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Ambiguity aversion and the degree of ambiguity

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Abstract
We empirically show that sample information not only moderates prospects' outcome ambiguity but also decision makers' revealed aversion of them. Since most natural prospects permit at least some sample inference, accounting for their degree of ambiguity improves prediction of aversion. The special case of full ambiguity, as in Ellsberg-type designs, is typically averted—yet many decision makers systematically like low degrees of ambiguity while disliking higher degrees. Ambiguity attitudes might thus usefully be characterized by not only their sensitivity to degrees of ambiguity but also such ambiguity thresholds. Just as people like some risks but not others, they have ambiguity attitudes that depend on how much ambiguity there is. We thus show how attitudes towards a degree of ambiguity are systematic, enabling prediction across sources of ambiguity.

Keywords Ambiguity preferences · Decision making under uncertainty · Risky choice · Confidence thresholds

1 Introduction

Why someone would invest in the Dow Jones but not the Nikkei index, even if she expected them to move similarly, is sometimes attributed to the source-specificity of attitudes towards ambiguity (Trautmann & Van De Kuilen, 2015). Such attitudes describe the extent to which someone likes prospects with outcome probabilities that cannot be fully specified. Ambiguity attitudes are understood to vary with each source of uncertainty, groups of events generated by the same uncertainty-generating mechanism (Abdellaoui et al., 2011, p. 699).

Inconsistency in ambiguity attitudes across sources is variously ascribed to people's comfort with the source, their biases towards familiar sources (home

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bias), or, more generally, their source preference (Abdellaoui et al., 2011; Chew & Sagi, 2008; Tversky & Fox, 1995). Source-specificity of ambiguity attitudes makes it hard to predict people's decisions under uncertainty. Policy makers or decision analysts might classify people by their attitudes towards risk, i.e., fully specified probabilities (Garagnani, 2020; Schildberg-Hörisch, 2018), but cannot easily do the same with attitudes towards ambiguity.

Work by Anantanasuwong et al. (2019) suggests that people's estimated aversion to ambiguity actually appears reasonably stable across four survey sources when simultaneously estimating people's perceptions of each source's ambiguity. Unfortunately, for policy makers and analysts, this insight offers no means to predicting aversion to ambiguity before observing it. And without reliable prediction, the practical value of eliciting ambiguity attitudes is limited, even if the research area has seen strong methodological advancements (Baillon et al., 2018, 2021; Dimmock et al., 2016b). Attitudes revealed for one source of uncertainty are poor predictors of how someone might like other sources of uncertainty. Knowing only that someone invested in the Dow would tell us little about her proclivity to invest in the Nikkei too.

We propose that qualifying sources of ambiguity with the amount of information available to the decision maker remedies this state of affairs. Such qualification permits prediction across sources and thus gets closer to capturing an aversion trait—a person-specific attitude towards a given amount of ambiguity.

We build on the insight that most natural prospects come with incomplete information, constraining the distribution of possible outcomes (Einhorn & Hogarth, 1986; Yates & Zukowski, 1976). Variation in such distributions—intuitively, the shape of what can possibly happen—can be formally expressed as an objective degree of ambiguity associated with a source of uncertainty (Chew et al., 2017; Epstein & Schneider, 2007; Izhakian, 2017; Marinacci, 2002). At low degrees of ambiguity, there is much information about what the outcome probabilities are (converging on fully specified risk); at high degrees of ambiguity, there is little information about what the outcome probabilities are (converging on Ellsberg-type full ambiguity, Ellsberg 1961).

Expressing ambiguity attitudes relative to the degree of ambiguity has face validity. Natural prospects involve varying amounts of ambiguity that is neither absolute nor absent. Outcome probabilities are somewhat known, albeit imperfectly. People who choose or not to invest in companies, insurance, property, education, and many other things besides, have had access to some information about these choices. Even those playing the lottery can gauge success rates on the basis of limited prior observation. But they cannot be sure about the probability of winning. Neither can investors fully specify the probability of a gain. People form their beliefs about probability on the basis of samples of personal experience or that of others; these samples are quantifiable. Rating the ambiguity of a prospect by the sample information available about it thus maps onto source-preference conceptions of the familiarity and comfort a person has with an ambiguous prospect (Fox & Tversky, 1995; Tversky & Fox, 1995). Incidentally, Ellsberg had already invoked the notion of confidence (Ellsberg, 1961, p. 659) to gauge ambiguity.

Prior literature lays out how samples afford confidence in outcomes (Marinacci, 2002). One way to express such statistical confidence is through the posterior probability distribution over the payoff of a prospect suggested by sample evidence, given a uniform prior (e.g., Bayes et al., 1763; Laplace, 1986; Hill, 1968). Because natural prospects offer varying levels of statistical confidence, it is not surprising that empirical studies of choice behavior sometimes detect traces of aversion to ambiguity (e.g., Dimmock et al., 2016a) and sometimes not (e.g., Dimmock et al., 2016b). People hold different amounts of information about different prospects and, consequently, might or might not invest in the Dow or Nikkei, for example.

Where samples display information about underlying likelihoods, people update their expectations about ambiguous events with unknown probabilities in a loosely Bayesian manner (Gigerenzer & Hoffrage, 1995). They appear to adjust their revealed preference for ambiguity strongly (Ert & Trautmann, 2014; Gigliotti & Sopher, 1996; Klingebiel & Zhu, 2022), mildly (Kutzner et al., 2017), or not at all (Baillon et al., 2017; Bricet, 2018). Inconsistent empirical designs make comparisons across such studies difficult; task difficulty and biased priors may have been partly responsible for inconclusive results. There is as of yet little consensus about how exactly decision makers adjust their preferences for ambiguous prospects with sample information.

Our experiment involves physical two-color Ellsberg urns (Ellsberg, 1961) with samples. The rigorous design is a first to systematically capture choices over a range of differently ambiguous prospects. Our results offer support for systematic aversion sensitivity to the degree of ambiguity. People dislike high degrees of ambiguity more than low degrees of ambiguity. In fact, many actually like low-ambiguity prospects and dislike high-ambiguity prospects. Such findings point to the existence of an individual trait that can be conceptualized as confidence preference, a threshold degree of ambiguity beyond which people switch from liking to disliking ambiguous prospects.

Our work makes two contributions. First, we reveal ambiguity attitudes to be systematically related to the degree of ambiguity. Accounting for sample information is a tractable way to objectively assess degrees of ambiguity, improving on alternatives such as surveying beliefs (e.g., Einhorn & Hogarth, 1985; Viscusi & Magat, 1992) or altering state spaces (e.g., Chew et al., 2017; Viscusi & Chesson, 1999). Most importantly, capturing systematic attitudes permits out-of-source predictions. Knowing an investor's experience with Nikkei investing, her aversion to this source can be predicted on the basis of her attitude towards the Dow, for example. Policy makers, say, could infer people's reactions to prospective pension schemes. Firms could anticipate whether or not their competitor is likely to launch an innovation given adoption levels in the wider market.

Second, we situate ambiguity-aversion thresholds. Such thresholds, or confidence preference, explain why the same person can be found to seek some and avert other sources of ambiguity such as Nikkei and Dow. Because our confidence preference derives from a systematic relationship between ambiguity aversion and the degree of ambiguity, we can now classify decision makers by where they would stop probing an ambiguous prospect and act (Hausmann-Thürig & Läge, 2008; Navarro-Martinez et al., 2018) or, to paraphrase an earlier notion, when they would stop fearing and start hoping (Viscusi & Chesson, 1999). People's ambiguity aversion can be expressed as a function of thresholds and sensitivity to the degree of ambiguity, leading to predictable decisions across a variety of uncertain situations.

2 Aversion to partial ambiguity

We adopt the common definition of ambiguity as uncertainty about outcome probabilities (Baillon et al., 2018). Here, ambiguity is a "distribution of probabilities other than a point estimate" (Becker & Brownson, 1964, p. 64). The probabilities are unknown but not unknowable, meaning they can be bounded and described. Ignorance, by contrast, refers to unknowable probabilities (Coombs et al., 1970). Ignorance is beyond the remit of our work.

We operationalize ambiguity through bets on draws from two-color Ellsberg urns, which more closely match naturally occurring ambiguity than bets on prospects that separate out the probabilities of compound risk (Abdellaoui et al., 2015; Halevy, 2007; Trautmann & Van De Kuilen, 2015). For urns containing a fixed number of 100 balls of two possible colors, outcome ambiguity can range from full (color proportions completely unknown), over partial (some color proportions more likely than others), to none (color proportions known precisely).

Drawing from a two-color urn constitutes a parsimonious partition of two mutually exclusive and collectively exhaustive events. Only one event occurs; the decision maker does not know which one *ex ante*. A bet x_E0 provides money amount x if event E happens and nothing otherwise (the complementary event E^c). The probabilities of events E and E^c are unknown; the bet x_E0 is then an ambiguous prospect. Bets may afford varying levels of precision with which the probabilities of E and E^c can be known *ex ante*. When the outcome probabilities are precisely stated, there is no ambiguity at all: a risky bet x_p0 provides money amount x with probability p and nothing otherwise. In our study, event E could be drawing a red ball from the urn, black otherwise (E^c), and the money amount x is always €20.

2.1 Degree of ambiguity

Our operationalization of the degree of ambiguity about the occurrences of events E and E^c builds on the expected utility with uncertain probabilities model (Izhakian, 2017, 2020). The model measure $\mho^2$ captures variation in prospects' ambiguity (see Appendix A for detail):

$$\mho^2(f) = \sum_i \mathbb{E}[\varphi_f(x_i)]\mathrm{Var}[\varphi_f(x_i)]. \tag{1}$$

The expectation $\mathbb{E}[\cdot]$ and the variance $\mathrm{Var}[\cdot]$ are based on the second-order probability measure on the set of first-order probability measures. $\varphi_f(\cdot)$ is the (uncertain) probability mass function. f is the act mapping events to possible outcomes. x_i specifies the outcome ($x_i \in \{€0, €20\}$ in this study). $\mho^2$ can be computed directly from the data and has the attractive property of being independent from both outcomes and attitudes. For a discussion of alternative measures such as those related to relative entropy see Izhakian (2020).

In our two-color urn setting, the probability p of drawing a red (analogously, black) ball from the urn is a random variable ranging from 0 to 1. The expectations for the probability of drawing a red and black ball, respectively, sum to 1 and the associated variances are identical. The ambiguity measure $\mho^2$ thus reduces to the variance in probability p (for details see Appendix A):

$$\mho^2 = \mathrm{Var}[p].$$

An uninformed uniform prior initially assigns the same probability to all possible values of p. In this case of full ambiguity, the variance of the uniform distribution $\mathcal{U}(0, 1)$ for an urn setting (e.g., Dekking et al., 2005) is

$$\mathrm{Var}[p]_{full} = \frac{1}{12}(1-0)^2 \approx 0.083.$$

Samples reshape the distribution of possible values of p and constrain the variance. Each sample draw with replacement from a two-color Ellsberg urn is a Bernoulli trial (Bernoulli, 1713); draws are independent from each other. After observing n sample draws producing y red balls, the probability p of getting a red ball with the next draw can be described by the mean of posterior probabilities, as per Bayes' theorem. In cases of continuous probabilities, p is beta distributed (e.g., Berger, 2013)

$$p \sim Beta(y+1, n-y+1);$$

and the posterior mean probability thus is

$$\mathbb{E}[p] = \frac{y+1}{n+2};$$

and the posterior variance is

$$\mathrm{Var}[p]_{partial} = \frac{(y+1)(n-y+1)}{(n+2)^2(n+3)}.$$

Cases of non-continuous probabilities, such as drawing a ball from an urn, follow this principle albeit with discrete values (Wald, 1947). We can thus use the equation for computing the degrees of ambiguity associated with the prospects in our experimental stimulus.

In sum, full ambiguity means any probability of winning is as likely as any other. The distribution of second-order probabilities then condenses with sample information. For any sample composition, smaller sample sizes yield a larger posterior variance Var[p]; the degree of ambiguity is high. Larger sample sizes yield a smaller posterior variance Var[p]; the degree of ambiguity is low (details in Appendix B). The greater the sample size, the more certain decision makers can be about the sample composition fully specifying the urn content and, therefore, the probability of drawing a particular ball color.

2.2 Ambiguity aversion

Ambiguity aversion indicates how much subjects dislike ambiguous bets x_E0 and $x_{E^c}0$ associated with drawing a ball from a two-color Ellsberg urn. Following established practice (Baillon et al., 2018), we construct a measure of *Aversion* by interpreting the matching probability *mp*—the point at which subjects reveal to be indifferent between an ambiguous bet x_E0 and a risky bet $x_{mp}0$. In our paper, $mp(E)$ and $mp(E^c)$ are the matching probabilities of betting, respectively, on red and on black when drawing a ball from the same urn. Matching probabilities fall within the closed interval [0, 1].

Elicited matching probabilities can reflect beliefs as well as attitudes. Beliefs about the likelihoods of event E and its complementary event E^c should sum to 1, however, which can be used to create a measure of *Aversion* that drops beliefs and more exclusively identifies ambiguity aversion (Baillon et al., 2018):

$$Aversion = 1 - mp(E) - mp(E^c).$$

Beyond aversion, ambiguity attitudes may additionally encompass ambiguity-generated likelihood insensitivity (e.g., Baillon et al., 2018; Dimmock et al., 2016b; Fox & Tversky, 1998; Li, 2017), or a-sensitivity. Insensitivity describes a common tendency to treat mid-range likelihoods as interchangeable. A-sensitivity is out of scope for our theoretical objective[1] and it cannot influence the aversion measure (Baillon et al., 2018). Incidentally, disregarding a-sensitivity means that the Baillon et al. (2018) method for capturing ambiguity aversion requires a state-space partition of two events only. This achieves an intuitive correspondence of our stimulus with real-world decision makers' classification of outcomes as either success or failure.

2.3 Prediction

Variation in the degree of ambiguity likely influences how much people like or dislike ambiguity. Since most natural prospects are partially ambiguous, with decision makers having access to varying amounts of information (Hertwig & Pleskac, 2010), showing that ambiguity aversion is a function of prospects' degree of ambiguity stands to foster understanding of decision-making behavior.

If some people avert the full ambiguity of one prospect (Ellsberg, 1961; Li et al., 2017; Trautmann & Van De Kuilen, 2015), on account of their lack of familiarity with that source of uncertainty, we might expect them to avert less the partial ambiguity of another prospect with whose source they are more familiar, as indicated by sample evidence. Prior empirical work could be seen to support such conjecture, even if not explicitly identifying a prospect's degree of ambiguity. For example, Chew et al. (2017) observe a relatively greater revealed preference for drawing a card from a deck with a smaller set of probabilities. Similarly, Ert and Trautmann (2014) report

[1] A recent study with a different premise (Anantanasuwong et al., 2019) suggests that fluctuations in a-sensitivity across sources may track variation in people's perception of ambiguity that typically goes unobserved. Our work examines aversion relative to a constant observable degree of ambiguity, alleviating the need to control for responses to unobserved variation.

a greater preference for ambiguous prospects when sampling narrows the possible distribution of probabilities.[2] Therefore, our paper examines whether the degree of ambiguity, as indicated by variance $Var[p]$, increases ambiguity aversion.

Hypothesis The degree of ambiguity amplifies ambiguity aversion.

The null would be indicated by a lack of systematic relationship between degrees of ambiguity and ambiguity aversion. If sample evidence did not increase familiarity with a source of uncertainty, it would not affect people's attitude towards it.

3 Method

3.1 Identification strategy

We require each subject to make a series of decisions about differently ambiguous urns so that we can examine whether or not the attitude measure *Aversion* is sensitive to the degree of ambiguity as indicated by variance $\mathrm{Var}[p]$. We then test for the statistical significance of the parameter β_1 in the following regression specification, clustering errors at the subject level:

$$Aversion = \beta_0 + \beta_1 \cdot \mathrm{Var}[p] + \epsilon. \tag{2}$$

Aiming for 100 subjects, with 11 ambiguous urns each, we registered to consider the hypothesis supported at a significance level of 5%. These parameters are in keeping with prior studies on decision making under uncertainty (e.g., Baillon et al., 2018; Dimmock et al., 2016b; Li et al., 2017).

3.2 Procedure

Our study offers binary bets with a winning payoff of €20 and nothing otherwise. All bets are associated with physical urns. Table 1 characterizes the choice structure. We ask subjects to make a series of binary choices between drawing a ball from a covered urn with ambiguous content and drawing a ball from an uncovered urn with stated contents as specified in the Table 1 (Holt & Laury, 2002 price-list format).[3] Figure 1 shows an example choice of urns.

All urns contain 100 balls. Eleven covered urns each contain either 0, 10, 20, 30, 40, 50, 60, 70, 80, 90, or 100 red balls, rest black. We elicit matching probabilities

[2] Earlier work such as that of Chew et al. (2017) and Ert and Trautmann (2014) did not separate beliefs from attitudes. Our study provides such separation in addition to the direct identification of degrees of ambiguity.

[3] We also elicited aversion to urns with non-ambiguous content but decided against using the data for the present analysis. This is because the preference-elicitation mechanism (certainty-equivalents for risky choices) differs from our main mechanism and complicates comparisons. Data are available upon request.

Table 1 Choice Structure

Covered Urn	Uncovered Alternatives
Proportion of red and black balls unknown (one of the eleven possibilities listed in the right-hand column)	100 red
	90 red + 10 black
	80 red + 20 black
	70 red + 30 black
	60 red + 40 black
	50 red + 50 black
	40 red + 60 black
with or without sample information	30 red + 70 black
	20 red + 80 black
	10 red + 90 black
	100 black

by matching a covered urn with uncovered urns that span its possible contents (Table 1). For example, when betting on drawing a red ball, a subject might prefer uncovered urns with 80 red balls or more, and she might prefer the covered urn if uncovered urns contain 70 red balls or fewer. Her matching probability of betting on drawing a red ball from the covered urn then is the midpoint of red-ball contents in the marginal uncovered urns: 75%.[4] Subjects bet on both colors: first red, then black. Using complementary events helps us separate beliefs from attitudes (Baillon et al., 2018) and guards against informed priors (Hey et al., 2010).

Only one subject was present at a time. The sequence of subjects was randomized through a computerized wheel of names. The procedure was that subjects:

1. read the task instructions (details in Appendix C Fig. 7);
2. draw three envelopes, randomly determining *ex ante* the decision relevant for payment;
3. make choices, betting first on red and then on black;
4. open the three envelopes;
5. draw a ball from the payment urn;
6. inspect ball color and receive payment if any.

Research assistants verbally solicited and recorded decisions on paper. They also provided answers to comprehension questions if asked. Research assistants were blind to the hypothesis.

[4] Following established practice (e.g., Baillon et al., 2018), research assistants imposed consistency by eliciting matching probabilities from one side only, beginning with the uncovered urn containing no balls of the bet color. We imputed the remainder.

Fig. 1 Example Choice Between a Covered Urn with Sample Information and an Uncovered Urn with Stated Contents

3.3 Ambiguity-generating mechanism

We generate ambiguity for each covered urn and subject prior to her entering the room. The process is random and there is no manipulation. Research assistants determine the content of subjects' covered urns by throwing a fair 12-sided die for each. The number shown on the top face of the die determines the color composition of the urn. If the die shows 1, the urn is filled with 10 red and 90 black balls. If the die shows 2, the urn is filled with 20 red and 80 black balls. And so on. If the die shows 11, the urn is filled with 0 red and 100 black balls. If the die shows 12, research assistants throw the die again. Verifiable randomization by die preserves uninformed priors and minimizes distrust in experimenters (Fox & Tversky, 1995).

One covered Urn *l* is a fully ambiguous, classic two-color Ellsberg urn. Subjects receive no information about this urn beyond that it has one of the 11 possible contents specified in the instructions. Ten covered urns are partially ambiguous. For these, subjects additionally receive information about a sample of balls drawn from each urn. Sample balls were independently drawn and replaced prior to subjects entering the room. The number of draws was randomly determined for each urn, throwing the aforementioned 12-sided die. If the die shows 1, the sample size is 1. If the die shows 2, the sample size is 2. And so on. Twelve is the maximum possible sample size. All ambiguity-generating mechanisms are clearly stated in the instructions (details in Appendix C Fig. 7).

3.4 Participants and incentivization

Ninety-eight participants from the student pool of a European business school, undergraduate as well as postgraduate, spent an average of 23 minutes on the task, ranging from a 14-minute minimum to a 38-minute maximum. Subjects received compensation through a randomized prior incentive system (Johnson et al., 2021). At the beginning of the experiment, subjects draw one envelope each from three piles. The envelope from the first pile determines the bet color relevant for payment. The envelope from the second pile determines the covered urn relevant for payment. The envelope from the third pile determines the uncovered alternative relevant for payment. Together,

Table 2 Construct Correlates

	Age	Female	Graduate
Degree of Ambiguity	0.007	0.055	0.027
	(.828)	(.073)	(.378)
Ambiguity Aversion	0.048	-0.054	0.083
	(.114)	(.079)	(.006)

The table contains pairwise Pearson correlation coefficients (*p* values) for 1,078 observations (98 subjects each decide on 11 covered urns)

the three envelopes identify the payment choice set. Subjects open the envelopes after having made all their choices and then draw a ball from the urn they chose in the randomly selected payment choice set. Subjects receive €20 if the drawn ball is of the bet color, nothing otherwise.[5] The experimental instructions clearly communicate this procedure.

4 Results

4.1 Randomization and representativeness

Degrees of ambiguity in our experiment vary exogenously at the urn level, through the randomization process described in Section 3.3. Subjects of different gender, age, and educational attainment experience similar variation of this nature (see Table 2). Orthogonality to personal characteristics is thus given, facilitating analyses of the effect of degrees of ambiguity.

Thirty-seven subjects identify as female; 61 subjects identify as male. Subjects are between 19 years and 41 years of age. The 30 subjects studying for a graduate degree appear to avert ambiguity more than the 68 subjects studying for undergraduate degrees. The composition of our subjects and their behavior are in line with prior studies on decision making under uncertainty (e.g., Epstein & Halevy, 2020; Halevy, 2007; Kutzner et al., 2017).

4.2 Main analysis

Table 3 contains descriptive statistics. Figure 2 shows the distribution of the raw data in binned scatter plots. Subjects on average choose matching probabilities that exceed low posterior probabilities of drawing a ball of the winning color. This tendency is more pronounced when the degree of ambiguity is higher. Figure 2 shows the best fit for a linear relationship between posterior mean and matching probability

[5] Note that the experiment instructions do not state an average payoff subjects could expect to receive. Calculating such expectations *ex-ante* would involve assumptions about the mean degree of ambiguity subjects could expect to face as well as their mean attitude to that degree. In the end, subjects took home €13.7 on average.

Table 3 Summary Statistics

	Mean	Standard Deviation	Min	Max	N
Choices: Without Samples					
Degree of Ambiguity[a]	0.10	0	0.10	0.10	98
Ambiguity Aversion	0.16	0.26	–0.50	0.70	98
Choices: With Samples					
Degree of Ambiguity	0.02	0.02	0.004	0.06	980
Ambiguity Aversion	0.09	0.19	–0.95	0.90	980
Subjects: With Samples					
Choice Consistency[b]	0.12	0.07	0	0.35	98
Ambiguity-Aversion Sensitivity[c]	3.16	4.47	–14.09	19.73	98
Ambiguity-Aversion Threshold[d]	0.02	0.01	0	0.056	40

[a]Notes: For a covered urn without samples (i.e., under full ambiguity), the variance of the discrete uniform distribution in our experiment $\frac{0}{100}, \frac{10}{100}, \ldots, \frac{90}{100}, \frac{100}{100}$ is $\mathrm{Var}[p]_{full} = \sum_{i=0}^{10} \frac{1}{11} \cdot (\frac{i}{10} - \frac{1}{11} \sum_{i=0}^{10} \frac{i}{10})^2 = 0.1$

[b]Choice Consistency is indicated by the root-mean square error for an estimation across 10 partially ambiguous urns that follows Eq. (2) at subject level

[c]Ambiguity-Aversion Sensitivity is indicated by the coefficient for posterior variance in the same estimation

[d]Ambiguity-Aversion Threshold is the degree of ambiguity at which the *Aversion* measure is zero. That is, decision makers dislike degrees of ambiguity above the threshold, and like them below the threshold

that is moderated by the degree of ambiguity. The lines in the graph exemplify the relationship under low- and high-ambiguity—the midpoints of the lower and upper tertiles of the posterior variance measure, respectively.

In a regression with 1,960 elicited matching probabilities (one for each ball color, sampled urn, and subject) and errors clustered at the subject level, the degree of ambiguity elevates the extent to which subjects' matching probabilities depart from Bayesian posteriors (p = .001). This is consistent with our hypothesis, albeit not yet a test. Replicating the exercise with sample proportions and sample size, instead of

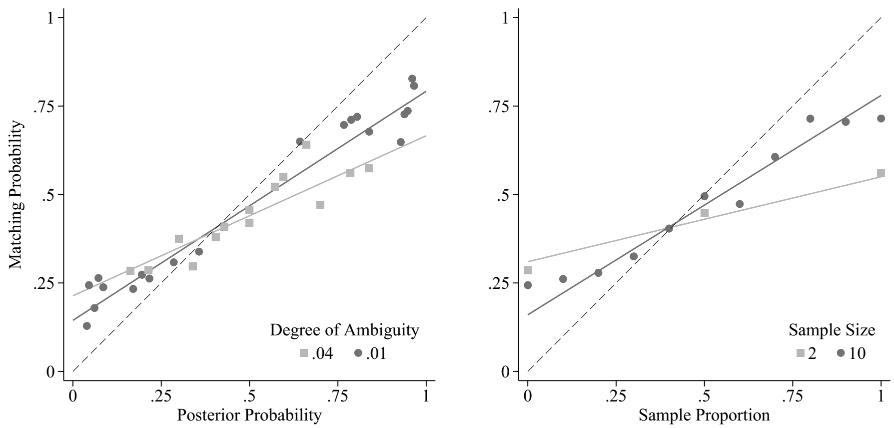


Fig. 2 Revealed Preferences Moderated by Partial Ambiguity

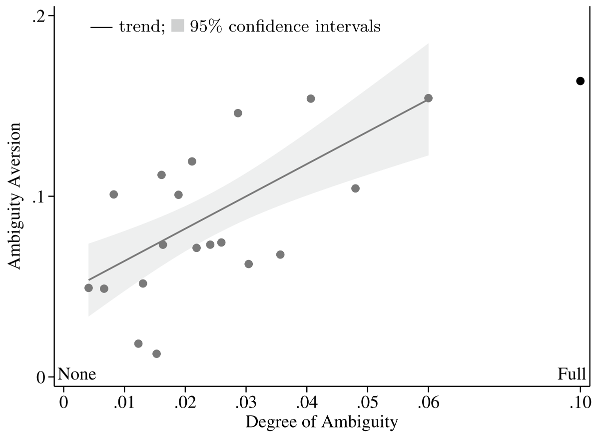


Fig. 3 Revealed Aversion to Degrees of Ambiguity

posterior probability and posterior variance, offers much the same insight: the difference between sample proportions and matching probabilities is sensitive to the information provided by sample size (Fig. 2).

Since matching probabilities can reflect attitudes as well as beliefs, we next inspect the *Aversion* measure. The binned raw data in Fig. 3 shows that subjects on average dislike ambiguity, and that such disliking attitude is greater when the degree of ambiguity is higher. Such sensitivity seems systematic as the 95% bounds in Fig. 3 indicate. Testing our hypothesis with a fixed effect regression (Model (5) in Table 4) shows the relationship between degree of ambiguity and ambiguity aversion being significant at the $p < .001$ level, supporting our hypothesis.

Figure 3 additionally charts the observed aversion to full ambiguity (average *Aversion* to the covered Urn l without samples). This data point would constitute the Ellsberg end of the aversion pattern shown for partially ambiguous urns. Aversion to full ambiguity indeed appears to be greater than what we observe for partial ambiguity.

4.3 Predictive validity

The systematic relationship between the degree of ambiguity and ambiguity aversion can be used for out-of-source prediction. We conduct cross-fold validation as follows. To predict a subject's ambiguity aversion to a covered Urn k with samples ($k \in \{1, ..., 10\}$), we first regress her ambiguity aversion on the degree of ambiguity of the other nine covered urns j with samples ($j = 1, ..., 10; j \neq k$). That is,

$$Aversion_j = a_k + b_k \cdot \mathrm{Var}[p]_j + \epsilon_j, \quad j = 1, ..., 10; j \neq k. \tag{3}$$

Then we use the estimated parameters a_k and b_k to predict ambiguity aversion to the partial ambiguity of the hold-out Urn k:

Table 4 Estimation Results

	Matching Probability			Ambiguity Aversion	
	(1)	(2)	(3)	(4)	(5)
Posterior Probability	0.60^{***}	0.71^{***}	0.71^{***}		
	(0.04)	(0.05)	(0.02)		
Degree of Ambiguity		2.2160^{*}	1.83^{**}	1.79^{**}	2.54^{***}
		(0.86)	(0.54)	(0.51)	(0.31)
Interaction		-6.20^{**}	-6.20^{***}		
		(1.76)	(0.95)		
Errors Clustered	Subject	Subject	no	Subject	no
Fixed Effects	no	no	Subject	no	Subject
Constant	0.15^{***}	0.12^{***}	0.13^{***}	0.05^{**}	0.03^{**}
	(0.02)	(0.03)	(0.01)	(0.02)	(0.01)
Observations	1,960	1,960	1,960	980	980
R^2	0.48	0.49	0.49	0.02	0.02

Notes: The table lists regression coefficients (standard errors). The interaction term is the product of posterior probability and degree of ambiguity

$^{*}p < 0.05$; $^{**}p < 0.01$; $^{***}p < 0.001$

$$\widehat{Aversion}_k = a_k + b_k \cdot \mathrm{Var}[p]_k.$$

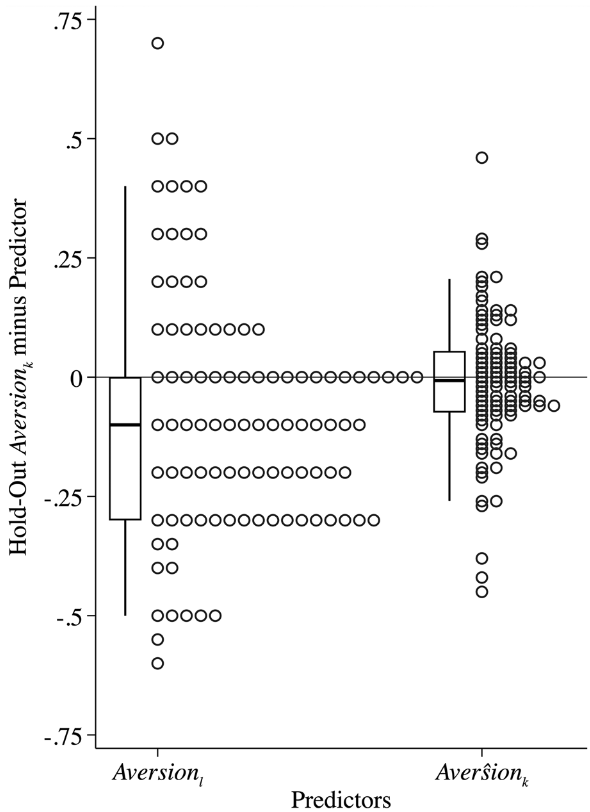
We compare the prediction error, i.e., the difference between observed $Aversion_k$ and predicted $\widehat{Aversion}_k$, to the error in predictions of aversion to the hold-out urn that are based on the covered Urn l without samples, i.e., the difference between observed $Aversion_k$ and observed $Aversion_l$.

The root mean squared errors for the $\widehat{Aversion}_k$ predictor range from 0.121 to 0.203, whereas those of the $Aversion_l$ predictor range from 0.255 to 0.292. The errors in the latter are larger for every hold-out urn. For illustration, Fig. 4 displays one example ($k = 10$) showing the accuracy of predicting subjects' $Aversion_k$ to the final urn. Subject's $Aversion_l$ to the full ambiguity of the covered urn without samples predicts the final-urn $Aversion_k$ poorly, with greater dispersion of predicted values around the actual value, and a systematic overestimation bias, given that full ambiguity exceeds partial ambiguity. Knowing subjects' aversion to a degree of ambiguity helps reduce dispersion and bias. It appears to account for much of the variation that would previously have been attributed to source preference.

4.4 Subject-level variation

We also inspect variation in individual behavior across the range of ambiguity by fitting a regression line for each subject. Fit between subjects' ambiguity aversion and the degree of ambiguity minimizes estimation error when a constant is added (see Appendix D Table 5 for detail on the goodness of fit for various model specifications), suggesting that aversion may not typically be zero when ambiguity is

Fig. 4 Prediction Quality



near zero. A polynomial specification does not meaningfully improve fit and we, therefore, describe subjects' aversion with a linear specification akin to that in Eq. (3), using the observations for all ten partially ambiguous urns.

The ambiguity aversion of 23 subjects slopes downward over the range of ambiguity. It stays flat for two subjects. Most of the subjects, however, dislike greater ambiguity: ambiguity aversion increases with degrees of ambiguity for 73 subjects. And it is not just variation in such sensitivity that is interesting. Our subject-level analysis reveals a sign change in ambiguity aversion for 49 subjects. Forty of those subjects like ambiguity when its degree is near zero but dislike it as it becomes larger. Figure 5 illustrates this with varying limits on choice consistency (model fit).

The average threshold, if any, at which subjects switch from liking to disliking ambiguity is at a posterior variance of 0.018. Some subjects, therefore, dislike ambiguity when given a sample of 2 red balls and 6 black balls ($\text{Var}[p] = 0.019$) but like ambiguity when given a sample of 2 red balls and 7 black balls ($\text{Var}[p] = 0.017$), for example. The minimum threshold is at 0.00008, and the maximum at 0.056. Whether or not such subjects like or dislike an ambiguous prospect is thus a function of the degree of ambiguity.

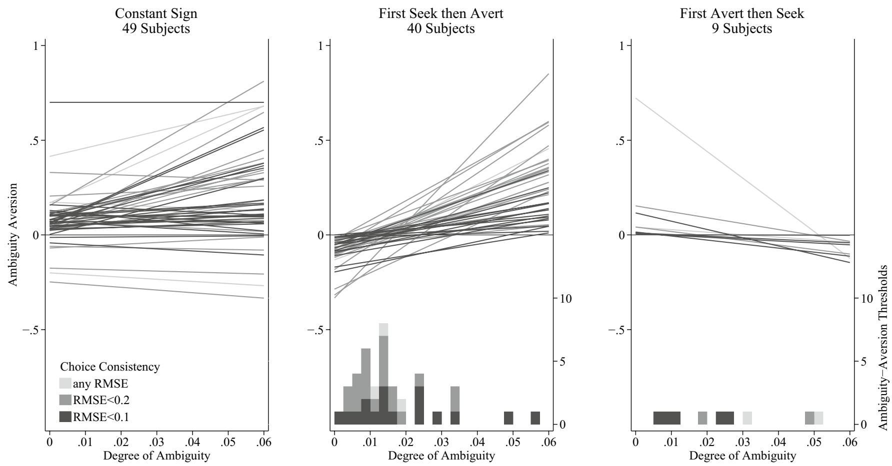


Fig. 5 Subject-Level Variation in Aversion Behavior

5 Discussion

Our work extends the literature on decision making under uncertainty by detailing the systematic nature of attitudes towards ambiguity. Recent advances in measuring ambiguity allow us to show how ambiguity aversion varies with the degree of ambiguity. Source—or urn, in our context—preference turns out to contain two principal components: an ambiguity threshold for aversion and an aversion sensitivity to the degree of ambiguity. Accounting for these components stands to help validate ambiguity attitudes (Trautmann & Van De Kuilen, 2015) and enables the incorporation of ambiguity attitudes into predictions of decision behavior.

5.1 Ambiguity-aversion sensitivity

The systematic relationship between ambiguity aversion and the degree of that ambiguity provides an explanation for the seeming puzzle that the same person might bet on the Dow Jones but not the Nikkei (Trautmann & Van De Kuilen, 2015), even if payoff probabilities are similar. If the person faced lower degrees of ambiguity for the Dow, on account of having access to more information than for the Nikkei, all else equal, then liking the Dow and disliking the Nikkei (or liking it less) is an understandable position. The degree of ambiguity of a prospect thus offers a causal explanation for variation in decisions under uncertainty.

Prior work (Hey et al., 2010; Kothiyal et al., 2014) had described such variation as source preference and inferred its presence from aggregate group observations. Prediction was limited to within sources. "The dependence of preferences on sources of uncertainty, with domestic stocks treated differently than foreign stocks, for instance, is an empirical fact that every ambiguity theory must accommodate" (Dimmock et al., 2016b, p.1366). Our approach of indexing aversion to the degree of ambiguity goes beyond accommodation: it identifies a structural component of source preference at the individual level. People's reactions to any ambiguous prospect might be governed by more than just its degree of ambiguity—yet this seems an important facet of a prospect's attractiveness, having explanatory power in our setting and outweighing the impact of a prospect's outcome space, for example, in other settings (Li et al., 2017).

Consistent with our newly tractable explanation, prior work in a financial-market context found ambiguity aversion to remain relatively stable when adjusting for perceived levels of ambiguity (Anantanasuwong et al., 2019). Rather than having to observe decision behavior to infer such perceptions, we show that deriving the degree of ambiguity from properties of the source alone is possible, facilitating prediction. We also complement work by Baillon et al. (2017) who observe that subjects avert financial options less when the movements of an underlying stock could be observed for longer. Our method of assessing the degree of ambiguity through the sample evidence available to decision makers offers such studies a means to predict aversion before observing it.

Sensitivity to degrees of ambiguity may also be present in the study of Chew et al. (2017), who find a higher preference for a less ambiguous prospect (50 possible probabilities) than for a more ambiguous one (100 possible probabilities), for example. Viscusi and Chesson (1999) observe an analogous result when two possible outcome probabilities of a hypothetical ambiguous prospect are closer to vs further from another. Relatedly, Gigliotti and Sopher (1996) see a higher preference for a larger than a smaller hypothetical urn, when a tenth of the urn content is provided as hypothetical samples. Our study extends these prior efforts by separating out beliefs, eliciting ambiguity preferences through complete choice lists (Dimmock et al., 2016b), covering a more naturally varying range of probabilities, and going to some length towards ensuring flat priors. The reliability of our methodology provides some confirmation of these earlier findings.

Our study also supports prior psychological enquiry. Einhorn and Hogarth (1985) manipulate ambiguous prospects in decision problems by varying state spaces and signal noise. When subjects report their impressions of ambiguity as being low, they display a greater preference for ambiguous prospects. While the logic is consistent with our study, Einhorn and Hogarth (1985) offer limited predictive validity, since ambiguity preference can be assessed only once subjective beliefs are disclosed. Nonetheless, the authors' measures of signal noise and state space increases could loosely translate to our degree of ambiguity measure. Similarly, Heath and Tversky (1991) have subjects rate their own knowledgeableness. Subjects prefer ambiguous prospects more when their knowledgeableness is high. Heath and Tversky (1991) refer to knowledgeableness as competence, which maps onto experience and familiarity—an expression of the amount of information available to the decision maker.

5.2 Ambiguity-aversion thresholds

The systematic sensitivity to degrees of ambiguity may not just attenuate the magnitude of ambiguity-aversion attitudes. It may involve a change of sign, too. A large number of our subjects' choice behavior is best described by a positive attitude towards ambiguity when its degree is low and a negative attitude towards ambiguity when its degree exceeds a threshold. This is consistent with an earlier observation in which subjects *hoped* to be better off with a hypothetical ambiguous prospect with a narrow range of outcome probabilities, and *feared* a prospect with a mean-preserving wider range Viscusi and Chesson (1999), (emphasis is on original terminology). Such ambiguity-aversion thresholds vary and may offer a useful classification of people when predicting their choices under uncertainty.

The thresholds may be understood as confidence preferences. Confidence is the inverse of the degree of ambiguity and people may require different degrees of ambiguity before liking ambiguous prospects. Variation in confidence preference is consistent with observations of (hypothetical) physicians' readiness to act on diagnoses, for example Jackson et al. (2017). Such confidence thresholds have been considered in analyses of when people stop sampling (Hausmann-Thürig & Läge, 2008; Navarro-Martinez et al., 2018; Ostwald et al., 2015). Our work extends these insights to the more common situations in which people cannot control sampling and instead work with an exogenously determined sample (Klingebiel & Zhu, 2022).

Such confidence preferences likely help explain decision making under uncertainty. The decision to opt into entrepreneurship is an example of a decision where experience is given and not extendable in the short term. In the case of entrepreneurial choices, confidence preferences may add substantially to what can be explained by risk preferences, which do not describe well the proclivity to engage in endeavors with unspecifiable probabilities of success (e.g., Chen et al., 2018; Gutierrez et al., 2020). Yet, such probabilities can be bounded and are not totally unknown. Ambiguity-aversion thresholds may go some way towards explaining decisions taken on the basis of such boundable probabilities, such as those in entrepreneurial entry.

Business decisions more generally are neither purely risky nor fully ambiguous, even if this is a popular bifurcation invoked in the literature (Miller, 2007). Instead, managers make decisions on the basis of limited samples, implying simultaneous consideration of probability and ambiguity. For example, firms might have access to early indications of an innovation's success potential before they invest irrevocably (Klingebiel & Esser, 2020), observe adoption signals before choosing one of two technology options (Eggers, 2014), monitor profitability updates during development (Klingebiel, 2018), or let posterior probabilities guide product launch and exit (Hitsch, 2006). Ambiguity thresholds could explain observed decision behavior in these domains. Future research may thus usefully examine to which extent confidence preferences can help predict choices in applied contexts.

6 Conclusion

We show that people's aversion of ambiguity systematically depends on how much ambiguity there is to avert. Studying people's choices allows for the approximation of a trait—people's aversion to a given degree of ambiguity. Such information can be used to improve prediction of future choices across sources. Policy makers and decision analysts thus stand to benefit by being able to more reliably gauge people's response to uncertainty. This includes accommodation for the observation that not all ambiguity is disliked. Many people like low degrees of ambiguity. The threshold at which someone switches from liking to disliking ambiguity describes a confidence preference, explaining why she perhaps invests in the Dow but not the Nikkei, even if she expects both indices to move similarly. Our method allows to identify such switching points. Since the degree of ambiguity for most natural prospects is neither zero nor absolute, including entrepreneurship or product innovation, for example, knowledge of the dynamics of partial-ambiguity aversion fills a gap between the existing notions of risk (zero ambiguity) and (full) ambiguity aversion.

Appendix A. Degree of ambiguity and posterior variance

In the Izhakian (2017, 2020) expected utility with uncertain probabilities model, there are two levels of uncertainty. The realized outcome is uncertain, and the probabilities of the outcomes are uncertain. A first-order probability measure P responds to the uncertain outcome and is additive. In our setting, the composition of the urns determines the first order probability measures P. The set of all additive first order probability measures is $\mathcal{P}$. The second-order probability measure ξ responds to the uncertain probabilities of the possible outcomes. In Eq. (1), the expectation $\mathbb{E}[\cdot]$ and the variance $\mathrm{Var}[\cdot]$ are based on the second-order probability measure ξ on the set $\mathcal{P}$ of the first-order probability measures. $\varphi_f(\cdot)$ is the uncertain probability mass function. In an infinite state space,

$$\mho^2(f) = \int_X \mathbb{E}[\varphi_f(x)]\mathrm{Var}[\varphi_f(x)]\, dx$$

where X is the set of all possible outcomes and $\varphi_f(\cdot)$ is the uncertain probability density function. This ambiguity measure $\mho^2$ is the expected volatility of probabilities across the relevant events. It is independent of the magnitude of the outcomes and of the attitudes towards risk and ambiguity. It can be directly computed from the data. In our setting, let the bet f offer €20 if drawing a red ball from a two-color Ellsberg urn and nothing otherwise (20_E0). After n sample balls drawn from this urn with replacement, the sample contains y red balls and $n-y$ black balls. Let p_r (p_b) be the probability of drawing a red (black) ball from the ambiguous urn. Based on Bayes' theorem, the first-order probability measures are beta distributed: $p_r \sim Beta(y+1, n-y+1)$ and $p_b \sim Beta(n-y+1, y+1)$ (e.g., Bernardo & Smith, 1994). Accordingly,

$$\begin{aligned}
\mathbb{E}[p_r] &= \frac{y+1}{n+2}, \\
\mathbb{E}[p_b] &= \frac{n-y+1}{n+2}, \quad \text{and} \\
\mathrm{Var}[p_r] = \mathrm{Var}[p_b] &= \frac{(y+1)(n-y+1)}{(n+2)^2(n+3)}.
\end{aligned}$$

Let $\mathrm{Var}[p]$ be the posterior variance of p_r and p_b. The ambiguity measure $\mho^2$ in our setting is then computed as

$$\begin{aligned}\mho^2(f) &= \mathbb{E}[\varphi_f(20)]\mathrm{Var}[\varphi_f(20)] + \mathbb{E}[\varphi_f(0)]\mathrm{Var}[\varphi_f(0)] \\ &= \mathbb{E}[p_r]\mathrm{Var}[p_r] + \mathbb{E}[p_b]\mathrm{Var}[p_b] \\ &= \mathbb{E}[p_r]\mathrm{Var}[p] + \mathbb{E}[p_b]\mathrm{Var}[p] \\ &= (\mathbb{E}[p_r] + \mathbb{E}[p_b])\mathrm{Var}[p] = \mathrm{Var}[p].\end{aligned}$$

Appendix B. Sample size and posterior variance

Based on beta distribution, the variance of the posterior distribution $\mathrm{Var}[p]$ is computed as

$$\mathrm{Var}[p] = \frac{(y+1)(n-y+1)}{(n+2)^2(n+3)}, \tag{4}$$

where p is the underlying winning probability, n is the sample size, and y is the occurrence of the winning event in the sample. Note that $n \geq 1$ and $n \geq y \geq 0$.

The posterior mean $\mathbb{E}[p]$ is computed as $\frac{y+1}{n+2}$. Thus, $y = \mathbb{E}[p](n+2) - 1$. Plugging into $\mathrm{Var}[p]$ results in

$$\mathrm{Var}[p] = \frac{\mathbb{E}[p](1-\mathbb{E}[p])}{n+3}. \tag{5}$$

If holding the posterior mean $\mathbb{E}[p]$ constant, based on Eq. (5), the posterior variance $\mathrm{Var}[p]$ increases (decreases) when the sample size n decrease (increases). Given any posterior mean $\mathbb{E}[p]$, the posterior variance $\mathrm{Var}[p]$ is the largest when the sample size $n = 1$. Plugging in $n = 1$, one can derive the maximum posterior variance for all ambiguous prospects with sample information when the underlying winning probability p is continuous: $\max \mathrm{Var}[p] = 0.055$. In our experiment, the winning probability p is discrete, i.e., $p \in \{0, \frac{10}{100}, \frac{20}{100}, ..., 1\}$. Hence, the maximum posterior variance is 0.06 in our experiment.

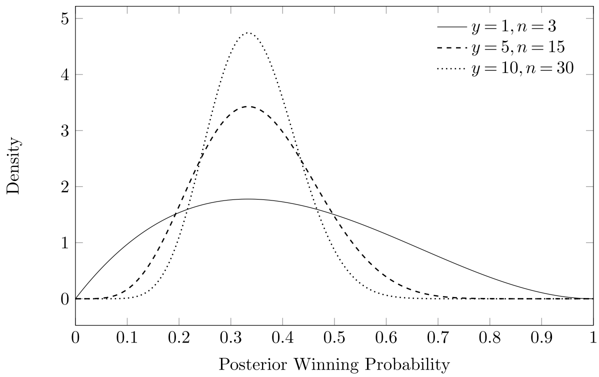


Fig. 6 Posterior Distribution with Different Sample Sizes

If holding the sample proportion $\hat{p} = \frac{y}{n}$ constant, based on Eq. (4),

$$\mathrm{Var}[p] = \frac{(\hat{p}n + 1)(n - \hat{p}n + 1)}{(n+2)^2(n+3)}.$$

Differentiating Var[p] with respect to n yields

$$\frac{\partial \mathrm{Var}[p]}{\partial n} = \frac{-\hat{p}(1-\hat{p})n^3 - (8\hat{p}(1-\hat{p}) + 2)n^2 - (6 - 22\hat{p}(1-\hat{p}))n - 2}{(n+2)^3(n+3)^2}.$$

Since the sample proportion $0 \le \hat{p} \le 1$, one can derive $0 \le \hat{p}(1-\hat{p}) \le 0.25$. Accordingly,

$$-\hat{p}(1-\hat{p})n^3 \le 0,$$
$$-(8\hat{p}(1-\hat{p}) + 2)n^2 < 0, \quad \text{and}$$
$$-(6 - 22\hat{p}(1-\hat{p}))n < 0.$$

With $(n+2)^3(n+3)^2 > 0$, one can see that $\frac{\partial \mathrm{Var}[p]}{\partial n} < 0$, indicating the posterior variance Var[p] increases (decreases) when the sample size n decreases (increases), holding the sample proportion constant. Figure 6 demonstrates the posterior distribution with different sample sizes.

Appendix C. Experimental instructions

Welcome to Our Decision-Making Study

You will be asked to make 14 choices. You can win money (€20 max). The money you win depends on your choices. These choices involve spinning cages, some covered, some uncovered. Try for yourself to draw a ball from the cage in front of you. You will see that it is a random process. All cages contain exactly 100 balls that are either red or black.

You decide whether you want to bet for money that the next ball drawn from a cage has the correct colour. If the bet is on red and the ball is red, you win €20. If the bet is on red and the ball is black, you win nothing. Vice versa for black.

You always choose between two options. For illustration, an example choice could look like this:

Playing red, which cage do you want to bet on?

The first 3 choices involve uncovered cages. You can visually inspect the cages and its colour composition is clearly stated for you. The alternatives are fixed amounts of Euros (see below for illustration). The experimenter will ask you whether you want to play the cages or get a fixed amount of money.

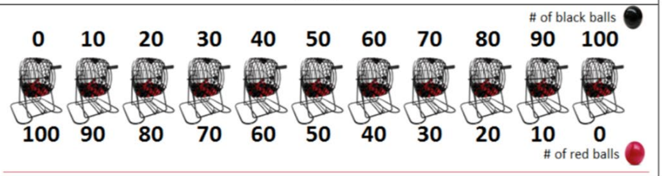
Choice Set #	1-3
Focal Cage	Uncovered
Alternatives	€0 €2 €4 €6 €8 €10 €12 €14 €16 €18 €20

(a) Page 1

Fig. 7 Instructions

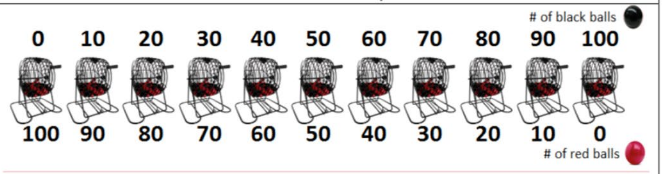
The 4th choice involves a covered cage and its composition unknown to you. It contains 100 balls of which 0, 10, 20, 30, 40, 50, 60, 70, 80, 90, or 100 balls are red (rest black). For each participant, the experimenter randomly determines the colour composition by the roll of a fair die*.

The alternatives are uncovered cages with stated colour composition (see below for illustration). The experimenter will ask you which cage you want to bet on, covered cage or uncovered cage.

Choice Set #	4
Focal Cage	Covered
Alternatives	

The remaining choices all involve covered cages with sample information. The sample balls, randomly drawn with replacement, help you better understand the composition of the covered cage.

The alternatives again are uncovered cages with stated colour composition. The experimenter will again ask you which cage you want to bet on, covered cage or uncovered cage.

Choice Set #	5-14
Focal Cage	Covered with samples
Alternatives	

One of your choices is randomly selected for payment. This is done before you begin making decisions and will be revealed after you are finished*. You need to treat each of your decisions as carefully as if it were your only one.

Now it is time to wait for your turn. An experimenter will invite you into the room when it is time to begin. Until then, feel free to explore the example die and cage on the table.

(b) Page 2

Fig. 7 (continued)

What is inside the covered cages?

Covered cages contain 100 balls, of which 0, 10, 20, 30, 40, 50, 60, 70, 80, 90, or 100 are red, with black balls making up the remainder. For each participant, the experimenter randomly determines a new composition by rolling a fair 12-sided die, such as the one on your table.

If the die lands on 1, the cage is to contain 10 red balls and 90 black balls. If the die lands on 2, the cage is to contain 20 red balls and 80 black balls. If the die lands on 3, the cage is to contain 30 red balls and 70 black balls …and so on. If the die lands on 11, the cage is to contain 0 red balls and 100 black balls. If the die lands on 12, the roll is repeated.

How are samples drawn from the covered cages?

For each participant and cage, the experimenter randomly determines the sample size by a roll of the same fair 12-sided die. A value of 1 means 1 ball, 2 means 2 balls, and so on. Sample balls are individually drawn from the cage and then replaced. For example, if the die shows a value of 3, the experimenter will repeat the following 3 times: randomly draw a ball, record its colour, and put the ball back into the cage.

Which choice is relevant for payment?

To randomly determine the choice for payment, we set up 3 boxes of envelopes from which you draw one each at the very beginning. You can inspect the boxes and envelopes if you like. The envelope from the first box will determine the betting colour (red/black), the envelope from the second box the choice set (#1-14), and the envelope from the third box the alternative (#1-11).

For example, suppose the first envelope says red. And the second says choice set #10. And the third says #3. In that case, we will check what you chose to play for colour red in choice set 10 (a covered cage) for the alternative #3 (an uncovered cage with 80 red balls, 20 black balls). We will draw a ball from the cage you chose. If the ball is red, you win €20. If it is black, you win nothing.

In the special cases when the second envelope indicates one of the first three choice sets, and you chose a fixed-amount of money, we will directly pay you the stated € amount. No balls drawn in that special case.

We open the envelopes only after you made all your decisions. This means you need to treat each of your decisions as carefully as if it were your only one.

(c) Page 3

Fig. 7 (continued)

Appendix D. Subject-level analysis

Table 5 Goodness of Model Fit for Subjects' Aversion to Degrees of Ambiguity

		Root Mean Square Error			
Specification	#	Mean	SD	Min	Max
$Aversion = b\text{Var}[p] + \epsilon$	98	0.126	0.074	0.042	0.449
$Aversion = a + b\text{Var}[p] + \epsilon$	98	0.115	0.066	0	0.348
$Aversion = b\text{Var}[p] + c\text{Var}[p]^2 + \epsilon$	98	0.117	0.069	0.029	0.364
$Aversion = a + b\text{Var}[p] + c\text{Var}[p]^2 + \epsilon$	98	0.112	0.067	0	0.346

is the number of subjects for each of which we estimate the aversion to ten covered urns with samples. The root mean square error refers to this subject-level estimation. SD is the standard deviation of the error

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