B exits the market when r increases from 0.8 to 0.9 ($N_0 = 0.3$, $\alpha = 1$, $\beta = 0.6$)**

In response to Platform A's increasing piggybacking advantage, Platform B reduces the magnitude of its consumer-side subsidy (see Figure 4). When provider-side network effects are stronger than consumer-side network effects (i.e., $\alpha > \beta$), Platform A takes advantage of an increased piggybacking share by raising the consumer-side price p_A^c (see the right-hand side of Figure 3). In this case, Platform B goes in the opposite direction by reducing the consumer-side price p_B^c. Overall, as summarized in Corollary 1c, in response to Platform A's increasing piggybacking advantage, rather than competing head on, the optimal pricing strategy for Platform B is to avoid competition by moving in the opposite price direction on each side.

So far we have assumed a fixed external pool N_0 in which platforms can compete. We have shown the strategic importance of developing superior piggybacking capability (higher r). We now turn to the strategic implications when the piggybacking pool N_0 changes. Given our finding in Corollary 1a, which states that the platform subsidization conditions are affected by network effects (α and β) but not affected by r. In what follows, we assume that Platform A can piggyback (i.e., $r=1$) whereas Platform B cannot.

We depict the comparative statics of Platform A's prices over N_0 in Figure 5 and the comparative statics of Platform B's prices over N_0 in Figure 6. The shaded (white) regions represent the parameter space in which equilibrium prices are decreasing (increasing) in N_0. Regions I through IV are defined in Table 5.

Region	Definition
I	$\alpha \geq \beta$ and $t < T_2$
II	$\alpha \geq \beta$ and $t \in [T_2, T_1)$
III	$\alpha < \beta$ and $t < T_1$
IV	$\alpha \geq \beta$ and $t \geq T_1$
V	$\alpha < \beta$ and $t \geq T_1$

Table 5 **Definition of Parameter Regions of** $\{t, \alpha, \beta\}$

Proposition 4. *Assuming a fixed r, then in equilibrium, when N_0 increases, as summarized in Table 5, we have*

(a) ***(Competition Avoidance)*** *In Regions I, II, and III: on the consumer side, platforms' prices move in the opposite direction;*

(b) ***(Competition Mitigation)*** *In Regions IV and V: on the consumer side, platforms' prices move up.*

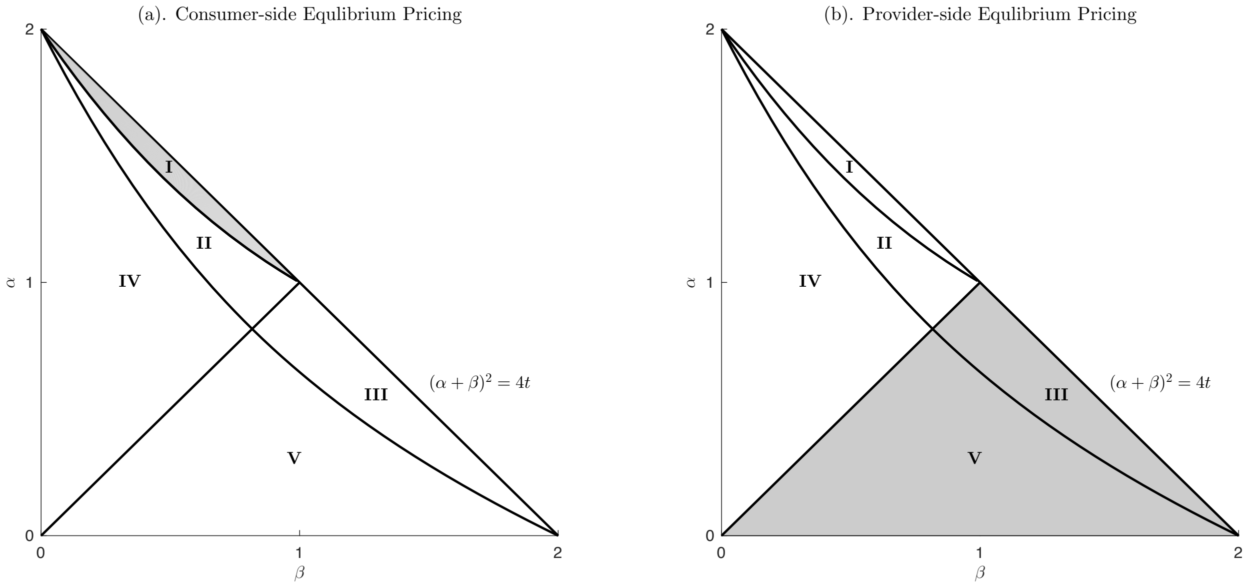

Figure 5 **Comparative Statics for Platform A's Equilibrium Prices. The shaded area represents regions where the price is decreasing in N_0. Regions are defined in Table 5.**

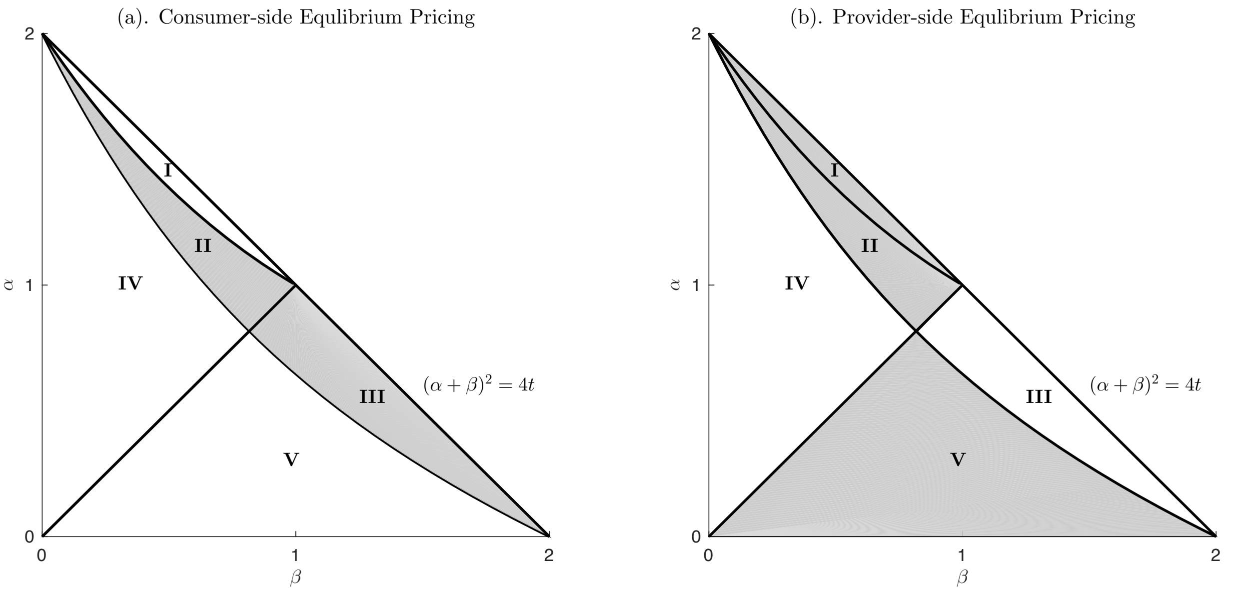

Figure 6 **Comparative Statics for Platform B's Equilibrium Prices. The shaded area represents regions where the price is decreasing in N_0. Regions are defined in Table 5.**

Proposition 4 (and Table 6) summarize our key findings. First, we note that, unlike Corollary 1c, in some regions – namely, Regions IV and V – platform competition is mitigated, that is, on each side, platform prices move in the same direction. So the effect of increasing N_0 (while fixing

Region	Consumer Side ($N_0 \uparrow$)	Provider Side ($N_0 \uparrow$)	
I	$(p_A^c)^* \downarrow$, $(p_B^c)^* \uparrow$	$(p_A^d)^* \uparrow$, $(p_B^d)^* \downarrow$	Competition Avoidance
II	$(p_A^c)^* \uparrow$, $(p_B^c)^* \downarrow$	$(p_A^d)^* \uparrow$, $(p_B^d)^* \downarrow$	
III	$(p_A^c)^* \uparrow$, $(p_B^c)^* \downarrow$	$(p_A^d)^* \downarrow$, $(p_B^d)^* \uparrow$	
IV	$(p_A^c)^* \uparrow$, $(p_B^c)^* \uparrow$	$(p_A^d)^* \uparrow$, $(p_B^d)^* \uparrow$	Competition Mitigation
V	$(p_A^c)^* \uparrow$, $(p_B^c)^* \uparrow$	$(p_A^d)^* \downarrow$, $(p_B^d)^* \downarrow$	

Table 6 **Comparative Statics on N_0**

r) is not the same as the effect of increasing r (while fixing N_0). Second, we note that, although it is optimal for both platforms to use the seesaw principle in Region V, the seesaw principle is no longer optimal in Region IV. In this case, on the consumer side, even the network effects are weaker (indicating smaller profitability for platforms – the "non-money side"), both platforms can raise prices, thanks to the increased number of piggybacking consumers on both platforms. Although these findings are different and new, intuitively, they are driven by an increased external pool. Third, and perhaps more interestingly, we note that Corollary 1c continues to hold true in the remaining regions, Regions I, II, and III. In these regions, platform competition is more intense, even though the external pool has become larger. These regions have interesting dynamics, as we briefly discuss below. Platform A's strategies are intuitive in Regions I and III, that is, the seesaw principle remains optimal. In Region II, however, the seesaw principle is no longer optimal. In this region, while Platform A raises price on the money side (the provider side), which is not surprising, it can take advantage of the increase in piggybacking customers, by raising (rather than decreasing) prices on the consumer side. In all three regions, rather than competing head on with Platform A, Platform B avoids competition by moving prices in the opposite direction on each side. In each case, Platform B reduces subsidies on the non-money side because of its increased disadvantages in harvesting on the money side. In other words, Platform B scales back on both sides to minimize profit loss in response to the rival's piggybacking advantage.

As we show in the extensions via Proposition 5, Corollary 4a is unique in the setting of a two-sided market where platforms have the flexibility to choose which market side to monetize. For example, in the video-streaming market, YouTube subsidizes content creators by sharing advertising revenue. Vimeo charges content creators for uploading content.[5] In this way, Vimeo has remained popular in recent years as measured by its traffic record.[6] Our model provides a theoretical framework for understanding Vimeo's strategic differentiation from YouTube.

4.3. Piggybacking and platform profits

Next we investigate the impact of piggybacking on platform profits. Again we assume that only Platform A can piggyback. It is intuitive that Platform A is always better off. However, Corollary 2 suggests that Platform B's profit is affected by piggybacking in a non-trivial way.

Corollary 2. *The following hold true in equilibrium:*

(a) Platform A's overall profit and number of adopters on both sides are always increasing in N_0;

(b) Platform B's overall profit and number of adopters on both sides are increasing in N_0 iff $t > T_1$;

(c) The profit gap between the two platforms is increasing and convex in N_0.

Interestingly, as illustrated in Figure 7, when network effects are not very strong, such that $T_1 < t$, even if Platform B has a disadvantage in piggybacking such that in the extreme case in which Platform B cannot directly benefit from piggybacking consumers (i.e., $r = 1$), it can still be better off as the external pool increases. Here is the intuition. In this case, Platform A tends to increase the price on the consumer side, leaving a larger consumer base for Platform B to serve. Thus Platform B has more pricing flexibility. But when network effects are strong enough to exceed a threshold (i.e., $T_1 \geq t$), Platform A finds it optimal to lower prices to leverage network effects. In this case, Platform B has little room for price adjustment. This, coupled with limited platform service differentiation (i.e., a small t), leads to a decrease in profit.

[5] https://vimeo.com/upgrade

[6] https://www.alexa.com/siteinfo/vimeo.com

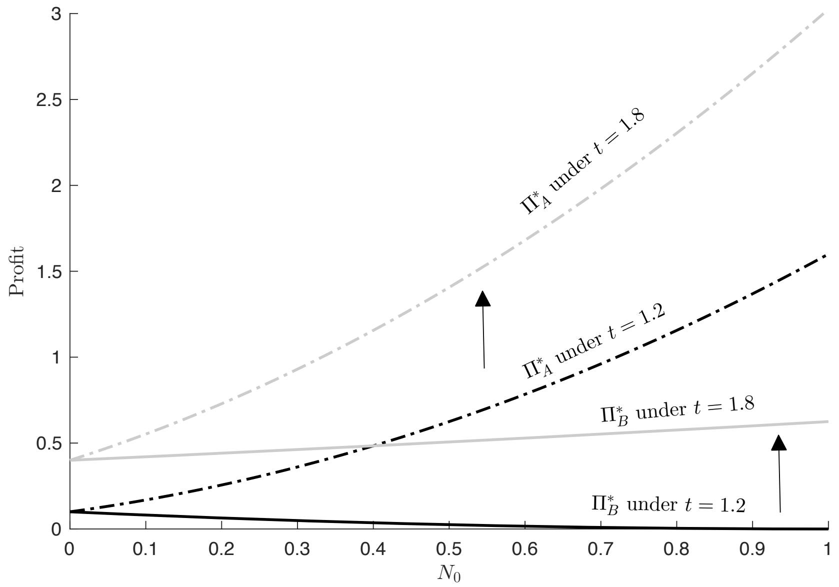

Figure 7 Profit Impacts of N_0 ($\alpha = \beta = 1, r = 1$)

5. Extensions

We extend the baseline model in three directions in this section: single-sided market piggybacking; endogenous piggybacking, in which N_0 is a platform decision; and fabricated piggybacking, in which platforms use "zombie" consumers to generate network effects but not profit.

5.1. Single-Sided Market Piggybacking

Our baseline two-sided model can be reduced to a single-sided model by letting $\alpha = \beta = 0$ in Equations (4) and (5). In this case, the provider side is gone, leaving only the consumer side, thus we can suppress the superscripts (i.e., d and c). Platform A optimizes p_A to maximize $\Pi_A = (rN_0 + N_A)p_A$. Simultaneously, Platform B optimizes $\Pi_B = [(1-r)N_0 + N_B]p_B$. Proposition 5 gives the equilibrium results.

Proposition 5. *In single-sided market competition with piggybacking, in equilibrium, optimal pricing strategies, market sizes, and profits are:*

(a) $p_A^* = t\left[1 + \frac{(1+r)N_0}{3}\right]$ *and* $p_B^* = t\left[1 + \frac{2(2-r)N_0}{3}\right]$*;*

(b) $N_A^* = \frac{1}{2} + \frac{(1-2r)N_0}{3}$ *and* $N_B^* = \frac{1}{2} + \frac{(2r-1)N_0}{3}$*;*

(c) $\Pi_A^* = \frac{t[3+2(1+r)N_0]^2}{18}$ *and* $\Pi_B^* = \frac{t[3+2(2-r)N_0]^2}{18}$.

Given Proposition 5, Corollaries 3 and Corollary 4 immediately follow.

Corollary 3. *Assume a fixed external piggybacking pool* N_0*. As* r *increases, the following hold true in equilibrium:*

(a) Platform A raises the price and platform B reduces the price, i.e., $\frac{\partial p_A^*}{\partial r} \geq 0$, $\frac{\partial p_B^*}{\partial r} < 0$;

(b) Platform A profits more and platform B profits less, i.e., $\frac{\partial \Pi_A^*}{\partial r} \geq 0$, $\frac{\partial \Pi_B^*}{\partial r} < 0$.

Corollary 4. *In single-sided market competition with piggybacking, given any fixed* $r \in [\frac{1}{2}, 1]$*, the following hold true in equilibrium:*

(a) ***(No Exit)*** *Both platforms remain in the market;*

(b) ***(Competition Mitigation)*** *Both platforms raise prices as* N_0 *increases, i.e.,* $\frac{\partial p_A^*}{\partial N_0} \geq 0$, $\frac{\partial p_B^*}{\partial N_0} \geq 0$;

(c) ***(Pareto Improvements)*** *Platform profit is increasing and convex in* N_0*, i.e.,* $\frac{\partial \Pi_A^*(p_A^*, p_B^*)}{\partial N_0} \geq 0$, $\frac{\partial^2 \Pi_A^*(p_A^*, p_B^*)}{\partial N_0^2} \geq 0$, $\frac{\partial \Pi_B^*(p_A^*, p_B^*)}{\partial N_0} \geq 0$, $\frac{\partial^2 \Pi_B(p_A^*, p_B^*)}{\partial N_0^2} \geq 0$;

(d) ***(Piggybacking Advantage)*** *The profit gap between platforms is increasing and convex in* N_0*, i.e.,* $\frac{\partial(\Pi_A^*(p_A^*, p_B^*) - \Pi_B^*(p_A^*, p_B^*))}{\partial N_0} \geq 0$, $\frac{\partial^2(\Pi_A^*(p_A^*, p_B^*) - \Pi_B^*(p_A^*, p_B^*)))}{\partial N_0^2} \geq 0$.

These single-sided market piggybacking results provide a benchmark so that we can understand the uniqueness of two-sided market piggybacking. First, in the presence of piggybacking, the platform with weaker piggybacking capability can be forced to exit the market (see conditions in Proposition 1), but this scenario would never happen in the single-sided setting. Second, when the piggybacking pool is fixed, an increase in Platform A's piggybacking capability r has similar profit implications in the single-sided setting and the two-sided setting, emphasizing the importance of piggybacking to platform's profits; however, the impacts on the platforms' pricing decisions in the two settings are different. Third, fixing r, when the piggybacking pool increases, the impact on both pricing decisions and platform profits in the two settings is strikingly different. We summarize the managerial implications of this uniqueness in Table 7.

5.2. Endogenous Piggybacking

In business practice, it is often costly for platforms to acquire piggybacking consumers from external networks, particularly when piggybacking is implemented on a non-collaborative basis. Examples

Market Type	Single-Sided	Two-Sided	
	Consumer	Consumer Side	Provider Side
Pricing impacts of $r\uparrow$	$p_A^*\uparrow$, $p_B^*\downarrow$	$(p_A^c)^*$: counter-clockwise rotating $(p_B^c)^*$: clockwise rotating	$(p_A^d)^*$ and $(p_B^d)^*$ depend on α and β
Profit impacts of $r\uparrow$	$\Pi_A^*\uparrow$, $\Pi_B^*\downarrow$	$\Pi_A^*\uparrow$, $\Pi_B^*\downarrow$	
Pricing impacts of $N_0\uparrow$	$p_A^*\uparrow$, $p_B^*\uparrow$	$(p_A^c)^*$ and $(p_B^c)^*$ depend on α and β	$(p_A^d)^*$ and $(p_B^d)^*$ depend on α and β
Profit impacts of $N_0\uparrow$	$\Pi_A^*\uparrow$, $\Pi_B^*\uparrow$	$\Pi_A^*\uparrow$, Π_B^* depends on α and β	

Table 7 Comparisons of Piggybacking Impacts: Single-Sided Market Setting vs. Two-Sided Market Setting

include when platforms use hacking (e.g., Airbnb hacked Craigslist users) or purchase directly from digital marketing services (as discussed in Section 1). In such cases, platforms tradeoff between the costs and benefits of piggybacking.

To capture this, we extend our baseline model to examine endogenous piggybacking. For simplicity, we assume that only Platform A piggybacks (i.e., $r=1$). Further, we assume a convex cost of bN_0^2 for acquiring N_0 piggybacking consumers, where b is the marginal cost of piggybacking. The convex cost assumption captures the fact that it is increasingly difficult for platforms to acquire consumers from external networks. To avoid the trivial solution of $N_0^*\to+\infty$ when b is too small, we assume that b is not so small that $b>\hat{b}$ ($\hat{b}$ is defined in the Appendix). The remaining assumptions are the same as in the baseline model. We modify Platform A's objective function (in Equation 4) and problem as follows.

$$\max_{p_A^c,p_A^d,N_0} \quad \Pi_A = p_A^c(N_0+N_A^c)+p_A^dN_A^d-bN_0^2. \tag{6}$$

Proposition 6 characterizes Platform A's optimal strategies.

Proposition 6. *Assume that the cost of piggybacking is bN_0^2 and only Platform A piggybacks (i.e., $r=1$). The following hold true in equilibrium when b increases:*
(a) N_0^* decreases (i.e., $\frac{N_0^*}{\partial b}<0$);

(b) Platform A raises the consumer-side price iff $t < T_2$;

(c) Platform A decreases the provider-side price iff $\alpha > \beta$.

Insights from Proposition 6 are the following. Findings from the exogenous piggybacking model (i.e., Corollaries 1a and 1b) are robust to endogenous piggybacking: Piggybacking complements subsidization if and only if $t < T_2$; the provider-side subsidization condition is determined only by cross-side network effects (i.e., the sign of $(\alpha - \beta)$). We illustrate these insights from Proposition 6 in Figure 8. When $t < T_2$ (e.g., this is the case when $\alpha = 1$, $\beta = 0.5$, $t = 0.6$ – the dotted line in Figure 8), as b *decreases*, N_0 increases (see Figure 8a). Rather than increasing prices on the consumer side, Platform A decreases prices (see Figure 8b, the dotted line when $\beta = 0.5$), illustrating the complementarity between piggybacking and subsidization. Platform A simultaneously increases subsidies when the marginal cost of piggybacking decreases. This is optimal, as Platform A can make the most profit by raising the price on the provide side (see Figure 8c). However, piggybacking no longer complements subsidization if condition $t < T_2$ is violated (e.g., this is the case when $\alpha = 1$, $\beta = 0.1$, $t = 0.6$ – the solid line in Figure 8). In this case, as b *decreases*, in addition to raising the price on the money side (which is the provider side because $\alpha = 1 > \beta = 0.1$), Platform A also raises the price on the consumer side (see Figure 8b) for two reasons: the consumer-side network effect is small, and there is a large consumer base to harvest, thanks to the *decreasing* piggybacking cost.

5.3. Fabricated Piggybacking

To capture the essence of fabricated piggybacking, we remove N_0 from the objective functions in Equations (4) and (5). Everything else remains the same as in our baseline setup. Proposition 7 examines the platforms' strategies when Platform A uses fabricated piggybacking to add N_0 zombie consumers. To facilitate comparison with our earlier results, we refer to the piggybacking strategy in the baseline model as "authentic piggybacking".

Proposition 7. *Assuming that only Platform A employs fabricated piggybacking (i.e., $r = 1$), the following hold true in equilibrium:*

(a) Platform B stays in the market when $N_0 < \frac{2(3t-2T_1)}{\alpha(\alpha+\beta)}$;

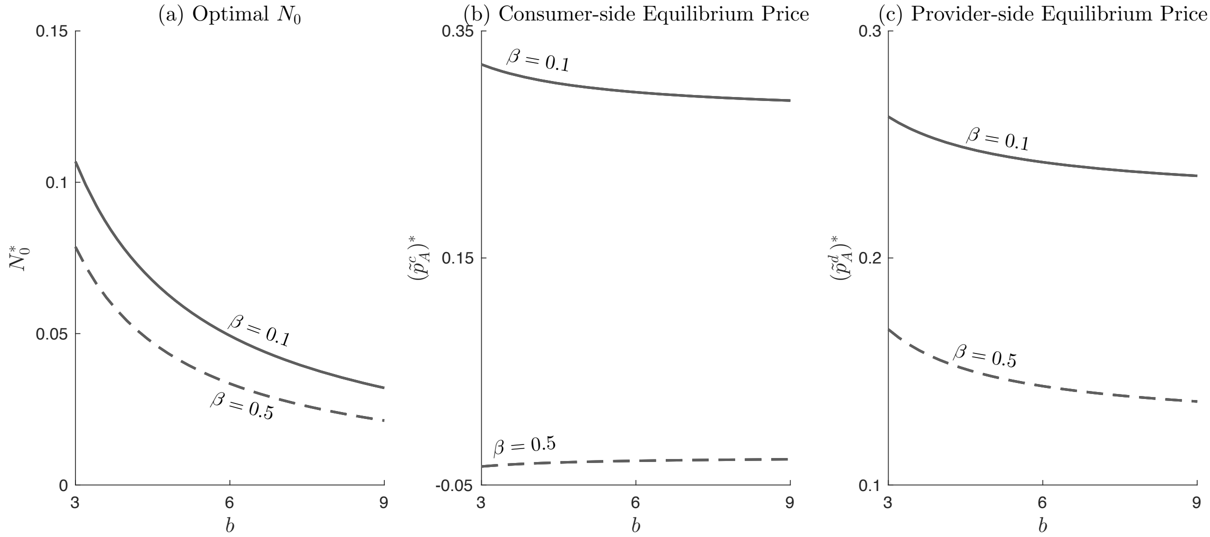

Figure 8 Comparative Statics of b ($\alpha = 1$, $t = 0.6$)

(b) Platform A's subsidization conditions are affected by N_0;

(c) Platform B's subsidization conditions are not affected by N_0.

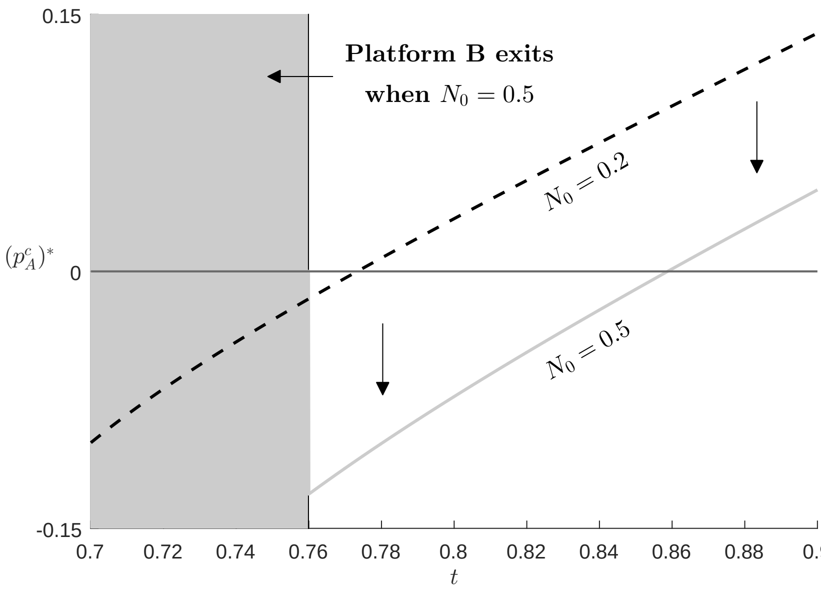

Figure 9 Platform A's Consumer-Side Equilibrium Price under Fabricated Piggybacking. The shaded area represents regions where Platform B exits the market when N_0 increases from 0.2 to 0.5 ($r = 1$, $\alpha = 1$, $\beta = 0.6$)

Proposition 7 reveals different platform strategies under fabricated piggybacking than under

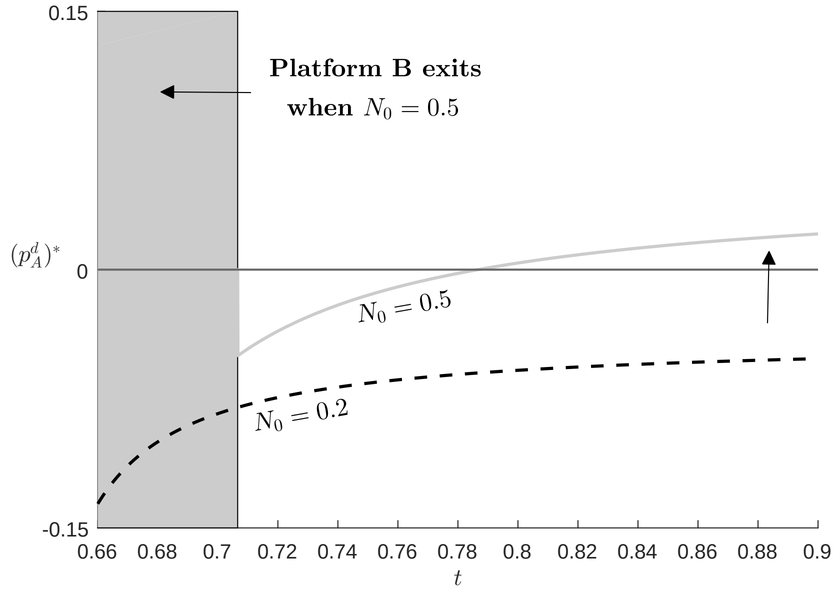

Figure 10 **Platform A's Provider-Side Equilibrium Price under Fabricated Piggybacking. The shaded area represents regions where Platform B exits the market when N_0 increases from 0.2 to 0.5 ($r = 1$, $\alpha = 0.6$, $\beta = 1$)**

authentic piggybacking. Although Platform A's fabricated piggybacking does not affect Platform B's subsidization condition, it does affect both the conditions and magnitude of Platform A's subsidy. This contrasts sharply with authentic piggybacking, in which Platform A's subsidization conditions remain unchanged under non-piggybacking, piggybacking, and endogenous piggybacking.

We use Figures 9 and 10 to illustrate further how N_0 affects Platform A's subsidization strategy under fabricated piggybacking. In contrast to the counter-clockwise rotation strategies in Figure 3 under authentic piggybacking, here Platform A finds it optimal to expand the region of subsidization as N_0 increases (see Figure 9), and subsidizing providers might no longer be optimal even if $\beta > \alpha$ (see Figure 10). The key message is that managing zombie consumers is non-trivial because doing so requires the platform to adjust pricing/subsidization strategies accordingly on both sides. The comparative statics of N_0 on platform pricing are summarized in Proposition 8.

Proposition 8. *Assuming that only Platform A employs fabricated piggybacking (i.e., $r = 1$), the following hold true in equilibrium:*

(a) $\frac{\partial (p_A^c)^*}{\partial N_0} < 0$ *iff* $\alpha(2T_1 - \beta^2) + 2t(\beta - 2\alpha) < 0$; $\frac{\partial (p_B^c)^*}{\partial N_0} < 0$ *iff* $t > T_2$;

(b) $\frac{\partial (p_A^d)^*}{\partial N_0} < 0$ *iff* $t < T_4 = \frac{\alpha^2+8\alpha\beta+3\beta^2}{12}$; $\frac{\partial (p_B^d)^*}{\partial N_0} < 0$ *iff* $\alpha > \beta$.

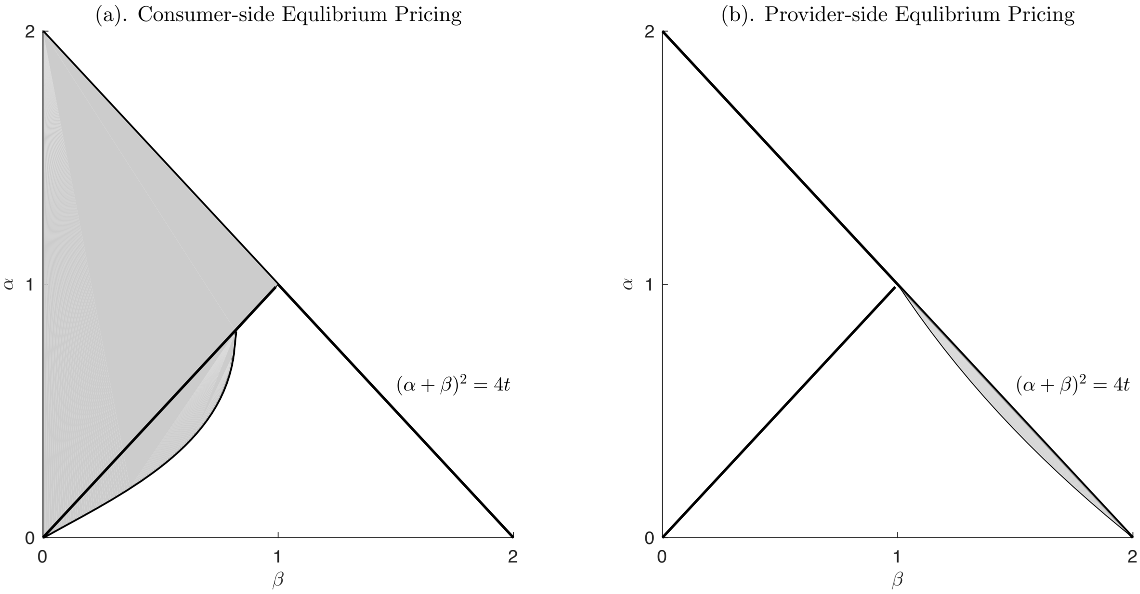

Figure 11 **Comparative Statics for Platform A's Equilibrium Prices under Fabricated Piggybacking. The shaded area represents regions where the price is decreasing in N_0.**

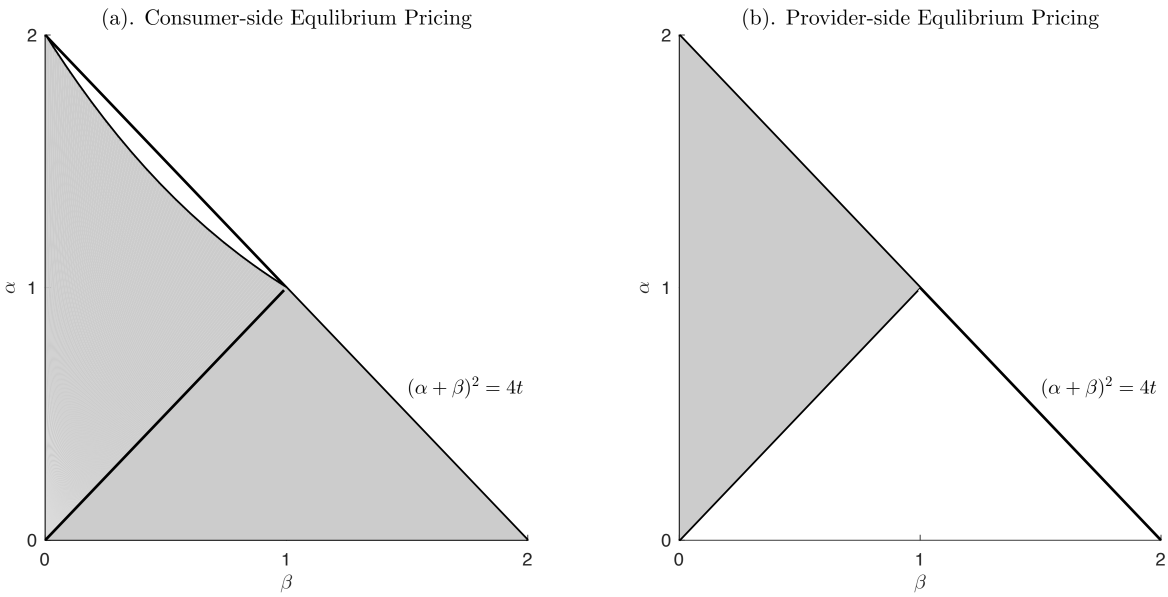

Figure 12 **Comparative Statics for Platform B's Equilibrium Prices under Fabricated Piggybacking. The shaded area represents regions where the price is decreasing in N_0.**

We illustrate the insights from Proposition 8 in Figures 11 and 12. We note a sharp contrast between the platforms' pricing strategies under authentic and fabricated piggybacking, compared to Figures 5 and 6. Two immediate differences, for example, are that Platform A by and large raises the price on the provide side (unlike Figure 5b), even if it is not the money side (i.e., when $\alpha < \beta$), but Platform B decreases the price (see Figure 11a) on the consumer side, even if it is the money side (i.e., when $\alpha < \beta$). Intuitively, this is because fabricated piggybacking consumers do not contribute any profit (when the price is positive) or loss (when the price is negative) to Platform A. Therefore, it may no longer be optimal to give away provider-side profit to increase the price on the consumer side — the seesaw principle no longer holds here.

Not surprisingly, the pricing impact under fabricated piggybacking on the rival Platform B's consumer side is largely negative (see Figure 12a). This is because Platform B is under greater pricing pressure from Platform A on the consumer side, and this pressure might be greater on the provider side (see Figure 12b) when $\alpha > \beta$. In other words, when $\alpha > \beta$, indicating that the provider side is the money side, although $(p_B^d)^* > 0$ holds, fabricated piggybacking undermines Platform B's ability to monetize – Platform B is forced to reduce prices to avoid direct competition. This is further reflected in the profits, which we summarize in Proposition 9.

Proposition 9. *Assuming that only Platform A employs fabricated piggybacking (i.e., $r = 1$), the following hold true in equilibrium:*

(a) Platform A is always better off as N_0 increases;

(b) Platform B is always worse off as N_0 increases;

(c) The profit gap between the platforms is increasing and convex in N_0.

Proposition 9 reveals that the profit impact on Platform B is different under fabricated piggybacking than authentic piggybacking. Under authentic piggybacking, per Corollary 2b, Platform B might benefit from N_0 when network effects are not very large. This is because Platform A will raise prices, giving Platform B greater pricing flexibility. However, under fabricated piggybacking, Platform A does not have such strong incentives for raising prices because zombie consumers do not contribute any profit directly. Consequently, Platform B suffers from Platform A's fabricated piggybacking because its pricing flexibility is narrowed.

6. Discussions and Conclusion

Piggybacking, as perhaps the most prominent traffic-based strategy, has become a critical and competitive strategy in practical platform launches. We explore formally the strategic implications of piggybacking and subsidization in a competitive setting with network effects. We reveal many insights, briefly summarized below. We identify conditions when piggybacking and subsidization can be complementary and when piggybacking can mitigate/intensify platforms' price competition. We show that, compared with the absence of piggybacking, platforms' subsidization conditions are not affected by piggybacking. In the extensions, we highlight the uniqueness of piggybacking on two-sided platforms versus single-sided platforms. We demonstrate that our insights are robust under endogenous piggybacking. Lastly, we show different platform strategies under fabricated piggybacking versus authentic piggybacking.

Practically, our results suggest that platforms can solve the "chicken-and-egg" problem via piggybacking, in conjunction with pricing. However, the interplay between piggybacking and pricing strategies is nontrivial, as the well-known seesaw principle may no longer be optimal in the presence of piggybacking. For example, a platform with stronger piggybacking capability might find it optimal to subsidize consumers (region I in Figure 5a) when the cross-side network effects are strong on the provider side. For instance, Zygna was a subsidiary platform on Facebook that was able to gain new users via viral acquisition. Nevertheless, it is reported that Zygna also made huge investments in paid marketing to harness its user adoption.[7] In yet another case example, Airbnb announced a free photography service for home owners to improve the pictures of their homes online, which eventually led to greater profit margins on the travelers' side.[8] This is consistent with our provider-side strategy as illustrated in Figure 5: Airbnb used the photography service to increase the attractiveness of the homes (i.e., a larger β). In this way, Airbnb can charge a higher price on the consumer side, while continuing to subsidize the provider side (free to home owners).

[7] http://www.ign.com/articles/2012/01/22/how-much-is-zynga-paying-for-new-gamers

[8] https://www.airbnb.com/info/photography

When piggybacking becomes costly, platform owners need to tradeoff between piggybacking and subsidization/pricing. We address this issue in Section 5.2. As it becomes more difficult or more costly to piggyback, it is optimal for the platform to lower the price on the provider side but not necessarily on the customer side (see Figure 8b). In 2010, when Zynga started to plan its spinning off from Facebook because of a weaker player acquisition effect,[9] it also set up a new payment program (called "Platinum Purchase Program") to focus on high-margin customers by encouraging them to recharge with more than $500 each time.[10] Our model suggests that this was an effort to remedy the loss due to weaker user acquisition from Facebook.

Finally, our model extension also sheds lights on fake user profiles and review fraud in digital platform competition. We find that pricing competition is more intense under fabricated piggybacking than under authentic piggybacking, because platforms have fewer incentives to raise prices under fabricated piggybacking. We leave it to future research to empirically test our model's predictions.

[9] https://techcrunch.com/2010/05/07/zynga-gunning-up-and-lawyering-up-for-war-against-facebook-with-zynga-live/

[10] http://gawker.com/5634379/the-secret-dealer-for-farmville-addicts

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Appendix

Proofs

Proof of Proposition 1. We show the existence of a duopoly equilibrium in two steps. First, we prove that each platform's profit maximization problem is jointly concave in the price decisions on both sides. This ensures that, given the rival platform's prices, each platform has a unique pair of prices to maximize overall profit. In other words, the equilibrium, if any, is unique. Second, we find the region where the focal market share is non-negative on both sides for both platforms. This helps us rule out the infeasible cases where the equilibrium market size is negative.

Step 1: Joint concavity of the profit-maximization problem. From the equilibrium conditions below:

$$N_A^c = \frac{1}{2} - \frac{p_A^c - p_B^c}{2t} + \frac{\beta(N_A^d - N_B^d)}{2t};$$

$$N_B^c = \frac{1}{2} - \frac{p_B^c - p_A^c}{2t} + \frac{\beta(N_B^d - N_A^d)}{2t};$$

$$N_A^d = \alpha(N_A^c + rN_0) - p_A^d,\ N_B^d = \alpha[N_B^c + (1-r)N_0] - p_B^d.$$

We have

$$N_A^c = \frac{1}{2} + \frac{p_B^c - p_A^c + \beta\,[p_B^d - p_A^d + (2r-1)\alpha N_0]}{2(t-\alpha\beta)}; \tag{A.1}$$

$$N_B^c = \frac{1}{2} - \frac{p_B^c - p_A^c + \beta\,[p_B^d - p_A^d + (2r-1)\alpha N_0]}{2(t-\alpha\beta)};$$

$$N_A^d = \frac{\alpha}{2} + \frac{\alpha\,[p_B^c - p_A^c + \beta(p_A^d + p_B^d)] - \alpha\,[\alpha\beta - 2rt]\,N_0 - 2tp_A^d}{2(t-\alpha\beta)};$$

$$N_B^d = \frac{\alpha}{2} + \frac{\alpha\,[p_A^c - p_B^c + \beta(p_A^d + p_B^d)] - \alpha\,[\alpha\beta - 2(1-r)t]\,N_0 - 2tp_B^d}{2(t-\alpha\beta)}.$$

Insert Equation (A.1) into the objective functions in Equations (4) and (5). Take Platform A as an example. The profit function is

$$\Pi_A = -\frac{1}{2(t-\alpha\beta)} \times (p_A^c)^2 + \left(\frac{1}{2} - \frac{(\alpha+\beta)p_A^d - p_B^c - \beta p_B^d - (2rt-\alpha\beta)N_0}{2(t-\alpha\beta)}\right) \times p_A^c$$
$$+\alpha\left(\frac{1}{2} + \frac{(2rt-\alpha\beta)N_0 + p_B^c + \beta p_B^d}{2(t-\alpha\beta)}\right) \times p_A^d - \left(1 + \frac{t}{t-\alpha\beta}\right) \times (p_A^d)^2.$$

If we take the second-order derivatives to p_A^c and p_A^d, respectively, we have

$$\frac{\partial^2 \Pi_A}{\partial (p_A^c)^2} = -\frac{1}{t-\alpha\beta},$$
$$\frac{\partial^2 \Pi_A}{\partial (p_A^d)^2} = -\left(1 + \frac{t}{t-\alpha\beta}\right),$$

both of which are negative due to $(\alpha+\beta)^2 < 4t$. The determinant of the Hessian matrix is

$$\det(\mathbf{H}_{p_A^c, p_A^d}(\Pi_A)) = \frac{8t - \alpha^2 - 6\alpha\beta - \beta^2}{4(t-\alpha\beta)^2},$$

which is positive when $(\alpha+\beta)^2 < 4t$ holds. Therefore, Π_A is jointly concave in p_A^c and p_A^d. Similarly, we can show that Π_B is jointly concave in p_B^c and p_B^d.

Step 2: The region of non-negative market size and the equilibrium. We first solve for equilibrium prices and reinsert them in Equation (A.1). Both the equilibrium prices and market size can be found in Proposition 2.

Under our assumptions of $(\alpha+\beta)^2 < 4t$ and $r \in [\frac{1}{2}, 1]$, it can be shown that N_A^c and N_A^d are both always non-negative. We next find regions of $N_B^c \geq 0$ and $N_B^d \geq 0$.

Region 1: $N_B^c = \frac{1}{2} + \frac{(2r-1)(t-T_1)}{3t-2T_1} \times N_0 < 0$:

N_B^c can be either increasing or decreasing in N_0, depending on the sign of $t - T_1$. When $t \geq T_1$, N_B^c is increasing in N_0. Thus $N_B^c \geq 0$ always holds because $N_B^c = \frac{1}{2} > 0$ when $N_0 = 0$. Otherwise when $t < T_1$, N_B^c is decreasing in N_0. There exists a threshold of N_0 above which $N_B^c < 0$. In this case, we solve for N_0 from $N_B^c = 0$:

$$\frac{1}{2} + \frac{(2r-1)(t-T_1)}{3t-2T_1} \times N_0 = 0 \rightarrow N_0 = \frac{3t-2T_1}{2(2r-1)(T_1-t)}.$$

Therefore, when $t < T_1$, $N_B^c \geq 0$ requires $N_0 \leq \frac{3t-2T_1}{2(2r-1)(T_1-t)}$ which is equivalent to $r \leq \bar{r} = \frac{1}{2} + \frac{2T_1-3t}{4(T_1-t)N_0}$. Region 1 is defined by $t < T_1$ and $r > \bar{r}$.

Region 2: $N_B^d = \frac{\alpha+\beta}{4}\left(1 + \frac{2[(2-r)t-T_1]}{3t-2T_1} \times N_0\right) < 0$:

Similarly, N_B^d can be either increasing or decreasing in N_0, depending on the sign of $(2-r)t - T_1$. We split our discussions into three cases: (1) $r < \frac{\alpha^2+\beta^2}{(\alpha+\beta)^2}$, (2) $r \geq \frac{\alpha^2+\beta^2}{(\alpha+\beta)^2}$ and $t < \frac{T_1}{2-r}$, and (3) $r \geq \frac{\alpha^2+\beta^2}{(\alpha+\beta)^2}$ and $t \geq \frac{T_1}{2-r}$. In cases 1 and 3, N_B^d is increasing in N_0, then $N_B^d \geq 0$ always holds because $N_B^d \geq 0$ when $N_0 = 0$. In case 2, a threshold exists for N_0 above which $N_B^d < 0$. We obtain N_0 by solving $N_B^d = 0$.

$$\frac{\alpha+\beta}{4}\left(1 + \frac{2[(2-r)t-T_1]}{3t-2T_1} \times N_0\right) = 0 \rightarrow N_0 = \frac{3t-2T_1}{2[(T_1-(2-r)t]}.$$

Therefore Region 2 (i.e., $N_B^d < 0$) is defined by $r \geq \frac{\alpha^2+\beta^2}{(\alpha+\beta)^2}$, $t < \frac{T_1}{2-r}$, and $N_0 > \frac{3t-2T_1}{2[(T_1-(2-r)t]}$.

Lastly, we merge Regions 1 and 2. It can be shown that Region 1 dominates Region 2. Therefore, the equilibrium exists everywhere outside Region 1. $\square$

Proof of Proposition 2. In the proof for Proposition 1, we have shown that both platforms' profit functions are jointly concave in prices. Therefore, when equilibrium exists, we can obtain the equilibrium by solving the following systems of first-order conditions (FOCs):

$$\begin{aligned}
\frac{\partial \Pi_A}{\partial p_A^c} &= -\frac{p_A^c}{t-\alpha\beta} + \left(\frac{1}{2} - \frac{(\alpha+\beta)p_A^d - p_B^c - \beta p_B^d - (2rt-\alpha\beta)N_0}{2(t-\alpha\beta)}\right) = 0;\\
\frac{\partial \Pi_A}{\partial p_A^d} &= -\left(1+\frac{t}{t-\alpha\beta}\right)p_A^d + \frac{\alpha\left[p_B^c + \beta p_B^d + N_0(2rt-\alpha\beta) + (t-\alpha\beta)\right] - (\alpha+\beta)p_A^c}{2(t-\alpha\beta)} = 0;\\
\frac{\partial \Pi_B}{\partial p_B^c} &= -\frac{p_B^c}{t-\alpha\beta} + \left(\frac{1}{2} - \frac{(\alpha+\beta)p_B^d - p_A^c - \beta p_A^d - (2(1-r)t-\alpha\beta)N_0}{2(t-\alpha\beta)}\right) = 0;\\
\frac{\partial \Pi_B}{\partial p_B^d} &= -\left(1+\frac{t}{t-\alpha\beta}\right)p_B^d + \frac{\alpha\left[p_A^c + \beta p_A^d + N_0(2(1-r)t-\alpha\beta) + (t-\alpha\beta)\right] - (\alpha+\beta)p_B^c}{2(t-\alpha\beta)} = 0.
\end{aligned}$$

The equilibrium market size and the corresponding profits can be obtained by inserting equilibrium prices in Equations (4) and (5). $\square$

Proof of Proposition 3. If we take the first-order partial derivatives with respect to equilibrium prices, we have

$$\frac{\partial(p_A^c)^*}{\partial r} = \frac{2N_0 t\,(t-T_2)}{3t-2T_1};$$
$$\frac{\partial(p_B^c)^*}{\partial r} = -\frac{2N_0 t\,(t-T_2)}{3t-2T_1};$$
$$\frac{\partial(p_A^d)^*}{\partial r} = \frac{N_0 t(\alpha-\beta)}{2(3t-2T_1)};$$
$$\frac{\partial(p_B^d)^*}{\partial r} = -\frac{N_0 t(\alpha-\beta)}{2(3t-2T_1)}.$$

Note that $3t-2T_1$ in the denominator is always positive if $4t > (\alpha+\beta)^2$. Therefore, the conditions of $\frac{\partial(p_A^d)^*}{\partial r} \geq 0$ and $\frac{\partial(p_B^d)^*}{\partial r} \geq 0$ depend only on the sign of $\alpha-\beta$.

Furthermore, $t-T_2$ is always positive if $\beta > \alpha$, which indicates that $\frac{\partial(p_A^c)^*}{\partial r} > 0$ and $\frac{\partial(p_B^c)^*}{\partial r} < 0$ always hold when $\beta > \alpha$. Otherwise, $\frac{\partial(p_A^c)^*}{\partial r} > 0$ and $\frac{\partial(p_B^c)^*}{\partial r} < 0$ if and only if $t > T_2$. □

Proof of Corollary 1.

(a) We show that Platform A's subsidization conditions are not affected by r. The conditions for Platform B can be shown in a similar way. On the consumer side, the subsidization condition for Platform A is

$$(p_A^c)^* < 0 \rightarrow (t-T_2)(1+\frac{2[(1+r)t-T_1]}{3t-2T_1}\times N_0) < 0.$$

Note that the assumption $r \geq \frac{1}{2}$ ensures that $(1+r)t \geq \frac{3t}{2} > T_1$. Thus the expression in the second bracket is positive. The subsidization condition reduces to $(p_A^c)^* < 0 \rightarrow t < T_2$, which is not affected by r.

(b) Combine the subsidization condition above with Proposition 3. If $t < T_2$, then $(p_A^c)^* < 0$, $\frac{\partial(p_A^c)^*}{\partial r} < 0$.

(c) This can immediately be observed from Table 4. □

Proof of Proposition 4. Define parameter region I through IV as in Table 5. We set $r = 1$ and take the first-order derivatives with respect to N_0 in equilibrium prices. Note that $T_1 > T_2$ always holds.

$$\frac{\partial (p_A^c)^*}{\partial N_0} = \frac{2\,(2t - T_1)\,(t - T_2)}{3t - 2T_1};$$
$$\frac{\partial (p_B^c)^*}{\partial N_0} = \frac{2\,(t - T_1)\,(t - T_2)}{3t - 2T_1};$$
$$\frac{\partial (p_A^d)^*}{\partial N_0} = \frac{(\alpha - \beta)\,(2t - T_1)}{2(3t - 2T_1)};$$
$$\frac{\partial (p_B^d)^*}{\partial N_0} = \frac{(\alpha - \beta)\,(t - T_1)}{2(3t - 2T_1)}.$$

(a) Region I satisfies $t < T_2$ and $\alpha > \beta$. Therefore we have $\frac{\partial (p_A^c)^*}{\partial N_0} < 0$, $\frac{\partial (p_A^d)^*}{\partial N_0} > 0$, $\frac{\partial (p_B^c)^*}{\partial N_0} > 0$, and $\frac{\partial (p_B^d)^*}{\partial N_0} < 0$. In contrast, Region III satisfies $t < T_1$ and $\alpha < \beta$, which result in $\frac{\partial (p_A^c)^*}{\partial N_0} > 0$, $\frac{\partial (p_A^d)^*}{\partial N_0} < 0$, $\frac{\partial (p_B^c)^*}{\partial N_0} < 0$, and $\frac{\partial (p_B^d)^*}{\partial N_0} > 0$. Region II satisfies $t \in [T_2, T_1)$ and $\alpha \geq \beta$. Therefore we have $\frac{\partial (p_B^c)^*}{\partial N_0} < 0$ and $\frac{\partial (p_B^d)^*}{\partial N_0} < 0$.

(b) Regions IV and V satisfy $t \geq T_1 \geq T_2$. Note that $3t > 2T_1$ always holds. Therefore we have $\frac{\partial (p_A^c)^*}{\partial N_0} > 0$ and $\frac{\partial (p_B^c)^*}{\partial N_0} > 0$. $\square$

Proof of Corollary 2.

(a) Take the first- and second-order derivatives with respect to N_0 in Π_A^*.

$$\frac{\partial (\Pi_A)^*}{\partial N_0} = \frac{(2t - T_1)\,(2t - T_3)\,((4N_0 + 3)\,t - 2\,(N_0 + 1)\,T_1)}{(3t - 2T_1)^{\,2}} > 0;$$
$$\frac{\partial^2 (\Pi_A)^*}{\partial N_0^2} = \frac{2\,(T_1 - 2t)^{\,2}\,(2t - T_3)}{(3t - 2T_1)^{\,2}} > 0.$$

$2t - T_3 \geq 0$ always holds if $4t > (\alpha + \beta)^2$. Then $(\Pi_A)^*$ is increasing and convex in N_0.

(b) Take the first- and second-order derivatives with respect to N_0 in Π_B^*.

$$\frac{\partial (\Pi_B)^*}{\partial N_0} = \frac{(t - T_1)\,(2t - T_3)\,[(2N_0 + 3)\,t - 2\,(N_0 + 1)\,T_1]}{(3t - 2T_1)^2};$$
$$\frac{\partial^2 (\Pi_B)^*}{\partial N_0^2} = \frac{2\,(t - T_1)^{\,2}\,(2t - T_3)}{(3t - 2T_1)^2} > 0.$$

In the parameter region defined by Proposition 1, $(2N_0 + 3)\,t - 2\,(N_0 + 1)\,T_1 \geq 0$ always holds. Therefore, it requires $t - T_1 > 0$ for $\frac{\partial (\Pi_B)^*}{\partial N_0}$ to be positive, and $(\Pi_B)^*$ is convex in N_0.

(c) The profit gap between the two platforms is given by

$$(\Pi_A)^* - (\Pi_B)^* = \frac{N_0\,(N_0+1)\,t\,(2t-T_3)}{3t-2T_1}.$$

Take the first- and second-order derivatives with respect to N_0 in Π_B^*.

$$\frac{\partial[(\Pi_A)^* - (\Pi_B)^*]}{\partial N_0} = \frac{(2N_0+1)\,t\,(2t-T_3)}{3t-2T_1} > 0;$$
$$\frac{\partial^2[(\Pi_A)^* - (\Pi_B)^*]}{\partial N_0^2} = \frac{2t\,(2t-T_3)}{3t-T_2} > 0.$$

Then the profit gap is increasing and convex in N_0. □

Proof of Proposition 5. Propositions 5 is a special case of Proposition 2 by setting $\alpha=\beta=0$. □

Proof of Corollary 3. This follows immediately from Proposition 5. □

Proof of Corollary 4. (a) The market size in Proposition 5b is always positive for both platforms for $N_0 \leq 1$. Therefore, the unique market equilibrium always exists.

Corollaries 4b through 4d can be immediately obtained by setting $\alpha=\beta=0$ in partial derivatives in the proofs of Corollary 2 and Proposition 2. □

Proof of Proposition 6. We show that for $b > \hat{b} = \max\left(\frac{(2t-\alpha\beta)(2t-T_1)}{2(3t-2T_1)}, \frac{(2t-\alpha\beta)^2}{4(2t-T_3)}\right)$, the objective function in Equation (6) is jointly concave in p_A^c, p_B^c, and N_0.

We first obtain the market size in equilibrium. The the first-order condition of N_0 gives

$$N_0^* = \frac{(2t-\alpha\beta)\,(p_A^c + \alpha p_A^d)}{4b(t-\alpha\beta)}.$$

We then reinsert it in the objective function. Following an approach similar to our proof for Proposition 1, we can show that the objective function is jointly concave in p_A^c and p_A^d if $b \geq \hat{b}$. The unique solution of the equilibrium gives

$$(p_A^c)^* = \frac{b\,(3t-2T_1)\,(t-T_2)}{2b\,(3t-2T_1)-(2t-T_1)\,(2t-\alpha\beta)};$$

$$(p_A^d)^* = \frac{b(\alpha-\beta)(3t-2T_1)}{2(2b(3t-2T_1)-(2t-T_1)(2t-\alpha\beta))};$$

$$N_0^* = \frac{(2t-\alpha\beta)(3t-2T_1)}{2(2b(3t-2T_1)-(2t-T_1)(2t-\alpha\beta))},$$

where $N_0^* \geq 0$ always holds when $b \geq \hat{b}$. Take the first-order derivatives with respect to b.

$$\frac{\partial(p_A^c)^*}{\partial b} = -\frac{2(2t-\alpha\beta)(3t-2T_1)(2t-T_1)(t-T_2)}{((2t-\alpha\beta)(2t-T_1)-2b(3t-2T_1))^2};$$

$$\frac{\partial(p_A^d)^*}{\partial b} = -\frac{(\alpha-\beta)(2t-\alpha\beta)(2t-T_1)(3t-2T_1)}{2((2t-\alpha\beta)(2t-T_1)-2b(3t-2T_1))^2};$$

$$\frac{\partial(N_0)^*}{\partial b} = -\frac{(2t-\alpha\beta)(3t-2T_1)^2}{((2t-\alpha\beta)(2t-T_1)-2b(3t-2T_1))^2} < 0.$$

(a) Note that $\frac{\partial(N_0)^*}{\partial b} < 0$ always holds.

(b) The sign of $\frac{\partial(p_A^c)^*}{\partial b}$ depends on the sign of $t-T_2$. Therefore, $\frac{\partial(p_A^c)^*}{\partial b} \geq 0$ if $t-T_2 < 0$, indicating that Platform A should subsidize less (i.e., $\frac{\partial(p_A^c)^*}{\partial b} > 0$ and $(p_A^c)^* < 0$) if less piggybacking occurs (i.e., $\frac{\partial(N_0)^*}{\partial b} < 0$), revealing a complementarity relationship between these two decisions.

(c) The sign of $\frac{\partial(p_A^d)^*}{\partial b}$ depends on $\alpha-\beta$. Therefore, $\frac{\partial(p_A^d)^*}{\partial b} < 0$ iff $\alpha > \beta$. $\square$

Proof of Proposition 7. As in the proof for Proposition 1, we first show that, the equilibrium, if any, is unique. Note that equilibrium numbers of focal market adopters are identical to those of Equation (A.1) because fabricated piggybacking does not change the role of network effects. However, the objective functions do not contain N_0 because fabricated piggybacking does not contribute any direct profit. Following an approach similar to the one we used in the proof for Proposition 2, we can derive the equilibrium prices as follows:

$$(p_A^c)^* = t-T_2+\frac{\alpha N_0(\alpha(2T_1-\beta^2)+2t(\beta-2\alpha))}{2(3t-2T_1)};$$

$$(p_A^d)^* = \frac{1}{4}\left(\alpha-\beta+\frac{(12-\alpha^2-8\alpha\beta-3\beta^2)\alpha N_0}{2(3t-2T_1)}\right);$$

$$(p_B^c)^* = (t-T_2)\left(1-\frac{\alpha(\alpha+\beta)N_0}{2(3t-2T_1)}\right);$$

$$(p_B^d)^* = \frac{1}{4}(\alpha - \beta)\left(1 - \frac{\alpha(\alpha+\beta)N_0}{2(3t - 2T_1)}\right).$$

(a) Reinsert equilibrium prices in the equilibrium market size in Equation (A.1). $N_B^c \geq 0$ and $N_B^d \geq 0$ requires an identical condition of $1 \geq \frac{\alpha(\alpha+\beta)N_0}{2(3t-2T_1)}$.

(b) It can be observed from the equilibrium prices above that the signs of prices are sensitive to N_0, meaning that subsidization conditions are affected by the magnitude of piggybacking.

(c) It can be observed from the equilibrium prices above that, when the equilibrium exists (i.e.,$1 \geq \frac{\alpha(\alpha+\beta)N_0}{2(3t-2T_1)}$), the signs of $(p_B^c)^*$ and $(p_B^d)^*$ are not affected by N_0. □

Proof of Proposition 8. If we take the first-order derivatives to equilibrium prices above, we have

$$\begin{aligned}
\frac{\partial (p_A^c)^*}{\partial N_0} &= \frac{\alpha\left(\alpha(2T_1 - \beta^2) + 2t(\beta - 2\alpha)\right)}{2(3t - 2T_1)}; \\
\frac{\partial (p_A^d)^*}{\partial N_0} &= \frac{3\alpha\,(t - T_4)}{2(3t - 2T_1)}; \\
\frac{\partial (p_B^c)^*}{\partial N_0} &= -\frac{\alpha(t - T_2)(\alpha+\beta)}{2(3t - 2T_1)}; \\
\frac{\partial (p_B^d)^*}{\partial N_0} &= -\frac{\alpha(\alpha-\beta)(\alpha+\beta)}{2(3t - 2T_1)}.
\end{aligned}$$

(a) The sign of $\frac{\partial (p_A^c)^*}{\partial N_0}$ depends on the sign of $\alpha(2T_1 - \beta^2) + 2t(\beta - 2\alpha)$. The sign of $\frac{\partial (p_B^c)^*}{\partial N_0}$ depends on the sign of $t - T_2$.

(b)The sign of $\frac{\partial (p_A^d)^*}{\partial N_0}$ depends on the sign of $t - T_4$. The sign of $\frac{\partial (p_B^d)^*}{\partial N_0}$ depends on the sign of $\alpha - \beta$. □

Proof of Proposition 9. (a) & (b) This can be obtained by taking the first- and second-order derivatives with respect to N_0 in equilibrium profits.

(c) The profit gap between the two platforms is

$$(\Pi_A)^* - (\Pi_B)^* = \frac{1}{4}\alpha N_0\left(\alpha N_0 + \frac{(\alpha+\beta)\left(\alpha^2 + 6\alpha\beta + \beta^2 - 8t\right)}{\alpha^2 + 4\alpha\beta + \beta^2 - 6t}\right),$$

which is increasing and convex in N_0. □